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COLLECTIVE RESONANCE FLUORESCENCE IN THE PRESENCE OF A THERMAL RESERVOIR

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I. INTRODUCTION

Resonance fluorescence of two-level atoms in the presence of a strong laser field has received a great deal of attention in recent years[1-10]. In the work[1] by Mollow, a three-peaked fluorescence spectrum in the interaction of a single two-level atom with a single-mode coherent state driving field has been predicted. This spectrum, a "dynamical stark" spectrum, was first observed in the work[2] by Schuda et al. Resently, the spectral[7,8,10], statistical[9,10] properties and nonclassical effects as squeezing, violation of the Cauchy - Schwarz inequality[11-14] in the collective resonance fluorescence have been discussed.

In this paper we consider the problem of collective resonance fluorescence of N two-level atoms driven by an intense coherent field in the presence of an additional incoherent field. This incoherent field is typically a Gaussian chaotic field and can be a thermal black-body field. In the case of intense external field the stationary solution for the density operator of the atomic system is given. Expansion of linewidths and asymmetry of a spectrum caused by a thermal black-body field are shown. Influence of the thermal field intensity on the photon statistics of spectrum components, cross-correlation between them and on the violation of the Cauchy - Schwarz inequality is investigated.

II. BASIC EQUATIONS

Let N two-level atoms be concentrated in a region small compared to the wave-lengh of all the relevant radiation modes (Dicke model). The atoms are driven by a single-mode coherent field of the frequency ω_L and coupled to a reservoir containing all modes of the radiation field. In treating the external field classically and using the Markov and rotating wave approximation for describing the coupling of the system with the thermal reservoir, one arrives at the following master equation for the reduced atomic density operator ρ [15]

$$\frac{\partial\rho}{\partial t} = -i\left[\frac{\delta}{2}(J_{22}-J_{11}) + G(J_{12}+J_{21}), \rho\right] - \frac{\gamma}{2}(\bar{n}+1)(J_{21}J_{12}\rho - J_{12}\rho J_{21} + H.C.) - \frac{\gamma}{2}\bar{n}(J_{12}J_{21}\rho - J_{21}\rho J_{12} + H.C.) \equiv L\rho \quad , \qquad (1)$$

where $\delta = \omega_{21} - \omega_L$ is the detuning of the laser frequency ω_L from the atomic resonance frequency ω_{21} ($\omega_{21} = \omega_2 - \omega_1$; $\hbar \equiv 1$); $G = -d_{21}E_0$ is the resonant Rabi frequency; γ is the radiative spontaneous transition rate from the excited level $|2\rangle$ to the ground state $|1\rangle$; $\bar{n} = \bar{n}(\omega_{21}) = [\exp(\omega_{21}/kT) - 1]^{-1}$ is the mean photon number in the broad-band thermal field provided by the reservoir at the atomic frequency ω_{21} ; J_{ij} ($i,j = 1,2$) are the collective operators (angular momenta) describing the atomic system and having in the Schwinger representation[9] the following form:

$$J_{ij} = C_i^+ C_j \qquad (i,j = 1,2) \quad ,$$

where the operators C_i and C_j^+ obey the boson commutation relations

$$[C_i, C_j] = \delta_{ij}$$

and can be treated as annihilation and creation operators for atoms populating the level $|i\rangle$.

Further, we restrict our consideration to a strong laser field or to a large detuning δ so that the Rabi frequency Ω satisfied the following relation

$$\Omega = \left(\frac{1}{4}\delta^2 + G^2\right)^{1/2} \gg N\gamma, \ \bar{n}\gamma \quad . \qquad (2)$$

After performing the canonical (dressing) transformation

$$C_1 = Q_1\cos\varphi + Q_2\sin\varphi \quad , \qquad (3)$$
$$C_2 = -Q_1\sin\varphi + Q_2\cos\varphi \quad ,$$

where $\operatorname{tg} 2\varphi = 2G/\delta$,

one can split the Liouville operator appearing in equation (1) into the slowly varying part and the terms oscillating at frequencies 2Ω and 4Ω . Since we assume that the Rabi frequency Ω is safficiently large according to the relation (2) the secular approximation[7] is justified and we retain only the slowly varying part of the Liouville operator and the master equation (1) reduces to

$$\frac{\partial\tilde{\rho}}{\partial t} = -i\,\Omega[R_3, \rho] + B(2R_3\tilde{\rho}R_3 - R_3^2\tilde{\rho} - \tilde{\rho}R_3^2) + X_1(2R_{12}\tilde{\rho}R_{21} - R_{21}R_{12}\tilde{\rho} - \tilde{\rho}R_{21}R_{12}) + X_2(2R_{21}\tilde{\rho}R_{12} - R_{12}R_{21}\tilde{\rho} - \tilde{\rho}R_{12}R_{21}) \ , \qquad (4)$$

where $\tilde{\rho} = U\rho U^+$, where U is the unitary operator representing the canonical transformation (3)

$$R_3 = R_{22} - R_{11} \quad , \qquad (5)$$

$$B = \gamma\left(\bar{n} + \frac{1}{2}\right)\sin^2\varphi\cos^2\varphi \quad , \qquad (6)$$

$$X_1 = \frac{\gamma}{2}(\bar{n}+1)\cos^4\varphi + \frac{\gamma}{2}\bar{n}\sin^4\varphi \quad , \qquad (7)$$

$$X_2 = \frac{\gamma}{2}(\bar{n}+1)\sin^4\varphi + \frac{\gamma}{2}\bar{n}\cos^4\varphi \quad , \qquad (8)$$

