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**LIGHT SQUEEZING  
IN TAVIS - CUMMINGS MODEL**

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For the last years, a large number of theoretical [1-3 and refs. therein] and experimental works [4-7] is concentrated on the problem of generation of squeezed light. Potential applications of the squeezed light in the detection of gravity waves is widely discussed [2,8-9].

The squeezing in the basic model of quantum optics - the Jaynes-Cummings-type model - has been considered in the papers [10-15], where the magnitude of squeezing is rather small.

In this letter we discuss the squeezing in the spectrum components of fluorescence from N two-level atoms interacting with a strong cavity mode. We show that in contrast with the collective resonance fluorescence in a free space, where the squeezing is absent in separate spectrum component [16], for the proper parameters of the system the squeezing is present only in one separate sideband. The conditions for receiving the large degree of the squeezing have been discussed.

We consider a system of N two-level atoms interacting with a single intense cavity mode at frequency  $\omega_L$  (Tavis-Cummings model). In the case when the cavity mode is intense so that it can be treated classically, the Hamiltonian of the system in the interaction representation has the following form [17]:

$$H = \frac{\delta}{2} (J_{2L} - J_{1L}) + G (J_{12} + J_{21}), \quad (1)$$

where  $\delta = \omega_{21} - \omega_L$  is the detuning of the cavity mode  $\omega_L$  from the atomic resonance frequency  $\omega_{21}$ ;  $G = -d_{21} E_L$  is the resonance Rabi frequency describing the interaction of the cavity mode with the atomic system;  $J_{kl}$  ( $k, l = 1, 2$ ) are the collective angular momenta of the atomic system having in the Schwinger representation the form  $J_{kl} = a_k^\dagger a_l$  ( $k, l = 1, 2$ ),

where the operators  $a_k$  and  $a_k^\dagger$  obey the boson commutation relation

$$[a_k, a_l^\dagger] = \delta_{kl}$$

and can be treated as annihilation and creation operators for the atoms being populated in the level  $|k\rangle$ .

We make the canonical transformation permitting us to move from trace two-level atoms to atoms "dressed" by the external field

$$\begin{aligned} a_1 &= \cos \varphi Q_1 + \sin \varphi Q_2, \\ a_2 &= -\sin \varphi Q_1 + \cos \varphi Q_2, \end{aligned} \quad (2)$$

where  $\operatorname{tg} 2\varphi = 2G/\delta$  and the operators  $Q_k$  and  $Q_k^\dagger$  satisfy the boson commutation relation

$$[Q_k, Q_l^\dagger] = \delta_{kl} \quad (k, l = 1, 2).$$

After the canonical transformation (2), the Hamiltonian (1) becomes

$$H = \Omega (Q_2^\dagger Q_2 - Q_1^\dagger Q_1), \quad (3)$$

where  $\Omega = (\frac{1}{4} \delta^2 + G^2)^{1/2}$ .

Using the Hamiltonian (3) one can find the explicit form for the operators  $Q_1(t)$ ,  $Q_2(t)$  and write the collective angular momenta of atoms in the form

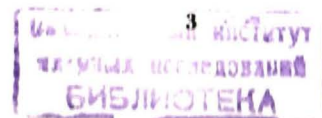
$$J_{21}(t) = S_0 + S_{21}^+ e^{2i\Omega t} + S_{21}^- e^{-2i\Omega t}, \quad (4)$$

$$J_{12}(t) = [J_{21}(t)]^\dagger, \quad (5)$$

where

$$S_0 = \sin \varphi \cos \varphi [\cos 2\varphi J_3 + \sin 2\varphi (J_{12} + J_{21})], \quad (6)$$

$$S_{11}^+ = \cos^2 \varphi [\cos^2 \varphi J_{21} - \sin^2 \varphi J_{12} - \frac{1}{2} \sin 2\varphi J_3], \quad (7)$$



$$S_{-1}^+ = \sin^2 \varphi \left[ \sin^2 \varphi J_{21} - \cos^2 \varphi J_{12} + \frac{1}{2} \sin 2\varphi J_3 \right], \quad (8)$$

$$J_3 = J_{22} - J_{11}. \quad (9)$$

Following refs. [18] we define the transient spectrum of the fluorescence into other modes as

$$S_F(\nu, T) = 2T \int_0^T dt_1 \int_0^T dt_2 \exp[-(\Gamma - i\nu)(T - t_1) - (\Gamma + i\nu)(T - t_2)] \\ \langle J_{21}(t_1) J_{12}(t_2) \rangle \exp[-i\omega_L(t_2 - t_1)], \quad (10)$$

where  $\Gamma$  is the bandwidth of the detector,  $T$  is the time at which the spectrum is evaluated. The  $\langle \dots \rangle$  indicates the average over the initial state of the atomic system.

