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TIME EVOLUTION OF LIGHT SQUEEZING IN A JAYNES – CUMMINGS MODEL

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The Jaynes-Cummings model of a two-level atom interacting with a single mode of the electromagnetic field in a cavity has been in recent years the object of intensive investigations. Stimulus for this considerable interest was provided by the development of the experimental techniques concerning Rydberg transitions [1-3] which have allowed one to test theoretical predictions on the features of the atom-field interaction depending explicitly on the quantum nature of the electromagnetic field. Among various possible nonclassical effects produced by atoms in cavities one finds the so-called collapses and revivals in the developments of the populations of the atomic levels [4]. Meystre and Zubairy [5] have demonstrated a coherent field interacting with an initially excited atom to bring out a squeezed state [6]. A large degree of squeezing (nearly perfect) has been obtained in such an exact system after certain time interval with increasing field intensity [7]. Since to produce squeezed electromagnetic fields in the laboratory [8-10] is now possible, it is instructive to analyze how the degree of squeezing of the field interacting with a single two-level atom will be altered. The interaction of squeezed vacuum with a two-photon generalized Jaynes-Cumings model [11] has been studied. It was shown that squeezing is initially revoked and recurs at later time. The higher the initial squeezing is the more regular the oscilations become.

In this paper we examine the field initially in a rather general squeezed state $|\beta, z\rangle$ that may be generated by first acting with the squeezed operator $S(z)$ on the vacuum followed by the displacement operator $D(\beta)$ [12]

$$|\beta, z\rangle = D(\beta)\, S(z)\, |0\rangle , \tag{1}$$

where

$$D(\beta) \equiv \exp(\beta a^{+} - \beta^{*} a)$$

and

$$S(z) \equiv \exp\left(\frac{z}{2} a^{+2} - \frac{z^{*}}{2} a^{2}\right). \tag{2}$$

(The squeezed vacuum considered by [11] corresponds to $\beta = 0$). We will show that no enhancement of squeezing at once after the onset of the interaction occurs if the atom is initially purely excited or de-excited as no initial phase exists.

The system Hamiltonian in the rotating wave approximation has the form

$$H = \hbar\omega a^{+} a + \hbar\omega_0 R^{z} + \hbar g (R^{+} a + R^{-} a^{+}). \tag{3}$$

where a^{+} and a are the Bose creation and annihilation operators for the photons at frequency ω . The two-level atom is described by the Pauli raising and lowering operators R^{+}, R^{-} and the inversion operator R^{z}, g is the coupling constant. We take for simplicity the resonant case $\omega = \omega_0$.

Let us consider the atom prior to the interaction is prepared in a coherent superpositions of excited and ground states [13]

$$|\psi_{atom}(t=0)\rangle = \cos(\tfrac{\theta}{2})|e\rangle + e^{i\phi} \sin(\tfrac{\theta}{2})|g\rangle . \tag{4}$$

Here ϕ is the relative phase between cos($\theta/2$) and sin($\theta/2$). We denote by $|n\rangle$ the Fock states of the electromagnetic field. The distribution $P_n = |q_n|^2 \equiv |\langle n|\beta, z\rangle|^2$ has been calculated by many authors (see for example [14,15]) and can be given as

$$q_n = (n!\cosh|z|)^{-1/2}\left(\frac{z}{2|z|}\tanh|z|\right)^{n/2}$$

$$\times \exp\left(-\frac{|\beta|^2}{2} + \frac{1}{2}\frac{z^*}{|z|}\tanh|z|\,\beta^2\right).H_n\left(\frac{\beta}{(2\frac{z}{|z|}\cosh|z|\sinh|z|)^{1/2}}\right), \tag{5}$$

where $H_n(x)$ is the nth hermite polynomial with complex arguments x which is defined as

$$H_n(x) = (-1)^n e^{x^2}\frac{d^n e^{-x^2}}{dx^n}, \quad n = 0, 1, 2, \ldots \tag{6}$$

