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FLUORESCENCE FROM N THREE-LEVEL ATOMS IN AN IDEAL CAVITY

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1. INTRODUCTION

In the last years, a large number of publications is centered upon the basic models of quantum optics, the Jayne's - Cummings and Tavis - Cummings models [1-4]. A number of interesting effects **such as the enhancement** and suppression of spontaneous emission [5-8], vacuum-field Rabi oscillation [9-11], collapse and revival [12] have been observed experimentally. In recent publications the spectrum of the fluorescence photons from a single or two atoms in an ideal and in finite - Q cavities has been calculated [13-18]. The applications of these models as sources of the nonclassical light are widely discussed [19-23].

In our previous work [26] the problem of the spectrum and photon statistics for the case of N two-level atoms interacting with an intense cavity mode has been discussed. In the present paper we consider the problem of N three-level atoms in a lossless cavity (Tavis - Cummings model) in the case when the cavity modes are intense so that it can be treated classically. Essential differences in intensities of the spectrum components from collective double resonance in a free space [27] have been discussed. We have shown that the sidebands can play the role of intense light ($\sim N^2$) which have sub-Poissonian photon statistics and reduce quantum fluctuations in one of the quadrature (squeezing).

II. SPECTRUM OF FLUORESCENCE FIELD

We consider a small system of N three level atoms (see the fig.) interacting with two resonant cavity modes (Tavis - Cummings model). This model proves to be a good theoretical model for experimental works of Rydberg atoms in high-Q cavity. For simplicity the field frequencies ω_1 and ω_2 are assumed to be in exact resonance with the atomic frequencies ω_{21} and ω_{32} of lower $|2\rangle \to |1\rangle$ and upper $|3\rangle \to |2\rangle$ transitions, respectively. The Hamiltonian of the system in the rotating wave approximation and in the interaction representation has the following form for the case when the field modes are intense so that it can be treated classically [2, 26]

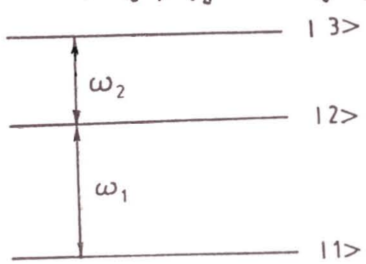
$$H = G_{21}\,(J_{12} + J_{21}) + G_{32}\,(J_{32} + J_{23}) \quad , \qquad (1)$$

where G_{21} and G_{32} are the resonant Rabi frequencies for the atomic transitions $|2\rangle \to |1\rangle$ and $|3\rangle \to |2\rangle$, respectively;

$$J_{ij} = \sum_{k=1}^{N} |i\rangle_k \langle j|_k \qquad (i,j = 1,2,3) \qquad (2)$$

are the collective angular operators for the atomic system. They satisfy the commutation relation

$$[\, J_{ij}\, ,\, J_{i'j'} \,] = J_{ij'}\,\delta_{ji'} - J_{i'j}\,\delta_{ij} \quad . \qquad (3)$$



Level scheme of the atomic system.

Following the previous works [26-28] , we introduce the Schwinger representation for the collective atomic angular operators

$$J_{ij} = a_i^{+} a_j \qquad (i,j = 1,2) \quad ,$$

where the operators a_i and a_i^{+} obey the boson commutation relations

$$[\, a_i\, ,\, a_j^{+} \,] = \delta_{ij}$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level $|i\rangle$.

After performing the canonical transformation

$$\begin{aligned} a_1 &= -\frac{1}{\sqrt{2}}\cos\alpha\, C_1 - \sin\alpha\, C_2 + \frac{1}{\sqrt{2}}\cos\alpha\, C_3 \quad , \\ a_2 &= \frac{1}{\sqrt{2}}\, C_1 + \frac{1}{\sqrt{2}}\, C_3 \quad , \qquad (4) \\ a_3 &= -\frac{1}{\sqrt{2}}\sin\alpha\, C_1 + \cos\alpha\, C_2 + \frac{1}{\sqrt{2}}\sin\alpha\, C_3 \quad , \end{aligned}$$