By using the definition of fluorescence spectrum (10) and relations (4)-(5) one shows that for the case of intense cavity mode the members in spectrum (8), which are proportional to  $\frac{1}{\Omega}$ , are ignored and the operators  $S_{-1}$ ,  $S_0$  and  $S_{+1}$  can be considered as the operator-sources of the spectrum components at frequencies  $\omega_L - 2\Omega$ ,  $\omega_L$  and  $\omega_L + 2\Omega$ , respectively [19].

Further, we discuss the squeezing in the spectrum components  $S_0$ ,  $S_{\pm 1}$ . As is easily seen from the relation (8), the operator  $S_0$  is the Hermitian operator  $S_0^+ = S_0$  and squeezing is absent for the central spectrum component  $S_0$ .

The squeezing properties of the two sidebands  $S_{-1}$  and  $S_{+1}$  may be studied by introducing the Hermitian operators

$$S_{-1} = L_1 + iL_2, \quad (11)$$

$$S_{+1} = R_1 + iR_2. \quad (12)$$

We speak of squeezing in the spectrum components  $S_{-1}$  if the normally-ordered variance of the operators  $L_1$  or  $L_2$  is less than zero

$$\langle :(\Delta L_1)^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta L_2)^2: \rangle < 0 \quad (13)$$

and in the spectrum component  $S_{+1}$  if the normally-ordered variance of the operators  $R_1$  or  $R_2$  is less than zero

$$\langle :(\Delta R_1)^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta R_2)^2: \rangle < 0. \quad (14)$$

Let the atoms be initially in the fully symmetric collective state

$$\rho_a = |n_1, n_2\rangle \langle n_2, n_1|, \quad (15)$$

where  $n_2$  and  $n_1$  are the population of the atoms in excited and ground states, respectively.

By using expressions (7)-(8) for the atomic operators  $S_{+1}$  and  $S_{-1}$ , one finds

$$\langle :(\Delta L_1)^2: \rangle = \frac{1}{4} \sin^4 \varphi \left[ \frac{1}{2} N^2 - \frac{1}{2} D^2 + N_- D \cos 2\varphi \right] \geq 0, \quad (16)$$

$$\langle :(\Delta L_2)^2: \rangle = -\frac{1}{8} \sin^4 \varphi \cos^2 2\varphi (D^2 + 2D \cos^{-1} 2\varphi - N^2 - 2N), \quad (17)$$

$$\langle :(\Delta R_1)^2: \rangle = \frac{1}{4} \cos^4 \varphi \left[ \frac{1}{2} (N^2 - D^2) + N + D \cos 2\varphi \right] \geq 0, \quad (18)$$

$$\langle :(\Delta R_2)^2: \rangle = -\frac{1}{8} \cos^4 \varphi \cos^2 2\varphi (D^2 - 2D \cos^{-1} 2\varphi - N^2 - 2N), \quad (19)$$

where  $D = n_2 - n_1$  is the initial population inversion of the atomic system. From the relations (16)-(19) one sees that the squeezing may be present only in out-of-phase components  $L_2$  and  $R_2$ . The variances  $\langle :(\Delta L_2)^2: \rangle$  and  $\langle :(\Delta R_2)^2: \rangle$  are dependent on the system parameters  $N, D$  and  $\cot g^2 \varphi$ . For the case of the exact resonance  $\cot g^2 \varphi = 1$  and for the case when the initial

atomic population inversion is equal to zero ( $D=0$ ), as is seen from the relations (16)-(19) the squeezing is absent for all spectrum components of the fluorescence field.

