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NONLINEAR EXCITATIONS
OF A DIATOMIC POLYMER

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In our previous paper (Fedyanin and Osipov 1987) we examined the ground-state properties and soliton excitations of $A B$ polymers within the framework of the finite band continuum scheme, first proposed by Gammer (1986) for the case of polyacetylene chains. In the present paper, we study the physical characteristics of $A B$ solitton pairs and polarons, both numerically and analytically. The aim of our paper is twofold. At first, we extend the Rice-Mele HamillIonian (Rice and Mile 1982) by adding electron transfers between second neighbour sites since their role in some $A B$ polymers could be considerably enhanced in comparison with the polyacetylene case. At second, we investigate numerically $\pi$-electron spectra of finite $A B$ chains to estimate effects caused by dicreteness of the lattice.

Let us consider the model Hamiltonian

$$
\begin{aligned}
& H=-\sum_{n, \sigma} \sum_{l=1}^{2} t_{n, n+1}\left(c_{n+l, \sigma}^{+} c_{n, \sigma}+h . c .\right)+ \\
& +\underset{n, \sigma}{a \sum_{n}(-1)^{n+1} c_{. .,-}^{+} c_{. ., n}+\frac{k}{2}-\sum_{n}\left(n_{n+2}-n_{n}\right)^{2}}
\end{aligned}
$$

for the linearly conjugated $A B$ polymer, where $t_{n, n+1}(n=1,2 \ldots N)$ denotes electron transfers between the first and second neighbours, respectively, $c_{n, \sigma}^{+}\left(c_{n, \alpha}\right)$ are the creation (annihilation) operators of relectrons with spin $\sigma$ at a $\varepsilon$ te $n$, a stands for an one-site potential, $K$ is a harmonic spring constant and $u_{n}$ are lattice displacements. We note that elastic interactions are restricted to the nearest-neighbour atomic cores. Also, the kinetic energy term is omitted in (1) because we are interested here in static lattice configurations only. It is assumed that there is one n-electron per site.

As usual, we adopt the linear forms for hopping matrix elements
$t_{n, n+1}=t_{0}-\gamma\left(u_{n+1}-u_{n}\right)$ and $t_{n, n+2}=t_{2 n}-\gamma_{2 n}\left(u_{n+2}-u_{n}\right)$,
where $t_{o}$ and $t_{2 n}$ are the electron transfer integrals between first and second neighbours in a regular chain. The corresponding derivatives of $t_{n, n+1}$ and $t_{n, n+2}$ with respect to intersite separation
a in the regular chain are denoted by $-\gamma$ and $-\gamma_{2 n}$. Whilst the parameters $t_{o} \equiv t_{1}^{A B}$ and $\gamma \equiv \gamma_{1}^{A B}$ do not depend on the site index, the hoppings $t_{2 n}$ and electron-lattice coupling constants $\gamma_{2 n}$ are equal to $t_{2}^{A(B B)} \equiv t_{2 A}\left(t_{2 B}\right)$ and $\gamma_{2}^{A A(B \theta)} \equiv \gamma_{2 A}\left(\gamma_{2 B}\right)$ for odd (even) sites, respectively.

It is convenient to rewrite the Hamiltonian (1) in terms of the dimensionless bond-length changes $v_{n}=\gamma\left(u_{n+1}-u_{n}\right) / t_{0}$

$$
\begin{aligned}
H & =t_{0}\left[a_{0} \sum_{n, \sigma}(-1)^{n+1} c_{n, \sigma}^{+} c_{n, \sigma}+\sum_{n, 0}\left(v_{n}-1\right)\left(c_{n+1, \sigma}^{+} c_{n, \sigma}+h . c .\right)+\right. \\
& \left.+\sum_{n, 0}\left(\hat{\gamma}_{n}\left(v_{n+1}+v_{n}\right)-\tau_{n}\right)\left(c_{n+2, \sigma}^{+} c_{n, \sigma}+\text { h.c. }\right)+\frac{1}{2 \beta} \sum_{n} v_{n}^{2}\right], ~(3)
\end{aligned}
$$

 $a_{0}=\alpha / t_{0} ; \hat{\gamma}_{n}=\gamma_{2 n} / \gamma ; \tau_{n}=t_{2 n} / t_{0}\left(\right.$ i.e. $\tau_{n}=\tau_{A}$ or $\tau_{B}$ ) and $\beta=\gamma^{2} / K t_{0}$.