where

$$tg\alpha = G_{32} / G_{21}$$

the Hamiltonian (1) reduces to the form

$$H = \Omega\, (R_{33} - R_{11}) \quad , \qquad (5)$$

where $\Omega = (\, G_{21}^2 + G_{32}^2 \,)^{1/2}$

and $R_{ij} = C_i^{+} C_j$ (i, j = 1,2,3) are the collective angular operators of the "dressed" atoms. The equations of motion for the operators $R_{ij}(t)$ are easily solved exactly, and by using the canonical transformation (4) one can find the collective atomic operators $J_{ij}(t)$ as

$$J_{21}(t) = S_0 + S_{-1}^+ e^{-i\Omega t} + S_{+1}^+ e^{i\Omega t} + S_{-2}^+ e^{-2i\Omega t} + S_{+2}^+ e^{2i\Omega t} ,$$
$$J_{12}(t) = J_{21}^+(t) , \qquad (6)$$

$$J_{32}(t) = Q_0 + Q_{-1}^+ e^{-i\Omega t} + Q_{+1}^+ e^{i\Omega t} + Q_{-2}^+ e^{-2i\Omega t} + Q_{+2}^+ e^{2i\Omega t} ,$$
$$J_{23}(t) = J_{32}^+(t) , \qquad (7)$$

where

$$S_0 = \frac{1}{2}\cos\alpha \left\{ \cos\alpha \, (J_{12} + J_{21}) + \sin\alpha \, (J_{23} + J_{32}) \right\} = S_0^+ , \qquad (8)$$

$$S_{-1}^+ = -\frac{1}{2}\sin\alpha \left\{ -\sin\alpha\cos\alpha \, J_3 - \cos^2\alpha \, J_{13} + \sin^2\alpha \, J_{31} - \sin\alpha \, J_{21} + \cos\alpha \, J_{23} \right\} ; \qquad (9)$$

$$S_{+1}^+ = -\frac{1}{2}\sin\alpha \left\{ \sin\alpha\cos\alpha \, J_3 + \cos^2\alpha \, J_{13} - \sin^2\alpha \, J_{31} - \sin\alpha \, J_{21} + \cos\alpha \, J_{23} \right\} , \qquad (10)$$

$$S_{-2}^+ = \frac{1}{4}\cos\alpha \left\{ J_{22} - \cos^2\alpha \, J_{11} - \sin^2 J_{33} + \cos\alpha \, (J_{21} - J_{12}) + \sin\alpha \, (J_{23} - J_{32}) - \sin\alpha\cos\alpha \, (J_{13} + J_{31}) \right\} , \qquad (11)$$

$$S_{+2}^+ = -\frac{1}{4}\cos\alpha \left\{ J_{22} - \cos^2\alpha \, J_{11} - \sin^2\alpha \, J_{33} + \cos\alpha \, (J_{12} - J_{21}) + \sin\alpha \, (J_{32} - J_{23}) - \sin\alpha\cos\alpha \, (J_{13} + J_{31}) \right\} , \qquad (12)$$

$$Q_0^+ = tg\alpha \, S_0 , \qquad (13)$$

$$Q_{-1}^+ = -ctg\alpha \, S_{+1} , \qquad (14)$$

$$Q_{+1}^+ = -ctg\alpha \, S_{-1} , \qquad (15)$$

$$Q_{-2}^+ = tg\alpha \, S_{-2}^+ , \qquad (16)$$

$$Q_{+2}^+ = tg\alpha \, S_{+2}^+ , \qquad (17)$$

$$J_3 = J_{33} - J_{11} . \qquad (18)$$

Further, we shall investigate the spectral and nonclassical properties of the fluorescence field in other modes. This fluorescence can be observed in the direction perpendicular to the cavity axis and its spectrum will be [13-15]

$$S_F(\nu, T) = 2\Gamma \int_0^T dt_1 \int_0^T dt_2 \, e^{-(\Gamma - i\nu)(T - t_1) - (\Gamma + i\nu)(T - t_2)} \langle J_{21}(t_1) J_{12}(t_2) \rangle e^{-i\omega_1 (t_2 - t_1)} , \qquad (19)$$

$$Q_F(\nu, T) = 2\Gamma \int_0^T dt_1 \int_0^T dt_2 \, e^{-(\Gamma - i\nu)(T - t_1) - (\Gamma + i\nu)(T - t_2)} \langle J_{32}(t_1) J_{23}(t_2) \rangle e^{-i\omega_2 (t_2 - t_1)} , \qquad (20)$$

where $S_F(\nu, T)$ and $Q_F(\nu, T)$ are the spectra **of the fluorescence fields, corresponding to** the lower and upper atomic transitions, respectively; 2Γ is the bandwidth of the detector, T is the time at which the spectrum is evaluated. The $\langle \dots \rangle$ stands for the average over the initial state of the atomic system.