$R_{ij} = Q_i^+ Q_j$ $(i,j = 1,2)$ are the collective operators of the dressed atoms. The operators Q_i and Q_j^+ satisfy the boson commutation relation

$$[Q_i, Q_j^+] = \delta_{ij} \quad , \qquad (9)$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij'}\delta_{i'j} - R_{i'j}\delta_{ij'} \quad . \qquad (10)$$

The exact stationary solution of equation (4) takes the form

$$\tilde{\rho}_{st} = Z^{-1}\sum_{n_1=0}^{N} X^{n_1}\, |n_1\rangle\langle n_1| \quad , \qquad (11)$$

where

$$X = \frac{X_1}{X_2} = \frac{(\bar{n}+1)\cos^4\varphi + \bar{n}\sin^4\varphi}{(\bar{n}+1)\sin^4\varphi + \bar{n}\cos^4\varphi} \quad , \qquad (12)$$

$$Z = \frac{X^{N+1} - 1}{X - 1} \quad , \qquad (13)$$

the state $|n_1\rangle$ is the eigenstate of the operators R_{11} and $R_{11} + R_{22}$. The solution (14) allows one to calculate all the stationary expectation values of the atomic observables. Some of the results that will be needed for our further consideration are given in Appendix.

In the case of the exact resonance $\cos^2\varphi = \frac{1}{2}$ the solution (11) reduces to

$$\tilde{\rho}_{st} = (N+1)^{-1}\sum_{n_1=0}^{N} |n_1\rangle\langle n_1| \quad . \qquad (14)$$

The solution (14) is independent of the mean photon number of the thermal field and has the same form for the case when the thermal reservoir is in the vacuum state (i.e. $T = 0$). Consequently in the exact resonance case all the one-time expectation values of the atomic observables are independent of $\bar{n}$.

III. FLUORESCENCE SPECTRUM

In this section we consider the effects that may arise in the collective steady-state fluorescence spectrum due to the thermal reservoir.

Following the works [6] , the steady-state spectrum of the fluorescent light has been calculated as the Fourier transform of the atomic correlation function

$$\langle J_{21}(\tau) J_{12}\rangle_s = \lim_{t\to\infty} \langle J_{21}(t+\tau) J_{12}(t)\rangle , \qquad (15)$$

where $\langle\ldots\rangle_s$ denotes an expectation value over the steady-state (11).

By using the transformation (3) one finds

$$J_{21}(t) = \sin\varphi\cos\varphi\, R_3(t) + \cos^2\varphi\, R_{21}(t) - \sin^2\varphi\, R_{12}(t) \quad . \qquad (16)$$

The equation of motion for $\langle R_{ij}(t)\rangle$ can be derived by using the master equation (4) and have the following forms:

$$\frac{d}{dt}\langle R_3(t)\rangle = -2(X_1 + X_2)\langle R_3(t)\rangle - (X_2 - X_1)\langle R_3^2(t)\rangle + (X_2 - X_1)(N^2 + 2N) \quad , \qquad (17)$$

$$\frac{d}{dt}\langle R_{12}(t)\rangle = -2i\Omega\langle R_{12}(t)\rangle - (4B + X_1 + X_2)\langle R_{12}(t)\rangle - \frac{1}{2}(X_2 - X_1)\langle\{R_{12}(t), R_3(t)\}\rangle \quad , \qquad (18)$$

$$\frac{d}{dt}\langle R_{21}(t)\rangle = \left(\frac{d}{dt}\langle R_{12}(t)\rangle\right)^{*} \quad , \qquad (19)$$

where $\{R_{ij}, R_3\} = R_{ij}R_3 + R_3 R_{ij}$.

Equations (17-19) are so far exact. They contain, however, terms with the products of operators which make them unsolvable in the general case.

For the one-atom case one can use the operator relation

$$R_{ij} R_{i'j'} = R_{ij'} \delta_{i'j} \qquad (i,j,i',j' = 1,2) \quad , \qquad (20)$$

then equations (17)-(19) reduce to the linear, exact solvable equations

$$\frac{d}{dt}\langle R_3(t)\rangle = -2(X_1+X_2)\langle R_3(t)\rangle + 2(X_2-X_1) \quad , \qquad (21)$$

$$\frac{d}{dt}\langle R_{12}(t)\rangle = -2i\Omega\langle R_{12}(t)\rangle - (4B+X_1+X_2)\langle R_{12}(t)\rangle \; , \qquad (22)$$

$$\frac{d}{dt}\langle R_{12}(t)\rangle = \frac{d}{dt}\langle R_{12}(t)\rangle^{*} \quad . \qquad (23)$$

Equations (21)-(23) are in agreement with the previous work [6] by Hildred et al. concerning resonance fluorescence of one atom in the presence of the thermal reservoir.

For the case of exact resonance $\cos^2\varphi = \frac{1}{2}$ we have $X_2 = X_1$ and the terms with the products of operators vanish; then all equations (17)-(29) reduce to the exact solvable linear differential equations.

For the off-resonance case, according to the works [8,10] , we use the decorrelation scheme

$$\langle\{R_3, R_{ij}\}\rangle = 2\langle R_3\rangle_s R_{ij} \quad . \qquad (24)$$

By using the density matrix (11) one can show that in the case of large N the decorrelation (24) yields a small error (with an order of $N^{-1/2}$) in the calculation of the steady-state fluorescent spectrum. With the approximation (24), equations (17)-(19) have simple exponential solutions.

