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ISING MODELS AND COUPLED ORDER PARAMETERS

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1. INTRODUCTION

There are several cases in which phase transitions are described by coupled Ising models. For example, in molecular crystals the orderings are of two types: translational and orientational orderings. They characterize the crystal phase, whereas the rotational ordering is broken in the plastic phase which takes place under some thermodynamic conditions [1]. Recently, the study of these crystals was resumed again [2]. The solid-plastic and the plastic-liquid phase transitions are described by two lattice gas models (Ising models), representing the translational (t_i) and the rotational (r_i) degrees of freedom at any site i with usual couplings in one-constant approximation ($J_{ij} = J$), and a mixed t-r coupling of the type $\Sigma\, t_i r_i$. There are arguments that under some circumstances one of the degrees of freedom, (t_i or r_i), can be considered as frosen variables and this leads to a simple description of the phase transitions in systems as NH_4Cl [2]. In general, the plastic phase, where $t = \langle t_i \rangle \neq 0$ and $r = \langle r_i \rangle = 0$, may disappear [3-5], and then the solid-liquid transition remains to be investigated. This is a case of interest in the present study. In fact, our consideration is somewhat more general. It may include a few other examples.

Several phase transitions in liquid solutions and binary alloys can also be studied by coupled Ising models. In this case the t-r coupling is more general, $\Sigma_{ij}\, t_i r_j$, and obviously, t_i and r_i have another meaning [6]. It is out of question that the investigation of coupled Ising models is of use for other complex systems as, say, some magnets [7,8] and ferroelectrics [9]. Besides, theoretical results from such type of models might be of principal interest for the theory of phase transitions. They are related with results for phenomenological mean-field or fluctuation free energies ("Hamiltonians") with two coupled order parameters ("fields").

In this paper, using mean-field and renormalization-group (RG) theories, we consider the model

$$H/T = -\frac{1}{2} \sum_{i \neq j} \left(\mathcal{J}^{t}_{ij} t_i t_j + \mathcal{J}^{r}_{ij} r_i r_j + 2h_0 t_i r_j \right), \qquad (1)$$

where $i,j = 1,\ldots,N$, N is the number of molecules ("spins") in the lattice, r_i and t_i take values ± 1. The original coupling constants are given by $J_{ij}^{t,r} = \mathcal{J}_{ij}^{t,r}/T$, and $H = h_0/T$. It is assumed here that $\mathcal{J}^{t,r} = \mathcal{J}_{ij}^{t,r}$ are symmetric matrices. In some cases the results will be given in the one-constant approximation ($J_{ij}^{t,r} = J^{t,r}$ for any i,j).

We present in section 2 a mean-field treatment of this model. The fluctuation Hamiltonian and the RG analysis are discussed in section 3. The results are summarized in section 4.

2. MEAN-FIELD TREATMENT

Using the well-known rules[10], the free energy $f = (F/N)$ is obtained in the form

$$f = T\{\frac{a_t}{2}t^2 + \frac{a_r}{2}r^2 + hrt - \ln 2\cosh(a_t t + hr) - \ln 2\cosh(a_r r + ht)\}, \quad (2)$$

where $t = \langle t_i \rangle$, and $r = \langle r_i \rangle$ are the order parameters. In eq.(2), the notations are:

$$h = h_0 \cdot \sum_{j(\neq i)} \cdot 1, \quad (3)$$

and

$$a_{t,r} = \sum_{j(\neq i)} \mathcal{J}_{ij}^{t,r}, \quad (4)$$

where the summation is over the interacting neighbours taken into consideration and the sums are presumed independent on the site i. If the neighbour spins act on the site i in one and the same way, introducing numbers z_h, $z_{t,r}$ of the neighbours for every coupling ($\hat{J}^{t,r}$ and h_0), we have $h = z_h h_0$ and $a_{t,r} = z_{t,r}\mathcal{J}^{t,r} \neq 0$ which is equivalent to the one-constant approximation. For the sake of simplicity we set below $z = z_h = z_{t,r}$.