Applying the relations (6-7) one shows that for the case of intense cavity modes the terms in the fluorescence spectra (19) and (20), which are proportional to Ω^{-1}, can be ignored (secular approximation) and the operators S_k and Q_k ($k = 0, \pm 1, \pm 2$)

can be considered as operators-sources of the spectrum components at frequencies $\omega_1 + k\Omega$ and $\omega_2 + k\Omega$, respectively. The line widths of all the spectrum components S_k and Q_k are equal to 2Γ.

Let the atoms be initially in the collective symmetric state

$$\rho_a = |n_1, n_2, n_3\rangle\langle n_3, n_2, n_1| \quad , \tag{21}$$

where

$$J_{11}|n_1, n_2, n_3\rangle = n_1 |n_1, n_2, n_3\rangle \quad ,$$
$$J_{22}|n_1, n_2, n_3\rangle = n_2 |n_1, n_2, n_3\rangle \quad ,$$
$$J_{33}|n_1, n_2, n_3\rangle = n_3 |n_1, n_2, n_3\rangle \quad ,$$

and n_1, n_2, n_3 are the initial atomic population in the lower $|1\rangle$, intermediate and upper states, respectively.

Using the relations (8-17) and (21) one finds that the intensities of each spectrum components S_k and Q_k ($k = 0, \pm 1, \pm 2$) are proportional to:

$$I_{S_0} = \langle S_0^2\rangle = \frac{1}{4}\cos^2\alpha\,[\cos^2\alpha\,(2n_1n_2 + n_1 + n_2) + \sin^2\alpha\,(2n_2n_3 + n_2 + n_3)] \quad , \tag{22}$$

$$I_{S_{-1}} = I_{S_{+1}} = \langle S_1^+ S_1\rangle = \frac{1}{4}\sin^2\alpha\,[\sin^2\alpha\cos^2\alpha\,(n_3 - n_1)^2 + \cos^4\alpha\, n_1(n_3+1) + \sin^4\alpha\, n_3(n_1+1) + \sin^2\alpha\, n_2(n_1+1) + \cos^2\alpha\, n_2(n_3+1)] \quad , \tag{23}$$

$$I_{S_{-2}} = I_{S_{+2}} = \langle S_2^+ S_2\rangle = \frac{1}{16}\cos^2\alpha\,[(n_2 - \cos^2\alpha\, n_1 - \sin^2\alpha\, n_3)^2 + \cos^2\alpha\,(2n_1n_2 + n_1 + n_2) + \sin^2\alpha\,(2n_2n_3 + n_2 + n_3) + \sin^2\alpha\cos^2\alpha\,(2n_1n_3 + n_1 + n_3)] \quad , \tag{24}$$

$$I_{Q_0} = tg^2\alpha\, I_{S_0} \quad , \tag{25}$$

$$I_{Q_{\pm 2}} = tg^2\alpha\, I_{S_{\pm 2}} \quad , \tag{26}$$

$$I_{S_{\pm 1}} = \frac{1}{4}\cos^2\alpha\,\{\sin^2\alpha\cos^2\alpha\,(n_3 - n_1)^2 + \cos^4\alpha\, n_3(n_1+1) + \sin^4\alpha\, n_1(n_3+1) + \sin^2\alpha\, n_1(n_2+1) + \cos^2\alpha\, n_3(n_2+1)\} \,. \tag{27}$$

As for the spectrum of double resonant fluorescence in a free space [29,27] the intensities of sidebands $I_{S_{-1}} = I_{S_{+1}}$, $I_{S_{-2}} = I_{S_{+2}}$, $I_{Q_{-1}} = I_{Q_{+1}}$; $I_{Q_{-2}} = I_{Q_{+2}}$, thus the fluorescence spectrum of the upper atomic transition $|3\rangle \to |2\rangle$ as of the lower atomic transition $|2\rangle \to |1\rangle$ is symmetric.