Applying relation (16), solutions of the equations (17)-(19) and using the quantum regression theorem [16] , one obtains the following expressions for the correlation function (15)

$$\begin{aligned}\langle J_{21}(\tau) J_{12}\rangle_s &= \sin^4\varphi\,\langle R_{12}R_{21}\rangle_s \exp(-\Gamma_1\tau - 2i\Omega\tau)\\ &+ \cos^4\varphi\,\langle R_{21}R_{12}\rangle_s \exp(-\Gamma_1\tau + 2i\Omega\tau)\\ &+ \sin^2\varphi\cos^2\varphi\,(\langle R_3^2\rangle_s - I_e)\exp(-\Gamma_0\tau)\\ &+ \sin^2\varphi\cos^2\varphi\, I_e\end{aligned} \qquad (25)$$

where

$$\begin{aligned}\Gamma_1 &= 4B + X_1 + X_2 + (X_2 - X_1)\langle R_3\rangle_s\\ &= \gamma(\bar{n}+\tfrac{1}{2})(1+2\cos^2\varphi\sin^2\varphi) + \tfrac{1}{2}(\sin^2\varphi - \cos^2\varphi)\langle R_3\rangle_s \; ,\end{aligned} \qquad (26)$$

$$\begin{aligned}\Gamma_0 &= 2(X_1+X_2) + (X_2 - X_1)\langle R_3\rangle_s\\ &= 2\gamma(\bar{n}+\tfrac{1}{2})(\cos^4\varphi + \sin^4\varphi) + \tfrac{\gamma}{2}(\sin^2\varphi - \cos^2\varphi)\langle R_3\rangle_s \quad ,\end{aligned} \qquad (27)$$

$$I_e = \frac{1}{2}(\sin^2\varphi - \cos^2\varphi)(N^2+2N)\langle R_3\rangle_s / \Gamma_0 \quad . \qquad (28)$$

The expressions for the weighting factors of the particular exponents $\langle R_{12}R_{21}\rangle_s$, $\langle R_{21}R_{12}\rangle_s$, $\langle R_3^2\rangle_s$ and value $\langle R_3\rangle_s$ are given in Appendix.

The steady-state spectrum of the fluorescence is proportional to the Fourier transform of the correlation function (25) and takes the form

$$S(\omega) \simeq \frac{1}{2} \mathrm{Re} \left\{ \int_0^\infty \langle J_{21}(\tau) J_{12} \rangle_s \exp[-i(\omega - \omega_L)\tau] \, d\tau \right\}$$

$$= \cos^4\varphi \, \langle R_{21} R_{12} \rangle_s \frac{\Gamma_1}{(\omega - \omega_L + 2\Omega)^2 + \Gamma_1^2}$$

$$+ \sin^4\varphi \, \langle R_{12} R_{21} \rangle_s \frac{\Gamma_1}{(\omega - \omega_L + 2\Omega)^2 + \Gamma_1^2}$$

$$+ \sin^2\varphi \cos^2\varphi \, (\langle R_3^2 \rangle_s - Ie) \frac{\Gamma_0}{(\omega - \omega_L)^2 + \Gamma_0^2} \qquad (29)$$

$$+ \frac{1}{2} \sin^2\varphi \cos^2\varphi \, Ie \, \delta(\omega - \omega_L) .$$

As for the case when the thermal reservoir is in the vacuum state[8] , the steady-state spectrum (29) contains three spectral lines centered at frequencies $\omega = \omega_L ; \omega_L \pm 2\Omega$. In the off-resonance case $\cos^2\varphi \neq \frac{1}{2}$ the central line contains the elastic component with the intensity I_e being proportional to N^2 (which vanishes for the exact resonance case) and the Lorentzian shaped component with the linewidth Γ_0 and intensity $\sin^2\varphi\cos^2\varphi \, (\langle R_3^2\rangle_s - Ie)$. The two sidebands are Lorentzians of the linewidth Γ_1 centered at frequencies $\omega = \omega_L - 2\Omega$ and $\omega = \omega_L + 2\Omega$, and having the intensities which are proportional to $\sin^4\varphi \langle R_{12} R_{21}\rangle_s$ and $\cos^4\varphi \langle R_{21} R_{12} \rangle_s$ respectively.

As is seen from the relations (26),(27), in the general case the linewidths of the three inelastic components are expanded due to the presence of the thermal field. The expansion of the linewidths Γ_0 , Γ_1 are sufficiently large for the case of $\bar{n} \gg 1$ and exact resonance when the collective part $[\frac{\gamma}{2}(\sin^2\varphi - \cos^2\varphi)\langle R_3 \rangle_s]$ of the linewidths vanishes. For the

off-resonance case and $N \gg 1$ from relation for $\langle R_3 \rangle_s$ given in Appendix one finds

$$\frac{1}{2} \gamma (\sin^2\varphi - \cos^2\varphi) \langle R_3 \rangle_s \approx \frac{\gamma}{2} N \, |\sin^2\varphi - \cos^2\varphi| \qquad (30)$$

$$\text{if} \quad \cos^2 \neq \frac{1}{2} .$$

Thus, for the case of $N \gg 1$ and $\cos^2\varphi \neq \frac{1}{2}$ the collective part of the spectrum linewidths is independent of $\bar{n}$ and dominantes over the other one-atom parts of the linewidths. Consequently, in this case the spectrum linewidths are approximately independent of the thermal field intensity $\bar{n}$.