The average photon number $\bar{n}$ is related to the parameters β and z according to $\bar{n} = |\beta|^2 + \sinh^2|z|$. The imaginary values of β and z create numerical problems, therefore, we consider further only the case in which β and z are real and positive. The initial atom-field state is then a product of the atomic superposition state and the squeezed field

$$|\psi(t=0)\rangle = \sum_{n=0}^{\infty} q_n\,|n\rangle\,|\psi_{atom}(t=0)\rangle. \tag{7}$$

The wave function of the total system in the interaction picture is found from Hamiltonian (3) to be

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} q_n\Big[\cos(\tfrac{\theta}{2})\cdot\big(A_{n+1}(t)\,|e;n\rangle + B_{n+1}(t)\,|g;n+1\rangle\big) + e^{i\phi}\sin(\tfrac{\theta}{2})\cdot\big(A_n(t)\,|g;n\rangle + B_n(t)\,|e;n-1\rangle\big)\Big], \tag{8}$$

where the coefficients $A_n(t)$ and $B_n(t)$ are

$$A_n(t) = \cos(gt\sqrt{n}),$$
$$B_n(t) = -i\sin(gt\sqrt{n}). \tag{9}$$

The observables of interest are the variances of the field quadratures which give information on squeezing. The quadratures and field operators are defined by

$$a_1 = \frac{1}{2}(a^+ + a)\;;\quad a_2 = \frac{1}{2i}(a - a^+)$$
$$a = a_1 + ia_2\;;\quad a^+ = a_1 - ia_2. \tag{10}$$

For convenience we introduce the relative variances

$$S_\alpha = \frac{(\Delta a_\alpha)^2 - (\Delta a_\alpha)^2_{coh,\,vac}}{(\Delta a_\alpha)^2_{coh,\,vac}} = 4(\Delta a_\alpha)^2 - 1 \tag{11}$$

since, as is well known, for the vacuum and coherent states of the field one has $(\Delta a_1)^2_{coh,\,vac} = (\Delta a_2)^2_{coh,\,vac} = \frac{1}{4}$. The condition for squeezing is

$$S_\alpha < 0. \tag{12}$$

We note that $S_\alpha > -1$ for an arbitrary field state and then the degree of squeezing is determined by $-S_\alpha = -100\,S_\alpha\,\%$. In terms of the photon operators we get for the squeezing factors S_α the expressions

$$S_1 = 2\langle a^+a\rangle + 2\,\mathrm{Re}\,\langle a^2\rangle - 4(\mathrm{Re}\,\langle a\rangle)^2,$$
$$S_2 = 2\langle a^+a\rangle - 2\,\mathrm{Re}\,\langle a^2\rangle - 4(\mathrm{Im}\,\langle a\rangle)^2. \tag{13}$$

We now proceed to calculate the short-time approximation when $gt \ll 1$ for the factors S_α. Using formulae (13) and (8) we obtain the expansions to order t^2 as follows:

$$S_1 = (e^{-2z} - 1) + \tau^2(e^{-2z}\cos\theta + 1 - \sin^2\theta\,\sin^2\phi), \tag{14a}$$
$$S_2 = (e^{2z} - 1) + \tau^2(e^{2z}\cos\theta + 1 - \sin^2\theta\,\cos^2\phi). \tag{14b}$$

With τ defined as the dimensionless time $\tau = gt$. For increasing squeezing initially presented in the first quadrature a_1, it is necessary that the coefficient of τ^2 (see (14a)) be less than zero. One can check this coefficient to achieve minimum at ϕ =90° and $\cos\theta = -e^{-2z}/2$; then we have

$$S_1 = (e^{-2z} - 1) - \tau^2\left(\frac{e^{-4z}}{4}\right). \tag{15}$$

The degree of squeezing increases quadratically from the onset of the interaction and the greater is initial squeezing the smaller is its relative enhancement. This feature holds true in the squeezed vacuum case too because eqs. (14), (15) show no dependence on the parameter β. As we shall see from the numerical results presented below, the most squeezing enhancement after the beginning of an atom-field interaction occurs just in the squeezed vacuum case. Let us consider two limiting situations when the atom is initially purely excited or de-excited. For the first situation we have $\theta/2 = 0$ and