The order parameters t and r are obtained by the conditions $(\partial f/\partial t) = 0$ and $(\partial f/\partial r) = 0$, which yield

$$a_t t + hr - a_t \cdot \mathrm{th}(a_t t + hr) - h \cdot \mathrm{th}(a_r r + ht) = 0 \quad (5a)$$

and

$$a_r r + ht - a_r \cdot \mathrm{th}(a_r r + ht) - h \cdot \mathrm{th}(a_t t + hr) = 0, \quad (5b)$$

or, after two obvious substractions of these equations,

$$a \cdot [t - \mathrm{th}(a_t t + hr)] = 0 \quad (6a)$$

and

$$a \cdot [r - \mathrm{th}(a_r r + ht)] = 0, \quad (6b)$$

where $a = a(h)$ is given by

$$a = a_t a_r - h^2. \quad (7)$$

The problem is to obtain the possible phases and their domains of stability. The interesting case is $h \neq 0$. If $h = 0$, the eqs. (5) or (6) decouple of two canonical mean-field equations with standard solutions. Further, we discuss the coupled model ($h \neq 0$).

The solution $t = r = 0$ (i.e. the para phase) always exists. One can obtain from eqs. (5) that the only allowed ordered phase is the mixed phase ($t \neq 0$, $r \neq 0$).

The form of eqs.(6) shows two distinct cases.

2.1. $\underline{a = 0}$. There is a degeneration which is clarified from eqs. (5). Replacing $a_r = h^2/a_t$ there we receive two identical equations, i.e. the equation

$$x - a_t \mathrm{th}\, x - h \cdot \mathrm{th}(hx/a_t) = 0 \quad (8)$$

for the (only) order parameter $x = a_t t + hr$. The phases $\pm x$ are physically indistinguishable. The eq.(8) can be analysed by Landau's expansion for $x \ll 1$ and $(hx/a_t) \ll 1$. It is obvious that the phase transition here is of second order for sufficiently small values of h^2. There are values of h and a_t however at which this transition may turn out of first order. The investigation of the last requires an expansion up to order x^2. Instead of these approximate ways of investigation, here one may successfully use numerical calculations.

2.2. $\underline{a \neq 0}$. It is convenient to change the variables to

$$x = a_t t + hr \quad (9a)$$

and

$$y = ht + a_r r. \tag{9b}$$

In terms of x and y, eqs. (5) are

$$x - a_t\,\mathrm{th}\,x - h\cdot\mathrm{th}\,y = 0 \tag{10a}$$

and

$$y - a_r\,\mathrm{th}\,y - h\cdot\mathrm{th}\,x = 0. \tag{10b}$$

After multiplications with h, a_t or a_r and two substractions of these equations each other, the system (10) becomes

$$a_r x - a\cdot\mathrm{th}\,x = hy,$$

and

$$a_t y - a\cdot\mathrm{th}\,y = hx,$$

and then, in an obvious way, one obtains two independent equations for x and y, i.e.

$$x - a_t\,\mathrm{th}\,x - h\cdot\mathrm{th}((a_r x - a\cdot\mathrm{th}\,x)/h) = 0 \tag{11}$$

and

$$y - a_r\,\mathrm{th}\,y - h\cdot\mathrm{th}((a_t y - a\cdot\mathrm{th}\,y)/h) = 0. \tag{12}$$

Note that the inversion $h \to -h$ changes nothing. It is sufficient to analyse one of this equations, for example, eq.(11). For h = 0 eq.(11) describes an ordinary second order phase transition. If $h \neq 0$, analytical calculations can be performed for $x \ll 1$ considering the h-dependent term as a perturbation. Strong sufficient conditions in which this procedure is valid are given by the inequalities $h < x$ and $h < a_t\,\mathrm{th}\,x$. In this domain of small h^2, eq.(11) is solved by iterations, i.e. setting

$$x(h) = x_0 + x(h), \tag{13}$$

where $x_0 = x(0)$. The solution $x_0 = 0$, which is stable for $a_t <$ < 1, is not of interest as it leads to $x(0) = 0$ within the first iteration (see eq.(11)). The solutions $\pm x_0$ ($\neq 0$) are identical and we shall discuss only $x_0 = x_0^+ > 0$; ($a_t > 1$). After the first iteration,

$$x - a_t\,\mathrm{th}\,x = h\cdot\mathrm{th}(hx_0/a_t), \tag{14}$$

using eq.(13), we obtain

$$\delta x = \frac{h\cdot\mathrm{th}(hx_0/a_t)}{q - a_t\,\mathrm{th}'x_0}. \tag{15}$$