In contrast with the spectrum line widths, the intensities of the spectrum components are independent of $\bar{n}$ for the exact resonance case. In this case, the elastic component vanishes and intensities of the three inelastic components have the superradiant behaviour ($\sim N^2$). For the off-resonance case $\cos^2\varphi \neq \frac{1}{2}$ the intensities of the spectrum components strongly depend on the thermal field intensity. The dependence of the intensities of the spectrum components on the thermal reservoir is shown in figs. 1,2 where the relative intensities of the sidebands (fig. 1a,b), i.e., the quantities $I_{-1}/N = \cos^4\varphi \, \langle R_{21} R_{12} \rangle / N$ (solid curves) and $I_{+1}/N = \cos^4\varphi \, \langle R_{21} R_{12} \rangle / N$ (dashed curves) and of the central line containing the elastic and inelastic components (fig. 2a,b), i.e., the quantities $I_0/N^2 = \sin^2\varphi\cos^2\varphi \, \langle R_3^2 \rangle_s / N^2$ are plotted as functions of the parameters $\cos^2\varphi$ for various values of N and $\bar{n}$. As is shown in fig. 1a,b, in the case of $\bar{n}$ = 0 (the curves ①) the intensities of the two sidebands are equal and spectrum is symmetric. In the case of $\bar{n} \neq 0$ (the curves ②) the intensities of two sidebands are quite different for the off-resonance case $\cos^2\varphi \neq \frac{1}{2}$ thus spectrum asymmetry is substantial. Analogous conclusions for the resonance fluorescence from

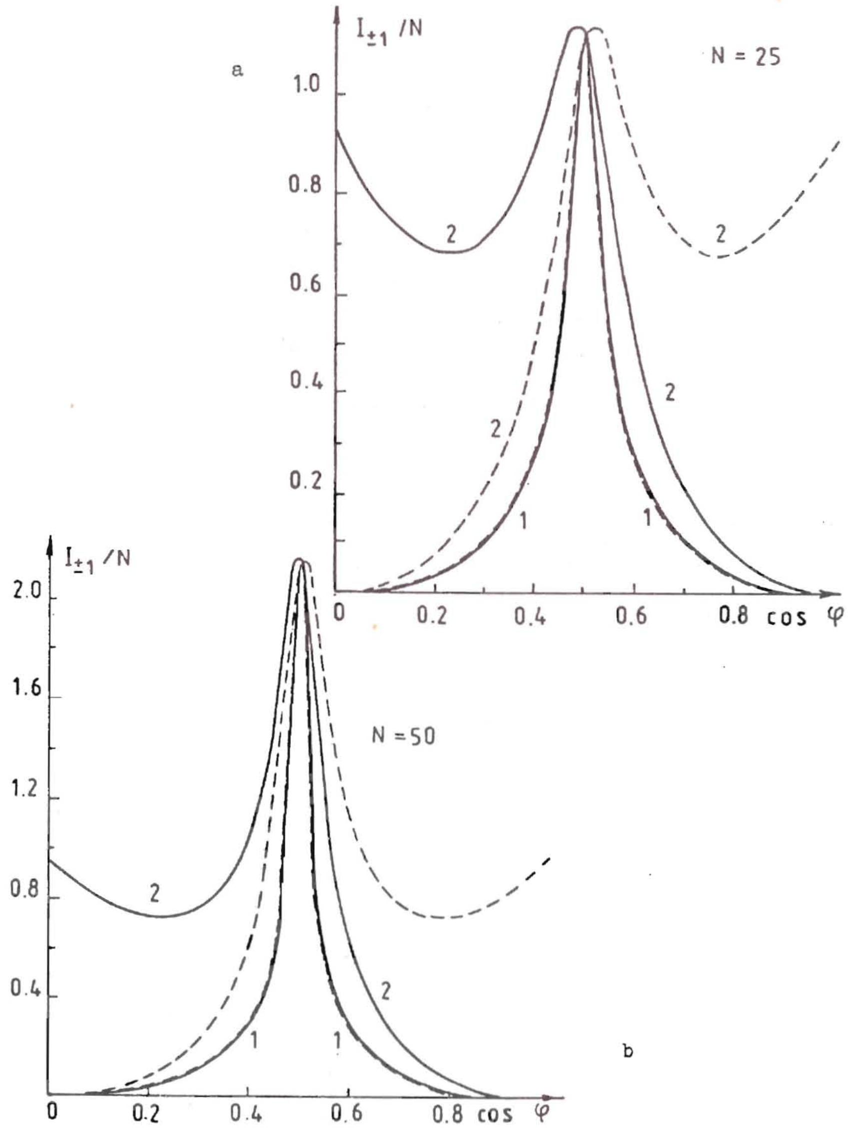


Fig. 1. The quantities I_{+1}/N (dashed curves) and I_{-1}/N (solid curves) as functions of $\cos^2\varphi$ for N = 25 (fig. 1a) and N = 50 (fig. 1b). The curves ①-② correspond to $\bar{n}$ = 0; 1.0, respectively.