For the case when all the atoms are initially in the ground state $|1\rangle$, i.e. $n_1 = N$, $n_2 = n_3 = 0$, eqs. (22)-(27) reduce to

$$I_{S_0} = \frac{1}{4}\cos^4\alpha\, N \quad , \tag{28}$$

$$I_{S_{\pm 1}} = \frac{1}{4}\sin^2\alpha\cos^2\alpha\, N(N\sin^2\alpha + \cos^2\alpha) \quad , \tag{29}$$

$$I_{S_{\pm 2}} = \frac{1}{16}\cos^4\alpha\, N(N\cos^2\alpha + \sin^2\alpha + 1) \quad , \tag{30}$$

$$I_{Q_{\pm 1}} = \frac{1}{4}\sin^2\alpha\cos^2\alpha\, N(N\cos^2\alpha + \sin^2\alpha + 1) = 4I_{Q_{\pm 2}} \quad , \tag{31}$$

$$I_{Q_0} = \frac{1}{4}\sin^2\alpha\cos^2 N \quad . \tag{32}$$

It means that the intensities of the sidebands $S_{\pm 1}$, $S_{\pm 2}$, $Q_{\pm 1}$ and $Q_{\pm 2}$ are proportional to N^2 while the center components S_0 and Q_0 have the intensities being proportional only to N . This result is essentially different from collective double resonance fluorescence in a free space [26] where the intensities of the central spectrum components S_0 and Q_0 are proportional to N^2.

For the case when all the atoms are initially in the intermediate state $|2\rangle$, i.e. $n_1=n_3=0$, $n_2=N$, the intensities (22)-(27) reduce to

$$I_{S_0} = \frac{1}{4}\cos^2\alpha\, N \quad , \tag{33}$$

$$I_{S_{\pm1}} = \frac{1}{4}\sin^2\alpha\, N \quad , \tag{34}$$

$$I_{S_{\pm2}} = \frac{1}{16}\cos^2\alpha\, N(N+1) \quad , \tag{35}$$

$$I_{Q_0} = \frac{1}{4}\sin^2\alpha\, N \quad , \tag{36}$$

$$I_{Q_{\pm1}} = 0 \quad , \tag{37}$$

$$I_{Q_{\pm2}} = \frac{1}{16}\sin^2\alpha\, N(N+1) \quad . \tag{38}$$

In this case only the extreme sidebands $S_{\pm2}$ and $Q_{\pm2}$ exhibit the superradiant behaviour ($I_{S_{\pm2}}, I_{Q_{\pm2}} \sim N^2$). The spectrum of fluorescence corresponding to the upper atomic transition $|3\rangle \rightarrow |2\rangle$ contains only three spectrum components Q_0 and $Q_{\pm2}$ centered at the frequencies ω_2 and $\omega_2 \pm 2\Omega$, respectively.

For the case when $n_1 \approx n_2 \approx n_3$ the intensities of all the spectrum components S_k and Q_k($k = 0, \pm1, \pm2$) are proportional to N^2 for any value of the parameter $\cos^2\alpha$ while the intensities of the sidebands $S_{\pm1}$ and $Q_{\pm1}$ of the double resonant fluorescence in a free space [26] exhibit superradiance bahaviour only for the case of $\cos^2\alpha = 1/2$.

To conclude this section we note that the intensities of the spectrum components, i.e. the spectrum picture, are strongly dependent on the system parameter $\cos^2\alpha$ and initial atomic population distribution and are essentially different from the case of collective double resonance fluorescence in a free space.

III. SUB-POISSONIAN STATISTICS

In this section we discuss the photon statistics of spectrum components. We show that in contrast with the double resonance in a free space[30] , the sidebands $S_{\pm1}$, $S_{\pm2}$ ($Q_{\pm1}$, $Q_{\pm2}$) have the sub- poissonian statistics for the collective case $N \gg 1$. It means that the fluorescence in a cavity can play the role of source for the high-intensity light with sub-poissonian statistics which has applications in the gravitational wave detector and in the quantum non-demolition measurement[31,32] . We limit the discussion to the photon statistics of the spectrum components S_k ($k = 0, \pm1, \pm2$) corresponding the lower atomic transition $|2\rangle \rightarrow |1\rangle$.