From eq.(14) one sees that $(\partial/\partial x)_{x_0}\cdot\mathrm{th}'x_0 = 1 - (x_0^2/a_t^2)$, and hence,

$$x = x_0 + \frac{h\cdot\mathrm{th}(hx_0/a_t)}{1 - a_t + (x_0^2/a_t)},\quad a_t > 1. \tag{16}$$

The result (16) is exact within the framework of our mean field consideration. The iteration correction δx is small, i.e. $\delta x \ll x_0$ if the above-mentioned restrictions for h are fulfilled. In Landau's approximation, $x_0^2 = 3(a_t - 1)/a_t$, and δx is small for $h^2 < a_t(a_t - 1)$. If $h^2 \ll 1$, eqs.(14) and (16) are simplified to

$$x - a\cdot\mathrm{th}\,x = h, \tag{17}$$

and

$$x = x_0\left(1 + \frac{h^2}{a_t(1 - a_t) + x_0^2}\right), \tag{18}$$

where $h_x = (h^2 x_0/a_t)$ plays the formal role of an "external field"/1/. Then one may introduce a "susceptibility" $\chi =$ $= (\partial x/\partial h^2)$ at h = 0 as a measure of the response to the appearance of the "field" h^2, i.e. the coupling. The quantity χ is easily calculated using eqs.(16) and (18). We shall explore this "external-field" analogy using the Landau approximation ($x \ll 1$).

In order to avoid some misunderstandings, note that in eqs. (14)-(16) and (18), x_0 is the exact solution of the canonical mean field equation $x_0 = a_t\cdot\mathrm{th}\,x_0$ and it would not be correct if one replaces into these equalities the approximate value $x_0 \approx 3(a_t - 1)/a_t$, which is valid for $a_t \gtrapprox 1$, i.e. $x_0 \ll 1$.

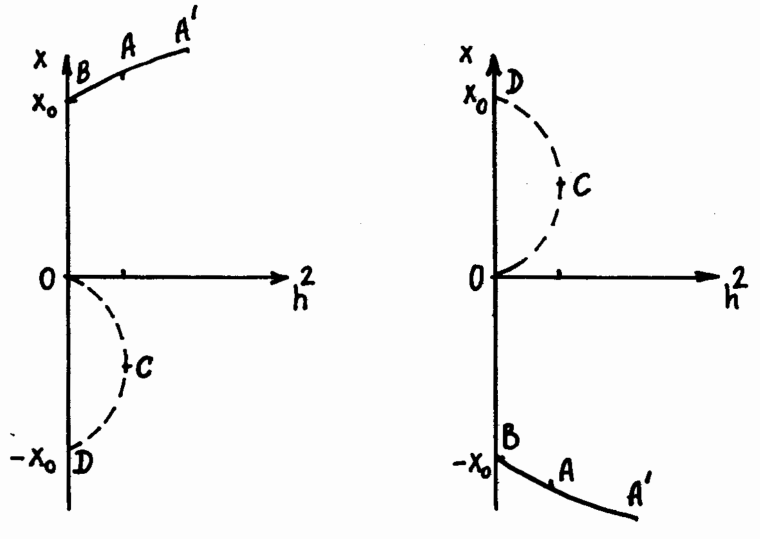
2.2.1. Landau approximation and external-field analogy. In fact we use the conditions $h < x \ll 1$ and $a_t \gtrapprox 1$. Then, having in mind that $x_0 \neq 0$ is given by $x_0^{\pm} = \pm(3(a_t - 1)/a_t)^{1/2}$, we have two equations for x:

$$(1 - a_t)x + \frac{1}{3}a_t x^3 = h_x^+, \tag{19a}$$

and

$$(1 - a_t)x + \frac{1}{3}a_t x^3 = h_x, \tag{19b}$$

where $h_x^{\pm} = (h^2 x_0^{\pm}/a_t)$. These equations give a description of the order parameter x in a full analogy with the case of a ϕ^4-model in an external field /11/. Following ref. /11/ one can analyze the function $h^2(x)$. This function has extrema at $x'_{\pm} = \pm x_0/\sqrt{3}$. There are however some differences as, here, $h^2(x) > 0$. From eq.(19a), one finds a maximum $h^2_{max} = 2a_t(a_t - 1)/3\sqrt{3} > 0$ of $h^2(x)$ at x'_-, and a dummy minimum $h^2_{min} = -h^2_{max}$ at x'_+ in the forbidden region $(0 < x < x_0)$ of values $h^2(x) < 0$. Correspondingly, from eq.(19b), $h^2(x)$ has a maximum $h^2_{max} = 2a_t(a_t - 1)/3\sqrt{3}$ 0 at $x = x'_+$. The curve $x(h^2)$ described by eq.(19a) is given for small h^2 and a fixed $a_t > 1$ on Figure 1a. The curve $x(h^2)$ from eq.(19b) is obtained by inversion $x \to -x$, giving an equivalent description (Figure 1b).



(a) (b)

Figure 1

Within the above approximation, the values of x in the vicinity of $\pm x_0$ are given by

$$x^{\pm} = \pm x_0\left(1 + \frac{h^2/a_t}{(1 - a_t) + a_t x_0^2}\right), \tag{20}$$

where the possibility for $x_0 = 0$ is included into consideration. In particular, for $x_0^2 = 3(a_t - 1)/a_t$,

$$x^{\pm} = \pm(3(a_t - 1)/a_t)^{1/2} \cdot \left(1 + \frac{h^2}{2a_t(a_t - 1)}\right). \tag{21}$$

It becomes clear from eqs.(20) and (21) that the susceptibilities $\chi^{\circ} = \chi(x_0 = 0)$, and $\chi^{\pm} = \chi(\pm x_0 \neq 0)$ at $h = 0$ are $\chi^{\circ} = 0$, and

$$\chi^{\pm} = \pm(3/a_t^3(a_t - 1))^{1/2}. \tag{22}$$

These formulae give some information about the qualitative features of the functions $x^{\pm}(h^2)$ depicted on Figures 1a,b. It is seen from these Figures that there are possible values of the order parameter x in the interval $0 < h^2 < h^2_{max}$. Which of them correspond to stable states is a question to be answered in the next subsections. The domains of x, where $\chi < 0$, cannot be straightforwardly identified as domains of instability as here χ is called a susceptibility in a special sense.

The picture described above is qualitatively valid in the framework of the exact treatment of eqs.(10), or (11), and in particular within the iterative treatment by eq.(14). This statement is verified through numerical calculations (see subsection 2.2.3.). Differences appear in the particular values of the parameters at which the function $x(h^2)$ has extrema.

We end this subsection with a note. The approximate results (20)-(22) cannot be obtained by the use of the exact eq.(16). If one replaces there $\text{thz} \approx z$ and the exact value x_0^2 with $x_0^2 \approx 3(a_t - 1)/a_t$, the obtaining of the results (20)-(22) requires some factors a_t to be neglected, which is correct only if $a_t \approx 1$. In fact, if such a procedure is performed straightforwardly, one will find from eq.(16) that

$$x^{\pm} = \pm x\left(1 + \frac{a_t h^2}{(a_t - 1)(3 - a_t^2)}\right), \tag{23}$$

instead of eq.(20). This incorrect formula describes a dummy singularity at $a_t = \sqrt{3}$. The correct correspondence requires the expansion of $th'x_0$ into eq.(15) to the order used in eqs.(19).

2.2.2. Location of phases. The point here is the location of the domains of stability of the para phase and the mixed phase. Obtaining the second derivatives of the free energy (2), one can determine the sufficient conditions for minima of the free energy in the form

$$a(1 - \alpha a_t - \beta a_r + \alpha\beta a) > 0 \tag{24}$$

and

$$a_t - \alpha a_t^2 - \beta h^2 > 0 \,, \tag{25}$$

where $\alpha^{-1} = \cosh^2(a_t t + hr)$, and $\beta^{-1} = \cosh^3(a_r r + ht)$, and we presume that $a \neq 0$; a_r, $a_t > 0$. The inverse inequality (25) and (24) present the conditions for maxima.