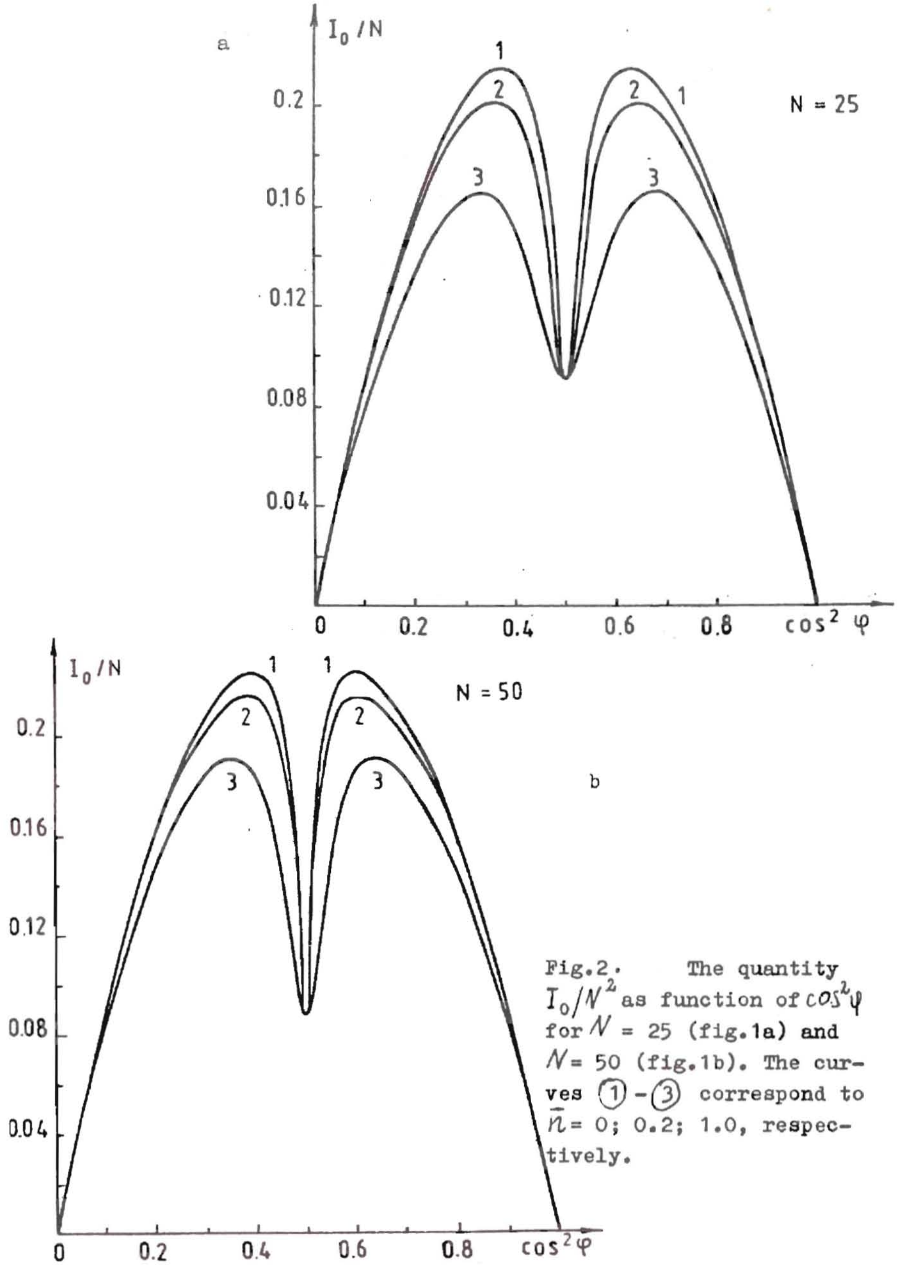


Fig.2. The quantity I_0/N^2 as function of $\cos^2\varphi$ for N = 25 (fig.1a) and N= 50 (fig.1b). The curves ①-③ correspond to $\bar{n}$= 0; 0.2; 1.0, respectively.

a single atom in the presence of thermal reservoir has been made in the work[6] by Hildred et al. Using the relations for $\langle R_{21} R_{12} \rangle_s$, $\langle R_{12} R_{21} \rangle_s$ and $\langle R_3^2 \rangle_s$ given in Appendix, one can show that for a large number of atoms $N \gg 1$ and off-resonance case the intensities of the spectrum components are strongly dependent on the thermal field intensity $\bar{n}$ and the fluorescence spectrum is quite asymmetric while the spectrum linewidths are independent of $\bar{n}$.

IV. STATISTICAL PROPERTIES

In the recent works[9,10,13] , the statistical properties of the collective resonance fluorescence in the case when the thermal reservoir is in vacuum state have been investigated. The anticorrelation between sidebands and central spectrum components[9] and the nonclassical correlation (violation of the Cauchy - Schwarz inequality) between two sidebands[13] have been predicted.

In this section, we discuss influence of the thermal reservoir on the statistical properties of the fluorescence field. We consider independence of the photon statistics of the spectrum component and nonclassical correlations between them on the thermal field intensity $\bar{n}$.

It is easy to see from the previous section and relation (16) that the operators $-\sin^2\varphi R_{12}$; $\sin\varphi \cos\varphi R_3$ and $\cos^2\varphi R_{21}$ can be considered as operator-sources of the spectrum components of the fluorescence field at frequencies $\omega_L - 2\Omega$, ω_L and $\omega_L + 2\Omega$, and for simplicity these operators will be denoted by S_1^+ , S_0^+ and S_1^+ , respectively.

As Loudon[17] , we define dht degree of second-order coherence between the spectrum components S_i and S_j in the form

$$G_{i,j}^{(2)} = \frac{\langle S_i^+ S_j^+ S_j S_i \rangle_s}{\langle S_i^+ S_i \rangle_s \langle S_j^+ S_j \rangle_s} \qquad (i,j = 0, \pm 1) \quad . \tag{31}$$

Since the operator S_i does not commute with the operator S_j in the general case, we have

$$G_{i,j}^{(2)} \neq G_{j,i}^{(2)} \qquad (i \neq j) \quad .$$

The correlation functions $G_{i,i}^{(2)}$ (i = 0 ± 1) describe the photon statistics of the spectrum components S_i and the correlation functions $G_{i,j}^{(2)}$ describe the cross-correlation between the spectrum components S_i and S_j .