$$S_1 = (e^{-2z} - 1) + \tau^2 (e^{-2z} + 1), \tag{16a}$$

$$S_2 = (e^{2z} - 1) + \tau^2 (e^{2z} + 1). \tag{16b}$$

For the second one $\theta/2 = 90°$, formulae (14a,b) are transformed into

$$S_1 = (e^{-2z} - 1) + \tau^2 (1 - e^{-2z}), \tag{17a}$$

$$S_2 = (e^{2z} - 1) + \tau^2 (1 - e^{2z}). \tag{17b}$$

We see from Eqs. (15a) and (16a) that there is no initial phase

present in both the situations and no enhancement of the degree of squeezing at once after the onset of interaction is possible.

The time-dependence of squeezing factors S_i to the fourth order is complicated. In fact, in a particular purely de-excited atom case $\theta/2 = 90°$, choosing phase angle $\phi = 90°$ as before we have

$$S_1 = (e^{-2z} - 1)(1 - \tau^2) + \frac{\tau^4}{3}\left[\beta^2(3e^{-4z} - 4e^{-2z}) + \frac{1}{4}(3e^{-4z} - 4e^{-2z} + 1)\right], \tag{18}$$

$$S_2 = (e^{2z} - 1)(1 - \tau^2) + \frac{\tau^4}{3}\left[\beta^2 + \frac{1}{4}(3e^{4z} - 4e^{2z} + 1)\right].$$

The β-dependence of factors S_i appears. For a field initially in a coherent state $z = 0$, Eqs. (18) are considerably simplified

$$S_1 = -\tau^4\frac{\beta^2}{3},$$

$$S_2 = \tau^4\frac{\beta^2}{3}, \tag{19}$$

and our result is in agreement with the result of [16] for one atom in the cavity and [17] for one-photon transitions which show the occurence of squeezing in the development of the a_1 quadrature.

In Fig. 1 we present the time evolution of S_1 computed numerically with the aid of Eqs. (5), (8) and (13) for various $\cos\theta$ equal to -1, $-e^{-2z}/2$ ($\simeq -0.45$ for z =0.05) -0.2, -0.1 and 0., θ taken = 90° anywhere below. It is evident that for $\cos\theta$ satisfying the condition $-e^{-2z} < \cos\theta < 0$; so the coefficient of τ^2 in formula (14a) is negative the degree of squeezing begins to increase to its maximum values. The optimum squeezing occurs at $\cos\theta = -e^{-2z}/2$.

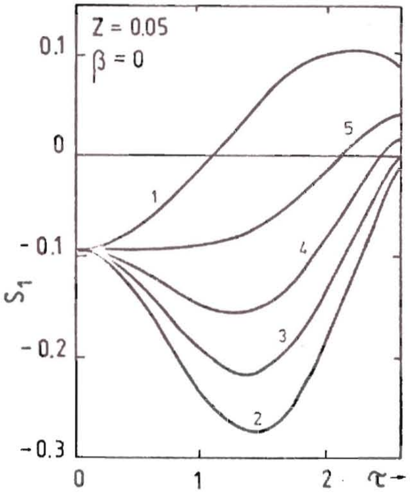

Fig. 1. Time behaviour of S_1 with the choice of cos θ (1) $\cos\theta = -1$, (2) $\cos\theta = -e^{-2z}/2$, (3) $\cos\theta = -0.2$, (4) $\cos\theta = -0.1$, (5) $\cos\theta = 0.$; $\phi = 90°$; $z = 0.05$; $\beta = 0$.