Following the works[33,34] , the spectrum component S_k($k = 0, \pm1, \pm2$) has the sub-poissonian statistics if the following relation is satisfied:

$$\langle :(\Delta I_{S_k})^2: \rangle = \langle S_k^+ S_k^+ S_k S_k \rangle - \langle S_k^+ S_k \rangle^2 < 0 \quad . \tag{39}$$

For the case when all the atoms are initially in the ground state $|1\rangle$, one finds from the relations (8-12) that

$$\langle :(\Delta I_{S_0})^2: \rangle = \frac{1}{16}\cos^6\alpha\,[2\cos^2\alpha\, N(N-1) + \sin^2\alpha\, N] > 0 , \tag{40}$$

$$\langle :(\Delta I_{S_{\pm 1}})^2: \rangle = \frac{1}{16}\cos^4\alpha \sin^4\alpha\, N \Big\{ 2(N-1)^2 \sin^2\alpha \cos 2\alpha + (N-1)(1 - 6\sin^2\alpha + 2\sin^4\alpha) - 1 \Big\} , \tag{41}$$

$$\langle :(\Delta I_{S_{\pm 2}})^2: \rangle = \frac{\cos^8\alpha}{4^4} \cdot \Big\{ -2N(N-1)(3 - 4\sin^2\alpha - \sin^4\alpha) - 4N \Big\} . \tag{42}$$

It is easy to see from the relation (40) that the central component has super-poissonian statistics without the case $N = 1$ and $\sin^2\alpha = 0$ (single two-level atom) when the central component S_0 has the poissonian statistics.

For the single-atom case one finds

$$\langle :(\Delta I_{S_{\pm 1}})^2: \rangle = -\frac{1}{16}\sin^4\alpha \cos^4\alpha , \tag{43}$$

$$\langle :(\Delta I_{S_{\pm 2}})^2: \rangle = -\frac{\cos^8\alpha}{64} , \tag{44}$$

thus all the sidebands $S_{\pm 1}$ and $S_{\pm 2}$ have the sub-poissonian statistics.

For the case of $N \gg 1$, as is seen from the relations (41) and (42), the intermediate sidebands $S_{\pm 1}$ have the sub-poissonian statistics for the case of $\cos^2\alpha \lesssim 1/2$ and the extreme sidebands $S_{\pm 2}$ have the sub-poissonian statistics for the case of $\cos^2\alpha \gtrsim 3 - \sqrt{7}$. It means that the sidebands of the fluorescence spectrum of N three-level atoms in a cavity are intense sources (the peak intensity being proportional to N^2) with sub-poissonian statistics which have impotant application in the detection of a very weak signal such as a gravitational wave [31] .

V. LIGHT SQUEEZING

In this section we discuss the squeezing in the spectrum components $S_k (k = 0, \pm 1, \pm 2)$.

The squeezing properties of the spectrum components S_k may be studied by introducing the Hermitian operators

$$S_i = L_1^{(k)} + i L_2^{(k)} \qquad (k = 0, \pm 1, \pm 2). \tag{45}$$

We speak of squeezing (phase squeezing) in the spectrum components S_k if the normally-ordered variance of the operators $L_1^{(k)}$ or $L_2^{(k)}$ is less than zero

$$\langle :(\Delta L_1^{(k)})^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta L_2^{(k)})^2: \rangle . \tag{46}$$

Let the atoms be initially in the collective symmetric state (21). As is easily seen from the relation (8), the operator S_0 is the Hermitian operator $S_0^+ = S_0$ and squeezing is absent for the central spectrum component S_0. The quantities $\langle :(\Delta L_{1,2}^{(k)})^2: \rangle$, $k = \pm 1, \pm 2$, can be derived by using the relations (9 - 12):

$$\langle :(\Delta L_1^{(\pm 1)})^2: \rangle = \frac{1}{8}\sin^2\alpha \Big\{ (\cos^2\alpha - \sin^2\alpha) n_1 n_3 + \sin^2\alpha\, n_2 (n_1 + 1) + \cos^2\alpha\, n_2 (n_3 + 1) + \cos^2\alpha \cdot (\cos^2\alpha - \sin^2\alpha) n_1 - \sin^2\alpha (\cos^2\alpha - \sin^2\alpha) n_3 \Big\} , \tag{47}$$

$$\langle :(\Delta L_2^{(\pm 1)})^2: \rangle = \frac{1}{8}\sin^2\alpha \Big\{ n_1 n_3 + \sin^2\alpha\, n_2 (n_1 + 1) + \cos^2\alpha\, n_2 (n_3 + 1) + \cos^2\alpha\, n_1 + \sin^2\alpha\, n_3 \Big\} \geq 0 , \tag{48}$$