By using the steady state density matrix (11) and commutation relation (9),(10) one finds the quantities $G_{i,i}^{(2)}$ (i =0,±1) in the form

$$G_{0,0}^{(2)} = \langle R_3^4 \rangle_s / \langle R_3^2 \rangle_s \quad , \tag{32}$$

$$G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = \langle R_{12} R_{12} R_{21} R_{21} \rangle_s / (\langle R_{12} R_{21} \rangle_s)^2 \quad , \tag{33}$$

where the values $\langle R_3^2 \rangle_s$, $\langle R_3^4 \rangle_s$, $\langle R_{12} R_{12} R_{21} R_{21} \rangle_s$, $\langle R_{12} R_{21} \rangle_s$ are given in Appendix .

For the one-atom case, by using the operator relation (20) one can show

$$G_{0,0}^{(2)} = \langle R_{11} + R_{22} \rangle_s / (\langle R_{11} + R_{22} \rangle_s)^2 = 1 \tag{34}$$

$$G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = 0 \tag{35}$$

thus the photon statistics of the central component remains Poissonian; the sidebands have sub-poissonian statistics and photon statistics of all three Mollow's spectrum components are independent of the thermal field intensity $\bar{n}$.

Contrary to the one-atom case the photon statistics is strongly dependent on the thermal field intensity $\bar{n}$; as is shown in figs. 3,4 where the correlation functions $G^{(2)}_{0,0}$, $G^{(2)}_{\pm1,\pm1}$ are plotted as functions of the parameter $\cos^2\varphi$ for fixed N =50 and for various values of $\bar{n}$. It is clear from figs. 3,4 that except for the point of exact resonance $\cos^2\varphi = \frac{1}{2}$, the thermal field intensity $\bar{n}$ plays an important role in the photon statistics for Mollow's triplet of the collective resonance fluorescence. By an analogous approach one finds the cross-correlation functions $G^{(2)}_{i,j}$ $(i \neq j)$ for the single-atom case in the following form

$$G^{(2)}_{0,1} = G^{(2)}_{1,0} = G^{(2)}_{0,-1} = G^{(2)}_{-1,0} = 1 , \qquad (36)$$

$$G^{(2)}_{1,-1} = X+1 > 1 , \qquad (37)$$

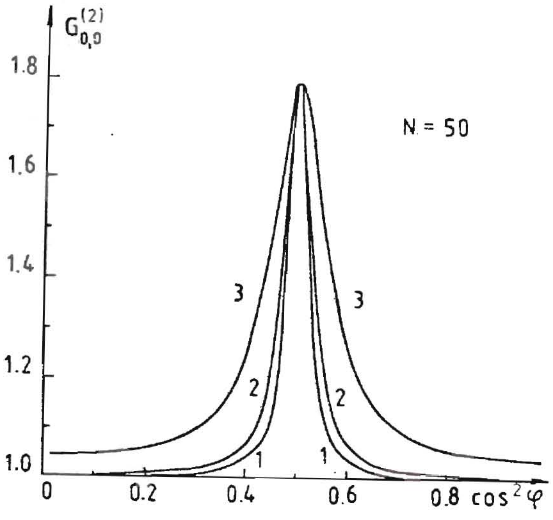
$$G^{(2)}_{-1,1} = 1+\frac{1}{X} > 1 . \qquad (38)$$

Thus, for the one-atom case the central component S_0 remains uncorrelated with the sidebands $S_{\pm1}$ in spite of the presence of the thermal field while the cross-correlation between the sidebands is directly dependent on the thermal field intensity $\bar{n}$.

For the collective case $N > 1$, by using the steady-state density matrix (11) one can show that except for the point of exact resonance $\cos^2\varphi = 1/2$, the cross-correlation function $G^{(2)}_{i,j}$ ($i \neq j$) is strongly dependent on the thermal field intensity.

Further, we fix our attention on the correlation between the sidebands and discuss the influence of the thermal field on the nonclassical effect-the violation of the Cauchy - Schwarz (C-S) inequality [17-22] that has been predicted for the collective resonance fluorescence in the work [13] .

Fig. 3. The quantity $G^{(2)}_{0,0}$ as function of $\cos^2\varphi$ for the case N = 50. The curves ①-③ correspond to $\bar{n}$ = 0; 0.2; 1.0, respectively.



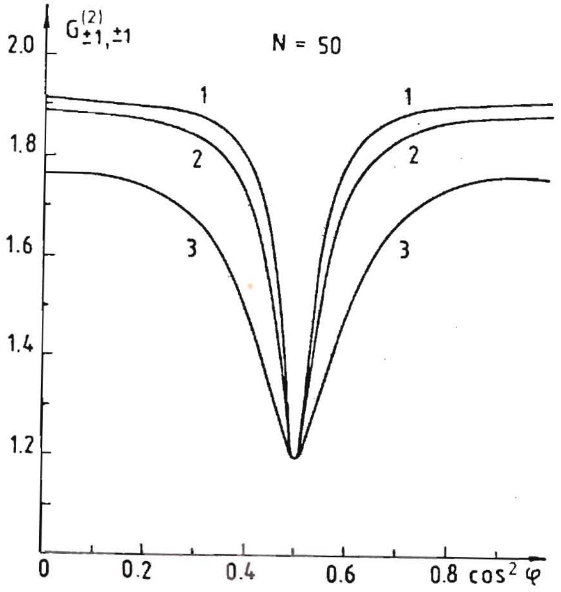


Fig. 4. The quantity $G^{(2)}_{\pm1,\pm1}$ as function of $\cos^2\varphi$ for the case N = 50. The curves ①-③ correspond to $\bar{n}$ = 0; 0.2; 1.0, respectively.