We restrict ourselves with some analytic calculations for the para phase ($t = r = 0$), where $\alpha = \beta = 1$ and ineqs.(24) and (25) are simpler, i.e.,

$$a(1 - a_t - a_r + a) > 0 \tag{26}$$

and

$$a_t - a_t^2 - h^2 > 0 \,. \tag{27}$$

Two cases are distinguished.

(1) The weak coupling case ($a > 0$). The para phase is a minimal state of the free energy in the domain

$$\begin{gathered} h^2 < 1/4 \,, \quad (a_t)_- < a_t < (a_t)_+ \,, \\ (h^2/a_t) < a_r < 1 - (h^2/(1 - a_t)) \,, \end{gathered} \tag{28}$$

where

$$(a_t)_\pm = \frac{1}{2}(1 \pm (1 - 4h^2)^{1/2}) \leq 1 \,. \tag{29}$$

It is seen that the domains of a_t and a_r are smaller than that for the decoupled case ($h = 0$): $0 < a_t$, $a_r < 1$. The narrowing depends on h^2.

If, as in ineqs.(28), $h^2 < (1/4)$, the para phase is a maximum of f for $a_t > 1$ and $a_r > h^2/(a_t - 1) + 1$.

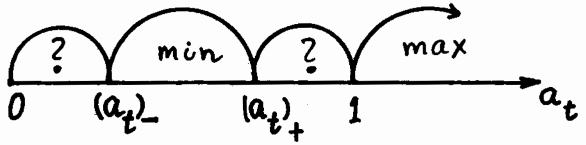


Figure 2

The situation for $h^2 < 1/4$ is shown on Figure 2. On this Figure we denote with (?) the regions where nothing is predicted by the present analysis.

If $h^2 > 1/4$, the para phase gives a maximum of f for $a_t > 1$ and $a_r > 1 + h^2/(a_t - 1)$.

For the very special circumstance $h^2 = 1/4$ nothing definite can be concluded.

(2). Strong coupling case. ($a < 0$). The para phase does not give minimal values of f. It is a maximizing phase for $h^2 < 1/4$ if

$$1 - (h^2/(1 - a_t)) < a_r < h^2/a_t \,, \tag{30a}$$

provided

$$0 < a_t < (a_t)_- \quad \text{or} \quad (a_t)_- < a_t < 1 \tag{30b}$$

or if, alternatively,

$$a_t > 1 \quad \text{and} \quad a_r < h^2/a_t \,. \tag{31}$$

This picture is shown by Figure 3.

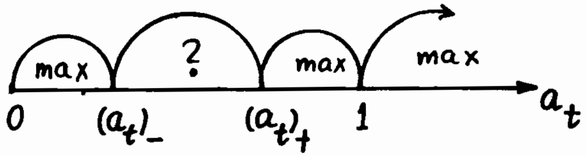


Figure 3

If $h^2 > 1/4$, the maxima of f are in the domains ($a_t > 1$, $a_r < h^2/a_t$) and ($a_t < 1$; $1 - h^2/(1 - a_t) < a_r < h\,/a_t$).

2.2.3. Numerical calculations. We have calculated $\bar{f} = f - f_p$, where f_p is the free energy of the para phase. This gives some information about the location of the phases. $\bar{f}$ has been calculated at fixed $y(h)$, a_t and a_r using a few values of h in the domain $h^2 < h^2_{max}$. It has been shown in this way that on the segments AB and DC of the curve $x(h^2)$, (see Figure 1), $\bar{f}$ has minima whereas the segment OC presents maximal values of $\bar{f}$. For any h^2 in the interval $(0, h^2_{max})$, the minima described by the segment DC are situated on the curve $\bar{f}(x)$ above the minima described by the curve AB. This means that the stable states are given by the values of $x(h^2)$ which lie on the segment AB, whereas DC describes metastable states.

This picture changes for $h^2 > h^2_{max}$. In this case only one minimum appears and it is situated on the segment AA'.

These considerations demonstrate that the description given in subsection 2.2.1. is qualitatively correct and that the inequality $\chi < 0$ really describes the regions of instability.

It has been shown that in the regions where the para phase is a minimal state, states of mixed phase are also possible, but the last ones give higher free energy, i.e. they are metastable states. Besides, it turna out that in the domains, where the para phase maximizes the free energy, there are small regions in which $\bar{f}$ takes positive values. The results demonstrate that the phase transition is of first order with regions of coexistence of stable and metastable phases.