In Fig.2 we plot the factor S_1 versus time for various values of the field amplitude β. As has been mentioned above, the first minimum of the field uncertainty in the $1\,th$ quadrature reaches its smallest value at $\beta = 0$. The larger the initial squeezing is, the more difficult its enhancement in the early period of the interaction. As the time goes on, the characteristic behaviour of S_1 exhibits still more dependence on β . Fig. 3a ($\beta = 0$) shows an apparently random time development of S_1 that is due to the thermal light-like statistical properties of squeezed vacuum. When β increases, the time behaviour of S_1 becomes similar to that obtained in [5,7] for the field initially in a coherent state (Fig. 3b). The larger β gives rise to stronger squeezing (up to 100%) in a longer duration.

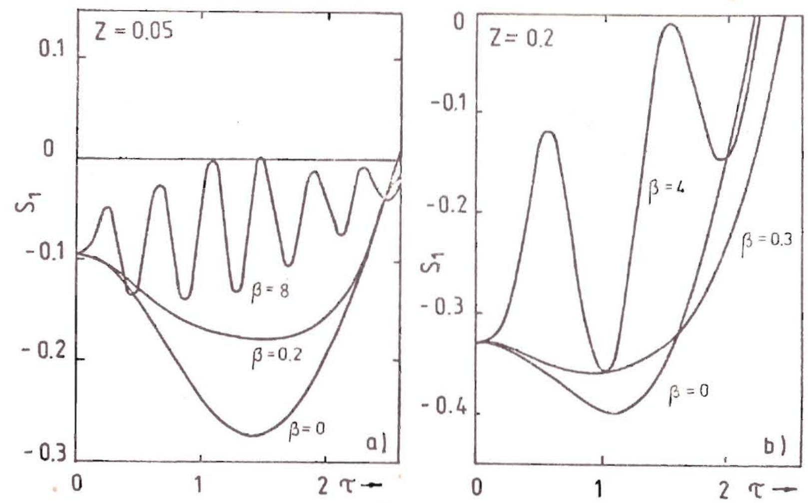

Fig. 2. Time behaviour of S_1 for different initial β . The initial squeezing and maximum value achieved at the first time after the onset of interaction approximately equal to (a) 9,5% and 27% with z =0.05, (b) 33% and 40% with z =0.2, respectively.

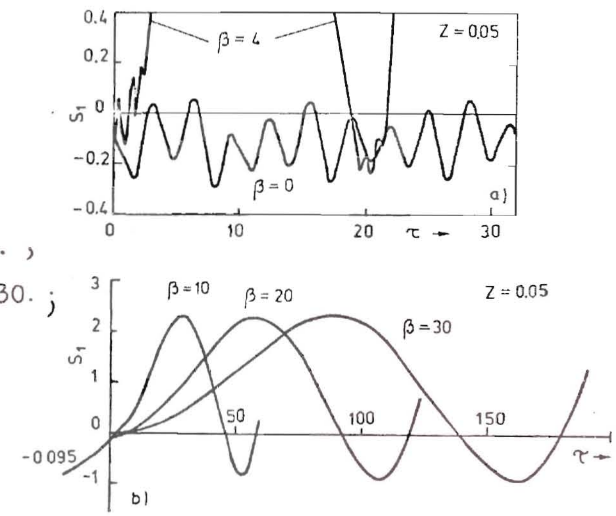

Fig. 3. Long time behaviour of S_1 , for (a) $\beta = 0.$, $4.$, (b) $\beta = 10.$, $20.$, $30.$; $z = 0.05$; $\cos\theta = -e^{-2z}/2$ and $\phi = 90°$.

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Эволюция сжатого света в модели Джейнса – Каммингса

Изучена эволюция флуктуаций в одномодовом сжатом свете, взаимодействующим с двухуровневым атомом, находящимся в состоянии когерентной суперпозиции нижнего и возбужденного уровней. Обсуждена зависимость от фазы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Time Evolution of Light Squeezing in a Jaynes – Cummings Model

Time evolution of fluctuations in a squeezed single-mode field interacting with a two-level atom initially in a coherent superposition of excited and ground states is treated. The phase dependence is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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