We speak that the violation of the Cauchy - Schwarz inequality holds for the correlation between the spectral components S_i and S_j if the following condition[13] is satisfied:

$$K_{i,j} = (G^{(2)}_{i,i} \cdot G^{(2)}_{j,j}) / (G^{(2)}_{i,j})^2 < 1 \quad . \qquad (39)$$

The factor $K_{i,j}$ describes the degree of the violation of the Cauchy - Schwarz inequality.

One shows by the relations (34)-(38) that $K_{i,j} = 0$ $(i \neq j \; ; \; i,j = 0, \pm 1)$ for the single-atom case. It means that the C-S inequality is violated for any two spectrum components from Mollow's triplet as in the case when the thermal field intensity $\bar{n}$ is equal to zero.

For the collective case ($N \geqslant 2$), as for the case of $\bar{n} = 0$[13] , the violation of the C-S inequality is presented only for two sidebands of the fluorescent spectrum while the factors $K_{1,-1}$ and $K_{-1,1}$, describing the degree of the violation of the C-S inequality between the sidebands $S_{\pm 1}$, is dependent on the thermal field intensity $\bar{n}$. One can write the factors $K_{1,-1}$ and $K_{-1,1}$ in the following form:

$$K_{1,-1} = \frac{\langle R_{21} R_{21} R_{12} R_{12} \rangle_s \langle R_{12} R_{12} R_{21} R_{21} \rangle_s}{(\langle R_{21} R_{12} R_{21} R_{12} \rangle_s)^2} \, , \qquad (40)$$

$$K_{-1,1} = \frac{\langle R_{21} R_{21} R_{12} R_{12} \rangle_s \langle R_{12} R_{12} R_{21} R_{21} \rangle_s}{(\langle R_{12} R_{21} R_{12} R_{21} \rangle_s)^2} \, , \qquad (41)$$

where the expectation values $\langle R_{21} R_{21} R_{12} R_{12} \rangle_s , \langle R_{21} R_{12} R_{21} R_{12} \rangle_s$ $\langle R_{12} R_{12} R_{21} R_{21} \rangle_s$ and $\langle R_{12} R_{21} R_{12} R_{21} \rangle_s$ can be found in Appen-

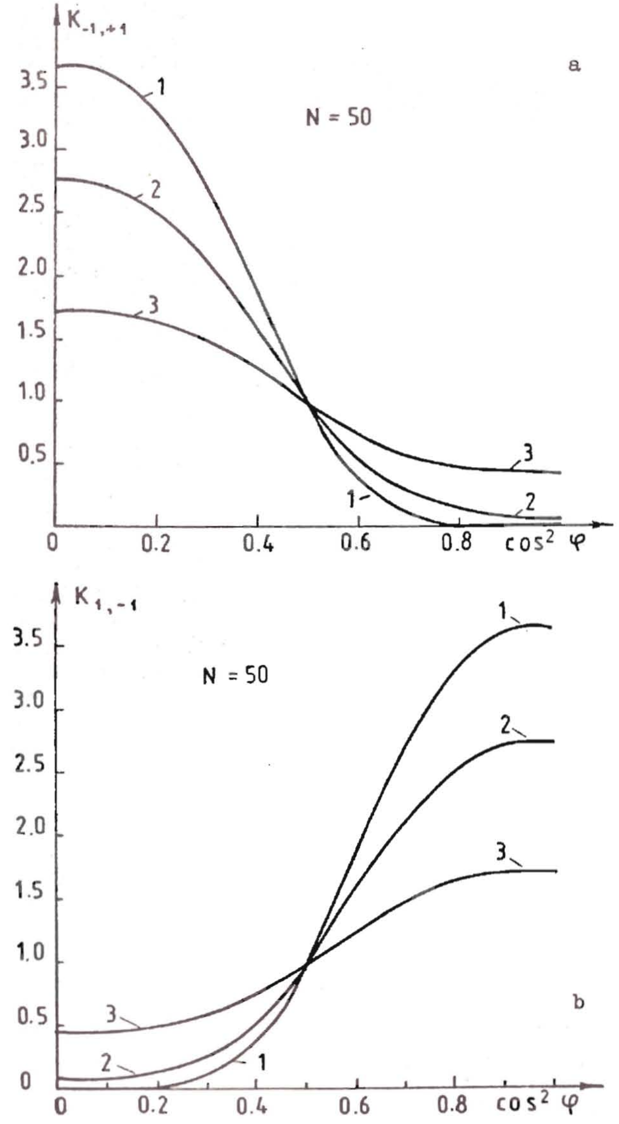


Fig. 5. The quantities $K_{-1,+1}$ and $K_{1,-1}$ as functions of $\cos^2\varphi$ for N = 50. The curves ①-③ correspond $\bar{n}$ = 0; 0.2; 1.0, respectively.

dix. The behaviour of the factors $K_{-1,1}$ and $K_{1,-1}$ as functions against the parameter $\cos^2\varphi$ for $N = 50$ and various values of $\bar{n}$ is plotted in figs. 5a,b, respectively. As is seen from fig. 5, the thermal reservoir reduces the factors $K_{-1,1}$ and $K_{1,-1}$ describing the degree of violation of the C-S inequality.