$$\langle :(\Delta L_1^{(\pm 2)})^2: \rangle = \frac{1}{16}\cos^4\alpha \sin^2\alpha\, (2 n_1 n_3 + n_1 + n_3) \geq 0 , \tag{49}$$

$$\langle :(\Delta L_2^{(\pm 2)})^2: \rangle = \frac{1}{16}\cos^2\alpha \Big\{ \cos^2\alpha\, (2 n_1 n_2 + n_1 + n_2) + \sin^2\alpha\, (2 n_2 n_3 + n_2 + n_3) \Big\} \geq 0 . \tag{50}$$

As is easily seen from the relations (49),(50), the squeezing is absent for the extreme sidebands $S_{\pm 2}$. For the intermediate spectrum components $S_{\pm 1}$ squeezing is present only in the in-phase component $L_1^{(\pm 1)}$. One can show from the relation (47) that the normally ordered variances $\langle :(\Delta L_1^{(\pm 1)})^2: \rangle$ reach the minimum if $n_1 = N$ (for the case $\cos^2\alpha < 1/2$) or $n_3 = N$ (for the case $\cos^2\alpha > 1/2$) and have the following form:

$$\langle :(\Delta L_1^{(\pm 1)})^2: \rangle = \begin{cases} \frac{1}{8}\sin^2\alpha\cos^2\alpha(\cos^2\alpha - \sin^2\alpha)N & \text{if } n_1 = N, \\ \frac{1}{8}\sin^4\alpha(\sin^2\alpha - \cos^2\alpha)N & \text{if } n_3 = N. \end{cases} \quad (51)$$

Thus, the variances $\langle :(\Delta L_1^{(\pm 1)})^2: \rangle$ are proportional to N.

Finally, we discuss the factor of squeezing for the atomic operators $L_1^{(\pm 1)}$ which have the following form:

$$F_{\pm 1} = \frac{\langle :(\Delta L_1^{(\pm 1)})^2: \rangle}{\frac{1}{2}|\langle [L_1^{(\pm 1)}, L_2^{(\pm 1)}]\rangle|}, \quad (52)$$

where

$$\frac{1}{2}|\langle [L_1^{(\pm 1)}, L_2^{(\pm 1)}]\rangle| = \frac{1}{16}\sin^2\alpha.$$
$$|(\cos^2\alpha - \sin^2\alpha)(n_1 - n_3) + \sin^2\alpha(n_2 - n_1) + \cos^2\alpha(n_2 - n_3)|. \quad (53)$$

For the case of $n_1 = N$ or $n_3 = N$ the relation (52) reduces to

$$F_{\pm 1} = \begin{cases} \frac{2\cos^2\alpha(\cos^2\alpha - \sin^2\alpha)}{|\cos^2\alpha - 2\sin^2\alpha|} & \text{if } n_1 = N, \\ \frac{2\sin^2\alpha(\sin^2\alpha - \cos^2\alpha)}{|\sin^2\alpha - 2\cos^2\alpha|} & \text{if } n_3 = N. \end{cases} \quad (54)$$

The factor of squeezing $F_{\pm 1}$ in equation (54) has the minimum value equal to $-2/9$ (i.e. $\sim 22\%$ of squeezing) for the points $\cos^2\alpha = 1/3$, $n_1 = N$ and $\cos^2\alpha = 2/3$, $n_3 = N$.

V. CONCLUSIONS

We have considered the problem of N three-level atoms interacting with two resonance modes in the lossless cavity. The superradiant behaviour of the spectrum components is discussed. Unlike the collective double resonance in a free space [30] where the sub-poissonian statistics of the spectral components is present only for the case of several atoms, in the cavity the sidebands of the fluorescence field have the sub-poissonian photon statistics for the case of large N. It means that the sidebands prove to be sources of the intense field ($\sim N^2$) which have the sub-poissonian statistics. We have shown that the squeezing is present only in the individual sidebands S_{-1} and S_{+1}. This result is different from the collective double resonance in a free space [35] where only two-mode squeezing is present in the mixture of two sidebands $S_{\pm 1}$ but not in the sidebands S_{-1} and S_{+1} taken separately.

The effects due to the quantum nature of the cavity modes and finite Q-value of the cavity request special investigations.

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