It is well known that the ground state of (3) ls the Peierls insulator possessing a spontaneous dimerization with a displacement field $u_{n}= \pm u_{o}$. The ground-state spectrum of (3) reads as $E_{k}=t_{o}\left\{-\left(\tau_{A}+\tau_{a}\right) \cos 2 k a \pm 2\left[\frac{1}{4}\left(\alpha_{0}+\Delta \tau \cos 2 k a\right)^{2}+\cos ^{2} k a+2^{2} \sin ^{2} k a\right]^{1 / 2}\right\}$, where $\Delta \tau=\tau_{B}-\tau_{A}, z=\Delta / 2 t_{0}=2 \gamma u_{0} / t_{0}$ and $-\pi / 2 a \leq k<\pi / 2 a$. The spectrum (5) exhibits the Peierls gap

$$
\begin{equation*}
E_{g}=2 \Delta_{0}=2 t_{0}\left[\left(a_{0}-\Delta \tau\right)^{2}+\left(\Delta / t_{0}\right)^{2}\right]^{1 / 2} \tag{6}
\end{equation*}
$$

at the Fermi wave vector $k_{F}=\pi / 2 a$. The parameter $z$ or, equivalently, the dimerization coordinate $u_{o}$,minimizes the ground-state energy of the chain, i.e., it is determined as the solution of

$$
\begin{equation*}
1 / \lambda \text { off }=\int_{0}^{\pi / 2} \frac{\sin ^{2} t d t}{\left[1 / 4\left(a_{0}+\Delta \tau \cos 2 t\right)^{2}+\cos ^{2} t+z^{2} \sin ^{2} t\right]^{1 / 2}}, \tag{7}
\end{equation*}
$$

where $\lambda_{\text {off }}=4 \gamma^{2} / \pi K t_{o}$ is the dimensionless electron-phonon coupling constant. As it foliows from (6), the dimerization exists for $\Delta_{0}>t_{0}\left(\alpha_{0}-\Delta \tau\right)$ only. Hence, for some $A B$ materials with a close to $\Delta_{o}$ and $t_{2 A} \not t_{2 B}$, the second neighbour interactions can easily suppress the dimerized structure of the ground state. In a general case, $\tau_{A}{ }^{\boldsymbol{\tau} \tau_{B}}$, equation (7) has to be solved numerically. For $\tau_{A}=\tau_{B}$ we find

$$
\begin{equation*}
1 / \lambda_{e f f}=\frac{\left(1+y^{2}\right)^{1 / 2}}{1-z^{2}}[K(m)-E(m)] \tag{8}
\end{equation*}
$$

where $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind, respectively, $m^{2}=\left(1-z^{2}\right) /\left(1+y^{2}\right)$ and $y=\alpha_{0} / 2$. Finally, for $\Delta \tau \ll 1$ and $y \ll 1, \Delta \tau \ll 1$, eq. 7 can be solved analytically even for $\tau_{A} \tau_{B}$ and we find

$$
\begin{equation*}
\Delta_{0}=2 t_{0}\left[\frac{1}{4}\left(a_{0}-\Delta \tau\right)^{2}+z^{2}\right]^{1 / 2}=8 t_{o} \exp \left(-1 / \lambda_{\text {off }}-1\right) . \tag{9}
\end{equation*}
$$

The charge transfer $2 e^{*}$ between the $A$ and $B$ atoms becomes

$$
\begin{equation*}
\mathrm{e}^{*} / \mathrm{e}=\frac{1}{\mathrm{~N}} \sum_{k, 0}^{\infty} \frac{a_{0}+\Delta \tau \cos 2 \mathrm{ka}}{E_{k}}, \tag{10}
\end{equation*}
$$

where $E_{k}$ is given by (5), the sum runs over occupied states and |el denotes the magnitude of the charge, $e$, on the electron.