For the case when all the atoms are initially in the ground state, i.e.  $D=-N$ , the relations (17) and (19) reduce to

$$\langle (\Delta L_z)^2 \rangle = + \frac{N}{4} \sin^4 \varphi \cos 2\varphi (1 + \cos 2\varphi) \quad , \quad (20)$$

$$\langle (\Delta R_z)^2 \rangle = - \frac{N}{4} \cos^4 \varphi \cos 2\varphi (1 - \cos 2\varphi) \quad . \quad (21)$$

Thus, the squeezing is present in the left sideband  $S_-$ , if  $0 < \text{ctg}^2 \varphi < 1$  (i.e.  $\delta > 0$ ) and it is present in the right sideband  $S_+$ , if  $\text{ctg}^2 \varphi > 1$  (i.e.  $\delta < 0$ ). In this case the variances  $\langle (\Delta L_z)^2 \rangle$  and  $\langle (\Delta R_z)^2 \rangle$  are proportional to  $N$ .

In the general case the conditions of the squeezing in the left sideband  $S_-$ , can be found from the relation (17) in the following form:

$$(i) \quad \langle (\Delta L_z)^2 \rangle < 0$$

$$\text{if: } \text{ctg}^2 \varphi > 1 \quad \text{and } D > -\cos^{-1} 2\varphi + D' \quad (22)$$

or

$$0 < \text{ctg}^2 \varphi < 1 \quad \text{and } D < -\cos^{-1} 2\varphi - D' \quad (23)$$

$$\text{where } D' = (\cos^{-2} 2\varphi + N^2 + 2N)^{1/2}.$$

(24)

Analogously, the conditions for the squeezing in the right sideband  $S_+$ , can be found from relation (19) in the form

$$(ii) \quad \langle (\Delta R_z)^2 \rangle < 0$$

$$\text{if } \text{ctg}^2 \varphi > 1 \quad \text{and } D < \cos^{-1} 2\varphi - D' \quad (25)$$

or

$$0 < \text{ctg}^2 \varphi < 1 \quad \text{and } D > \cos^{-1} 2\varphi + D' \quad , \quad (26)$$

where  $D$  is found from eq. (24).

Comparing relations (22) with (25) and relations (23) with (26) one shows that for the proper system parameters  $\text{ctg}^2 \varphi$  and  $D$  the squeezing is present only in one of the sidebands but not simultaneously in the two sidebands. These results are different from the steady-state collective resonance fluorescence in an intense laser field and in a free space [16] where the squeezing is absent in all the three spectrum components taken separately and it is present only in the mixture of the two sidebands as a result of the quantum correlation between them. In the other words, in the fluorescence field in the Tavis-Cummings model the one-mode squeezing is presented while in the steady-state collective resonance fluorescence in a free space only the two-mode squeezing is presented [16].

The detailed behaviour of the relative variances  $V_L = \langle (\Delta L_z)^2 \rangle / N$  (solid curves) and  $V_R = \langle (\Delta R_z)^2 \rangle / N$  (dashed curves) as functions against the parameters  $\text{ctg}^2 \varphi$  for  $N=10, D=10$  (curves ①) and  $D=-8$  (curves ②) is plotted in the figure. As is seen from the figure the squeezing is most large if all the atoms are initially in the ground (or excited) state. The reduction of the value  $|0\rangle$  entails the reduction of the squeezing.

Finally, we discuss the factors of squeezing for the atomic operators  $L_z$  and  $R_z$  which have the following form [16]

$$F_L = \frac{\langle (\Delta L_z)^2 \rangle}{\frac{1}{2} |\langle [L_1, L_2] \rangle|} \quad (27)$$

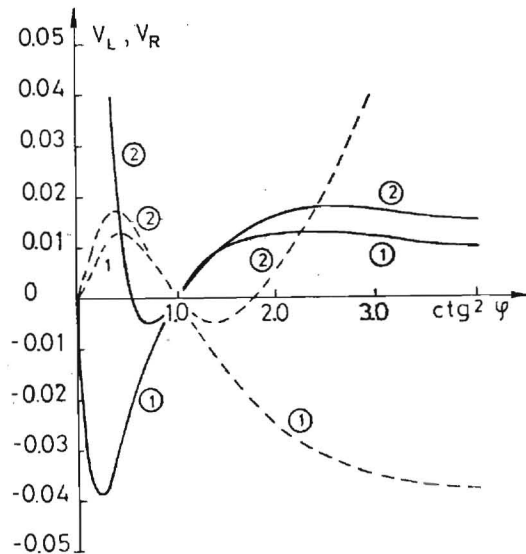
$$F_R = \frac{\langle (\Delta R_z)^2 \rangle}{\frac{1}{2} |\langle [R_1, R_2] \rangle|} \quad (28)$$