3. FLUCTUATION HAMILTONIAN AND RENORMALIZATION GROUP

We shall somewhat simplify the model (1) replacing the tr-term with $h_0 \sum r_i t_i$. The integral transformation

$$\int_{-\infty}^{\infty} \prod_{i=1}^{N} \cdot dx_i \cdot \exp\left(-\frac{1}{2} x_i V_{ij}^{-1} x_j + \sigma_i x_i\right) = C^{-1} \exp\left(\frac{1}{2} \sigma_i V_{ij} \sigma_j\right) \qquad (32)$$

is used in order to obtain the fluctuation Hamiltonian. In eq. (32) a summation is presumed over repeating indices and $C = (\mathrm{Det}(\hat{V}_{ij}^{-1}/2))^{-1/2}$. Applying the transformation two times with respect to t_i and r_i we obtain an effective Hamiltonian in the form:

$$H_{eff} = \int d^d x \Big(\frac{1}{2} r_1 \phi_1^2 + \frac{C_1}{2} (\nabla \phi_1)^2 + \frac{1}{2} r_2 \phi_2^2 + \frac{C_2}{2} (\nabla \phi_2)^2 + d_0 \phi_1 \phi_2 +$$
$$+ d_1 (\nabla \phi_1) \cdot (\nabla \phi_2) + u_1 \phi^4 + u_2 \phi_2^4 + v_1 \phi_1^3 \phi_2 + v_2 \phi_1 \phi_2^3 + w \phi_1^2 \phi_2^2\Big), \qquad (33)$$

where $\phi_1(x)$ and $\phi_2(x)$ are one-component fluctuation fields and d is the spatial dimensionality. The parameters r_1, r_2, c_1, etc., are related to the original parameters h_0 and $J_{ij}^{t,r}$. We shall not present here these relations. We restrict ourselves mentioning that $r_1 \approx (T - T_{c_1}^{\circ})$ and $r_2 \approx (T - T_{c_2}^{\circ})$, where $T_{c_1}^{\circ}$ and $T_{c_2}^{\circ}$ depend on h_0 and $J_{ij}^{t,r}$. The parameters c_1, c_2, u_1, u_2 and w are positive near T_{c_1} and T_{c_2}, whereas d_0, d_1, v_1 and v_2 may change sign for some values of h_0 and the exchange parameters.

The RG treatment is performed in the framework of the Wilson-Fisher recursion relations /12/. The parameter d_1 is irrelevant. The recursions for d_0, r_1 and r_2 are:

$$d_0' = b^2 (d_0 + 3 v_1 I_1(r_1) + 3 v_2 I_1(r_2)),$$

$$r_1' = b^2 (r_1 + 12 u_1 I_1(r_1) + 2 w I_1(r_2)), \qquad (34)$$

and

$$r_2' = b^2 (r_2 + 12 u_2 I_1(r_2) + 2 w I_1(r_1)),$$

where $I_n(r_i) = K_d \cdot \int dk \cdot k^{d-1} (k^2 + r_i)^{-n}$, $(i, n = 1, 2)$ and $K_d^{-1} = 2^{d-1} \cdot \pi^{d/2} \cdot \Gamma(d/2)$, and the integration is over momenta in the shell $b^{-1} < k < 1$; $b > 1$. The rescaling factor is denoted by b. The Fisher exponent $\eta = 0$ in this approximation. Introducing new parameters $x_i = K_d u_i$, $y_i = K_d v_i$ and $z = K_d w$, the remaining part of the recursion relations is

$$x_1' = b^{\epsilon} (x_1 - (36 x_1^2 + 18 y_1^2 + z^2) \ln b),$$

$$x_2' = b^{\epsilon} (x_2 - (36 x_2^2 + 18 y_2^2 + z^2) \ln b),$$

$$y_1' = b^{\epsilon} (y_1 - (36 x_1 y_1 + 12 y_1 z + 6 y_2 z) \ln b),$$

$$y_2' = b^{\epsilon} (y_2 - (36 x_2 y_2 + 12 y_2 z + 6 y_1 z) \ln b), \qquad (35)$$

and

$$z' = b^{\epsilon} (z - (36 y_1^2 + 36 y_2^2 + 8 z^2 + 12 x_1 z + 12 x_2 z + 36 y_1 y_2) \ln b),$$

where $\epsilon = 4 - d$.

