объединенный
институт
ядерных
исследований
дубна

E17-88-941

S.S.Lapushkin

DYNAMICS OF PARTICLE-LIKE EXCITATIONS IN XY-MODEL WITH EXTERNAL ACTION

Submitted to "DAN SSSR"

1988

1. Let us consider the system described by the following Hamiltonian

$$H=-\Omega\sum_i S_i^x-J\sum_i S_i^z S_{i+1}^z-B\sum_i S_i^x S_{i+1}^x-\sum_i f_i S_i^z-\Delta\sum_i U_i S_i^x, \qquad (1)$$

where i is changeable particle interaction, Ω is the tunnelling parameter (tunnelling frequency), "B" is the tunnelling movement interaction in neighbouring cells, f_i is the interaction with external field, $\Delta=\partial_{U_i}\Omega(X_i^0+U_i)|_{U_i=0}$, U_i is the shift of the i-th particle from the equilibrium position X_i^0.

It is assumed that "X"-axis is directed along the arrangement of spin mass center, $S=\frac{1}{2}$.

Let us pass to the Fermi-representation by using the usual formulae /1/:

$$S_i^z=\frac{1}{2}-n_i,\quad S_i^x=\frac{1}{2}\prod_{k=1}^{i-1}(1-2n_k)(a_i^+ + a_i),\quad S_i^y=\frac{1}{2}\prod_{k=1}^{i-1}(1-2n_k)(a_i^+ - a_i);$$

$$a_f^+ a_g + a_f a_g^+ = \delta_{fg},\quad n_k=a_k^+ a_k,\quad (1-2n_k)\equiv e^{i\pi n_k} \qquad (2)$$

We note that just this choice of the spin operators S_i^x, S_i^y, S_i^z in terms of the Fermi operators a_i follows from hierarchy of Hamiltonian parameters (1): $J\gg\Omega\gg B$; (below all the calculations are valid for rigid lattice $\Delta=0$).

The external action $f_i(t)$ has an impulse character an the solution of the corresponding equation in "free regime" (as will be seen below) should join the solution at $f_i\neq 0$. But this will be the subject of a special research.

Substituting (2) into (1) we obtain after usual but bulky calculations the following operator equation for $a_p(t)$:

$$i\dot{a}_p=\left[a_p H\right]_-=\left(J+f_p\right)a_p-\frac{B}{4}\left[a_{p+1}+a_{p-1}\right]-\frac{B}{4}\left[a_{p+1}^+ - a_{p-1}^+\right]-J\left(n_{p+1}+n_{p-1}\right)a_p+$$
$$+\frac{\Omega}{2}\left[\Pi_p+2\sum_i\Theta(i-p)(a_i^+ + a_i)\Pi_{i-1}(1-a_p-2n_p)\right]+\frac{\Delta}{2}\left[U_p\Pi_p+2\sum_i\Theta(i-p)(a_i^+ + a_i)U_i\times\right.$$
$$\left.\times\Pi_{i-1}(1-a_p-2n_p)\right],\qquad \Pi_i=\prod_{k=1}^{i}e^{i\pi n_k}\equiv\prod_{k=1}^{i}(1-2n_k) \qquad (3)$$

2. Passing to the differential equation for the probability amplitudes $\varphi(x,t)$ in the continuum limit $\varphi_f(t)=\langle 0|a_f|\psi\rangle\to\varphi(x,t)$, $\sum_f\to\frac{1}{a_0}\int dx$, we use the "reduction procedure" /2/.

Using the leadings terms according to their unlinearity and dispersion we obtain

$$i\hbar\dot{\psi}(x,t)=\Big[J+f(x,t)\Big]\varphi(x,t)-\frac{B}{4}\Big[2\varphi(x,t)+a_0^2\varphi_{xx}\Big]-2J\Big[|\varphi|^2+\dots\Big]\varphi(x,t)+$$
$$+\frac{\Omega}{2}\Pi^{(1)}+\frac{\Delta}{2}\Pi^{(2)}-\frac{Ba_0}{4}\varphi^{*}_{x}\ ,\quad \Pi^{(i)}=\Pi^{(i)}\Big\{\varphi(x,t),U(x,t);\ \varphi(\xi,t)\Big\}\ . \qquad (4)$$

$\Pi^{(i)}$, $i=1,2$ is an integral analogue of two last terms in (3) will be written down below after some scaling.