V. CONCLUSIONS

We have considered the problem of collective resonance fluorescence of the small system of N two-level atoms driven by an intense coherent driving field in the presence of an additional incoherent field. In the case of an intense external field the stationary solution for the density operator of the atomic system is given. We have shown the expansion of linewidths and asymmetry of the collective resonance spectrum caused by the thermal field. It has been shown that except for the point of exact resonance $\cos^2\varphi = \frac{1}{2}$ the thermal field intensity $\bar{n}$ plays an important role in determining the photon statistics of the spectrum component, cross-correlation between them. The thermal reservoir reduces the degree of the violation of the C-S inequality for the correlations between the sidebands.

APPENDIX

In this Appendix we give the explicit expressions for the steady-state averages of the atomic operators that can be calculated with the use of the density matrix (11)

$$\langle R_{11}\rangle_s = Z^{-1}[NX^{N+2} - (N+1)X^{N+1} + X]/(X-1)^2 \tag{A.1}$$

$$\begin{aligned}\langle R_{11}^2\rangle_s = Z^{-1}[&N^2X^{N+3} - (2N^2+2N-1)X^{N+2}\\ &+ (N+1)^2X^{N+1} - X^2 - X]/(X-1)^3\end{aligned} \tag{A.2}$$

$$\begin{aligned}\langle R_{11}^3\rangle_s = Z^{-1}[&N^3X^{N+4}(3N^3+3N^2-3N+1)X^{N+3}\\ &+ (3N^2+6N-4)X^{N+2} - (N^3+3N^2+3N+1)X^{N+1}\\ &+ X^3+4X^2+X]/(X-1)^4\end{aligned} \tag{A.3}$$

$$\begin{aligned}\langle R_{11}^4\rangle_s = Z^{-1}[&N^4X^{N+5} - (4N^4+4N^3-6N^2+6N-1)X^{N+4}\\ &+ (6N^4+12N^3-6N^2-12N+1)X^{N+3}\\ &- (4N^4+12N^3+6N^2-12N-11)X^{N+2} + (N^4+4N^3\\ &+ 6N^2+4N+1)X^{N+1} - X^4 - 11X^3 - 11X^2 - X]/(X-1)^5\end{aligned} \tag{A.4}$$

$$\langle R_3\rangle_s = N - 2\langle R_{11}\rangle_s \tag{A.5}$$

$$\langle R_3^2\rangle_s = 4\langle R_{11}^2\rangle_s - 4N\langle R_{11}\rangle_s + N^2 \tag{A.6}$$

$$\langle R_{12}R_{21}\rangle_s = -\langle R_{11}^2\rangle_s + (N+1)\langle R_{11}\rangle_s \tag{A.7}$$

$$\langle R_{21}R_{12}\rangle_s = -\langle R_{11}^2\rangle_s + (N-1)\langle R_{11}\rangle_s + N \tag{A.8}$$

$$\begin{aligned}\langle R_3^4\rangle_s = {}&16\langle R_{11}^4\rangle_s - 32N\langle R_{11}^3\rangle_s + 24N^2\langle R_{11}^2\rangle_s\\ &- 8N^3\langle R_{11}\rangle_s + N^4\end{aligned} \tag{A.9}$$

$$\begin{aligned}\langle R_{12}R_{12}R_{21}R_{21}\rangle_s = {}&\langle R_{11}^4\rangle_s - 2\cdot(N+2)\langle R_{11}^3\rangle_s\\ &+ (N^2+5N+5)\langle R_{11}^2\rangle_s - (N^2+3N+2)\langle R_{11}\rangle_s\end{aligned} \tag{A.10}$$

$$\begin{aligned}\langle R_{21}R_{21}R_{12}R_{12}\rangle_s = {}&\langle R_{11}^4\rangle_s - (2N-4)\langle R_{11}^3\rangle_s\\ &+ (N^2-7N+5)\langle R_{11}^2\rangle_s + (3N^2-7N+2)\langle R_{11}\rangle_s\\ &+ 2N^2 - 2N\end{aligned} \tag{A.11}$$

$$\begin{aligned}\langle R_{21}R_{12}R_{21}R_{12}\rangle_s = {}&\langle R_{11}^4\rangle_s - (2N-2)\langle R_{11}^3\rangle_s\\ &+ (N^2-4N+1)\langle R_{11}^2\rangle_s + (2N^2-2N)\langle R_{11}\rangle_s + N^2\end{aligned} \tag{A.12}$$

$$\begin{aligned}\langle R_{12}R_{21}R_{12}R_{21}\rangle_s = {}&\langle R_{11}^4\rangle_s - 2(N+1)\langle R_{11}^3\rangle_s\\ &+ (N+1)^2\langle R_{11}^2\rangle_s\end{aligned} \tag{A.13}$$

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Ле Хонг Лан, А.С.Шумовский, Чан Куанг E17-88-753
Коллективная резонансная флуоресценция
в присутствии теплового термостата

Обсуждены спектральные и статистические свойства коллективной резонансной флуоресценции в присутствии теплового термостата. Показано влияние интенсивности теплового поля на расширение спектральных ширин и на асимметрию спектра. Изучены фотонная статистика спектральных компонент, кросс-корреляция между ними и нарушение неравенства Коши - Шварца /КШ/ в зависимости от интенсивности теплового поля.

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Le Hong Lan, Shumovsky A.S., Tran Quang E17-88-753
Collective Resonance Fluorescence in the
Presence of a Thermal Reservoir

Spectral and statistical properties of the collective resonance fluorescence in the presence of a thermal reservoir are discussed. Influence of the thermal field intensity on expansion of the spectrum linewidths and on symmetry of the spectrum is shown. Photon statistics of the spectrum components, cross-correlation between them and violation of the Cauchy - Schwarz (C S) inequality in the dependence of the thermal field intensity are investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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