One can analyse these recursions using known rules /12/.

There are two decoupled fixed points (fps) characterized by the coordinate $\bar{z} = 0$ of z. They are: the Gaussian fp $\bar{x}_i = \bar{y}_i = 0$, (i=1,2) and an Ising fp ($\bar{y}_i = 0$, $\bar{x}_i = 1/36$), describing two decoupled Ising models.

Another type of generic fps is given by the coordinates $\bar{y}_i = 0$ and $\bar{z} \neq 0$. In this case one obtains the well-known Heisenberg fp, describing bicritical points and the so-called biconical fp, which is related to the tetracritical behaviour [13].

Besides there are a few essentially new fps. Firstly, symmetric fps $\bar{x}_1 = \bar{x}_2 = 1/144$, $\bar{z} = 1/24$ and $\bar{y}_1 = \bar{y}_2 = 1/72$ or $\bar{y}_1 = \bar{y}_2 = -1/72$. Secondly, two fps with somewhat more complicated coordinates take place:

$$\bar{x}_1 = A^- , \quad \bar{x}_2 = A^+$$

and

$$\bar{y}_1 = (2zA^-/3)^{1/2}, \quad \bar{y}_2 = (2zA^+/3)^{1/2}$$

or

$$\bar{y}_1 = -\bar{y}_1 \quad \text{and} \quad \bar{y}_2 = -\bar{y}_2 ,$$

where

$$A^{\pm} = \frac{1}{72}(1 - 12z \pm (1 - 24z)^{1/2}).$$

The second pair of fps has an exceptional property. The coordinates $\bar{x}_i$ and $\bar{y}_i$ depend on the coordinate $\bar{z}(=z)$ and this is a consequence of the degeneration of the system of equations for the fps in this case. The parameter z varies from zero to 1/24 and this leads to the description of two lines of fps. If $z > 1/24$ or $z < 0$, the last couple of fps does not exist.

The generic fps describe known exponents [12, 13]. It has been shown by numerical calculations that the symmetric fps always give two positive exponents, while in the case of the last couple of fps one has three positive exponents for any $z \in (0,1/24)$. One may therefore conjecture that the new fps, obtained here, describe first order phase transition.

4. CONCLUSIONS

We have investigated in this paper the critical behaviour of coupled Ising models. We have shown within the mean-field approximation that this complicated model describes a first order phase transition in both the weak coupling and the strong coupling cases. The renormalization group analysis has revealed a richer behaviour. A new types of critical behaviour are given by this model related to critical and multicritical points, which are previously known in the literature. They are described here by a few generic fps of the model. The RG has shown four new fps, two of them of very special kind. All of these fps are unstable and describe first order transitions in a support to the mean-field results. The properties of these new phase transitions of first order can be understood to some extent studying the renormalization group flows in the parametric space of the system.

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Коручева Е.Р., Узунов Д.И. E17-88-467
Модели Изинга и связанные параметры порядка

С помощью среднеполевого приближения и метода ренормализационной группы изучена статистическая система, состоящая из двух связанных моделей Изинга. В среднеполевом приближении описан фазовый переход первого рода из неупорядоченной фазы в смешанную фазу с двумя параметрами порядка. Ренормгрупповой подход дает несколько неподвижных точек, описывающих известные критические и мультикритические точки, а также и четыре новые неустойчивые подвижные точки, относящиеся к описанию фазовых переходоа первого рода. Эти результаты можно применить при описании фазовых переходов в молекулярных кристаллах, двухкомпонентных жидкостях и сплавах и некоторых магнетиках.

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Ising Models and Coupled Order Parameters

Using both mean-field and renormalization group approaches a statistical system consisting of two coupled Ising models is studied. A mean-field description of a first order phase transition from a disordered phase to a mixed phase, described by two order parameters, is presented. The renormalization group study gives several fixed points related to the well-known critical and multicritical points as well as four new unstable fixed points, describing first order transitions. The results might be useful in discussions of phase transitions in molecular crystals, binary liquids, binary alloys and some magnets.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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