3. For the following discussion of mathematical structure (4) we shall perform the scaling

$$t=a\tau;\ x=bz;\ \varphi=r\phi;\ h=f/(J-^{B}/_2)\ . \qquad (5)$$

With the use of the parameters according to (7) equation (4) becomes

$$i\dot{\Phi}_{\tau}=(1+h(z,\tau))\phi-\phi_{zz}-|\phi|^2\phi-\alpha\phi^{*}_{z}+\omega P_1+\delta P_2\ , \qquad (6)$$

where

$$a=\frac{\hbar}{J-\frac{B}{2}}\approx\frac{\hbar}{J}\ ,\quad b=\frac{a_0}{2}\left(\frac{B}{J-\frac{B}{2}}\right)^{\frac{1}{2}}\approx\frac{a_0}{2}\left(\frac{B}{J-\frac{B}{2}}\right)^{\frac{1}{2}},\quad r=\left(\frac{J-\frac{B}{2}}{2J}\right)^{\frac{1}{2}}\approx\frac{1}{\sqrt{2}}$$

$$d=\frac{1}{2}\left(\frac{J-\frac{B}{2}}{2J}\right)^{\frac{1}{2}}\approx\frac{1}{2}\left(\frac{B}{J}\right)^{\frac{1}{2}},\quad \omega=\frac{2\Omega\sqrt{J}}{(2J-B)}^{3/2}\approx\frac{\Omega}{J\sqrt{2}}\ ,\quad \delta=\frac{2\Delta\sqrt{J}}{(2J-B)}^{3/2}\approx\frac{\Delta}{J\sqrt{2}}\ ;$$

$$P_1=\Big\{1-\alpha\int_0^{(z-1)\alpha}|\Phi|^2d\xi+\alpha\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{(\xi-z-1)\alpha}e^{\varepsilon\alpha\xi'}\delta(\xi')d\xi'\Big[1-|\phi(\xi',t)|^2\Big]\frac{\phi^{*}(\xi',t)+\phi(\xi',t)}{\sqrt{2}}\times$$
$$\times\Big(1-\frac{\phi(z,t)}{\sqrt{2}}-|\phi(z,t)|^2\Big)\Big\} \qquad (7)$$
$$P_2=\Big\{\Big(U(z,t)-1\Big)\Big(1-\alpha\int_0^{(z-1)\alpha}|\phi(\xi,t)|^2d\xi\Big)+P_1\Big\}\ ,\qquad \varepsilon>0.$$

With the adiabatic approximation /2/ it is not difficult to derive the equation for the field shift $u(x,t)$ but as it has been mentioned above the main features are indicated for the rigid lattice as well ($u=0$).

3. From the hierarchy of the above-mentioned Hamiltonian parameters it is possible to conclude that

$$\alpha\ll1;\ \omega\ll\alpha;\ \Delta\ll\omega\ . \qquad (8)$$

As far as we are interested in particle-like solutions the last two terms in (6) may be omitted for the first approximation. certainly, the validity of this procedure may be checked by substituting the corresponding solutions and evaluating integrals. As a result we obtain

$$i\dot{\Phi}=(1+h(z,\tau))\phi-\phi_{zz}-|\phi|^2\phi-\alpha\phi^{*}_{z} \qquad (9)$$

For $h=0$ a similar equation was obtained in /3/. It was shown (vide supra /2/ and vide infra notes to this paper) that "small" perturbation $\propto\alpha\phi^{*}_{z}$ changes cardinally the spectrum of solutions (9) at $\alpha=0$. For the case $h=0$ in /4/ a partial solution was found (9). It is represented by the formula

$$i\Phi(z,\tau)=\lambda_0\operatorname{sech}\Big[\frac{\lambda_0}{\sqrt{2}}(z+\frac{\alpha}{2}\cos2\tau)\Big]\exp\Big\{-i\Big(1-\frac{\lambda_0^2}{2}\Big)\tau-\frac{i\alpha\lambda_0}{2\sqrt{2}}\operatorname{th}\Big(\frac{\lambda_0 z}{\sqrt{2}}\Big)\sin2\tau\Big\}\ . \qquad (10)$$

It is appropriate to note that this solution is valid for

$$\alpha\lambda\ll1,\quad \lambda^2\ll1,\quad \lambda=\sqrt{2(1-\omega)}\ , \qquad (11)$$

where ω is the parameter and it cannot be restored even in the frame of the perturbation theory generalized variant /5/.

4. The function $h(z,\tau)$ may be approximated in evolutional equation theory either by the step function or by narrow concentrated Gaussian distribution.

The solution (9), where $h(z,\tau)\neq0$ must be "joined" with (10) and may be used in physical applications.

It gives me pleasure to thank V.K.Fedyanin for discussing some details of the paper.

REFERENCES

/1/. Huang K. Statistical Mechanics, Wiley, New York, 1963.
/2/. V.G.Makhankov, V.K.Fedyanin, Phys.Rep., 1984, 104, N1, p.1-84.
/3/. V.G.Makhankov, V.K.Fedyanin, L.V.Yakushievitch, Phys.Let., 1977, 61A, N4, p.256-256.
/4/. V.G.Makhankov, V.K.Fedyanin, Phys.Lett., 1978, 68A, N2, p.169-171.
/5/. V.I.Karpman, E.M.Maslov, JETP, 1977, 73, N8, p.537.

Received by Publishing Department
on December 30, 1988.