where

$$\frac{1}{2} |\langle [L_1, L_2] \rangle| = \frac{1}{4} \sin^4 \varphi |D| \cdot |\cos 2\varphi| \quad (29)$$

$$\frac{1}{2} |\langle [R_1, R_2] \rangle| = \frac{1}{4} \cos^4 \varphi |D| \cdot |\cos 2\varphi| \quad (30)$$





The values  $V_L = \langle : (\Delta L_2)^2 : \rangle / N$   
 (solid curves) and  $V_R = \langle : (\Delta R_2)^2 : \rangle / N$   
 (dashed curves) as functions against the parameter  $\text{ctg}^2 \varphi$   
 for  $N=10$ ,  $D=-10$  (curves ①) and  $D=-8$  (curves ②).

For the case  $D=-N$ , using the relations (20)-(21) one finds

$$F_L = \begin{cases} 1 + \cos 2\varphi & \text{if } \text{ctg}^2 \varphi > 1 \\ -(1 + \cos 2\varphi) & \text{if } 0 < \text{ctg}^2 \varphi < 1 \end{cases} \quad (31)$$

$$F_R = \begin{cases} -(1 - \cos 2\varphi) & \text{if } \text{ctg}^2 \varphi > 1 \\ 1 - \cos 2\varphi & \text{if } 0 < \text{ctg}^2 \varphi < 1 \end{cases} \quad (32)$$

Thus, the factor of squeezing  $F_L$  can tend to the limited value  $F_L = -1$  (i.e. the 100% of squeezing) in the small region  $\text{ctg}^2 \varphi \lesssim 1$

and the factor of squeezing  $F_R$  can tend to the limited value  $F_R = -1$  in the small region  $\text{ctg}^2 \varphi \gtrsim 1$ . We note that in the case of exact resonance  $\text{ctg}^2 \varphi = 1$  one shows  $\langle [L_1, L_2] \rangle = 0$ ,  $\langle [R_1, R_2] \rangle = 0$  and in this case the concept of squeezing for the commuting operators  $L_1, L_2$  and  $R_1, R_2$  loses sense. One can show that though the normally-ordered variances

$\langle : (\Delta L_2)^2 : \rangle$  and  $\langle : (\Delta R_2)^2 : \rangle$  are the normally-ordered variances of the spectrum components of the fluorescence field, the factor  $F_L$  and  $F_R$  for the atomic operators are not coincident with the factors of squeezing for the corresponding field operators due the influence of the free parts of the fluorescence field on the commutators of the field operators. However, for the off-resonance case  $\text{ctg}^2 \varphi \neq 1$  and  $N \gg 1$  this influence is small and the described system can give the fluorescence sidebands with the nearly perfect squeezing. The squeezing for the whole fluorescence field can be investigated analogously and the results are in agreement with the previous work [20,21].

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Рассмотрено сжатие света в спектральных компонентах флуоресценции  $N$  атомов в идеальном резонаторе /модель Тависа - Каммингса/. Показано, что в отличие от коллективной резонансной флуоресценции в свободном пространстве<sup>/16/</sup> для определенного подбора параметров системы сжатие существует только в отдельной крайней компоненте флуоресцентного спектра. Обсуждено условие получения большой степени сжатия.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Shumovsky A.S., Tanas R., Tran Quang  
Light Squeezing in Tavis - Cummings  
Model

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The light squeezing in the fluorescence spectrum components from  $N$  two-level atoms in a cavity (Tavis - Cummings model) is considered. It is shown that in contrast with the collective resonance fluorescence in a space<sup>/16/</sup> for the proper parameters of the system the squeezing is present only in one separate sideband of the fluorescence spectrum. The conditions for receiving the large degree of the squeezing have been discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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