The soliton excitations or polaron states of (3) can be found exactly for finite $A B$ chains by direct numerical calculation or analytically taking the continum limit of (3) which, however, provides an accurate description for $z \ll 1$ only. We shall consider here both the methods.

Let us discuss first the analytical approach. The continumm description of an $A B$ polymer has first been reported by Rice and

Mele (1982). Recently, their linearized scheme has been extended by Fedyanin and Osipov (1987) by including the electronic dispersion. Since the finite band scheme involves the Rice-Mele approach as a epecial case we start with finite band field equations (for details see Fedyanin and Osipov 1987) for eigenvalues $E_{k}$, eigenstate amplitudes $A_{k}(x), B_{k}(x)$ and the local dimerization gap $\Delta(x)$ ( $\dagger=1$ )

$$
\begin{align*}
& \left(E_{k}-a-t_{2 k}^{A}\right) A_{k}(x)=-i v_{F}(k) \frac{\partial B_{k}(x)}{\partial x}-\left(\varepsilon(k)-i \Delta_{k}(x)\right) B_{k}(x) \\
& \left(E_{k}+\alpha-t_{2 k}^{\mathbf{B}}\right) B_{k}(x)=-i v_{F}(k) \frac{\partial A_{k}(x)}{\partial x}-\left(\varepsilon(k)+i \Delta_{k}(x)\right) A_{k}(x) \tag{11}
\end{align*}
$$

and

$$
\begin{equation*}
\Delta(x)=-\frac{i 4 r^{2} a}{K} \sum_{k . \sigma}^{o c e}\left(A_{k}^{*}(x) B_{k}(x)-c . c\right) \operatorname{coska} \tag{12}
\end{equation*}
$$

where the eigenvector corresponding to the eigenvalue $E_{k}$ is the
 according to $\underset{-L / 2}{L / 2} d x\left(\left|\psi_{k}^{A}(x)\right|^{2}+\left|\psi_{k}^{B}(x)\right|^{2}\right)=1 ; \Delta_{k}(x)=\Delta(x) \cos k a ; \quad v_{F}(k)=$ $\mathbf{v}_{\mathbf{F}}$ coska, where $\mathbf{v}_{\mathbf{F}}$ denotes the Fermi velocity $\mathbf{v}_{\mathbf{F}}=2 \mathrm{ta} ; \boldsymbol{c}(\mathrm{k})=2 \mathrm{t}_{\mathrm{o}}$ sinka; $t_{2 k}^{A(B)}=2 t_{2 A(2 B)} \cos 2 k a$ and finally, the length of the chain $\mathrm{L}=\mathrm{Na} \rightarrow \infty$. Only linear terms in $a / \xi_{0}\left(\xi_{0}=v_{F} / \Delta\right)$ are retained in (11) and (12) and, therefore, the derivatives $\gamma_{\text {2A(2B) }}$ (which appear in terms of an order of $\left(a / \xi_{0}\right)^{2}$ ) are absent in (11) and (12). We should note that here and hereafter the wavevector $k$ is measured relative to $k_{F}=\pi / 2 a$.

For the ground state $\Delta(x)=$ const, the solutions of (11) and (12) are plane waves with constant amplitudes and yield the eigenvalues given by (5) with $k \rightarrow k+\pi / 2 a$. In periodic chains the topologic constraints imply that the solitons can be created from the ground state only in the form of $A B$ soliton pairs.

Let us consider firet the case of $t_{2 k} \equiv t_{2 k}^{A}=t_{2 k}^{8}$. Since now the effect of the second neighbour hoppings results in a shift of the chemical potential only,it is convenient to introduce new energy variable $\hat{E}_{k}=E_{k}-t_{2 k}$. For $a<\Delta_{0}$ we have

$$
\begin{equation*}
\Delta(x)= \pm \Delta \operatorname{th}\left(x / \xi_{0}\right) \tag{13}
\end{equation*}
$$

where the plus refers to an $A$ soliton with the energy $\hat{E}_{A}=+\infty$ whereas the minus to a $B$ soliton with the energy $\hat{E}_{B}=-\alpha$. The continuum states have similar enereies $\hat{E}_{k}$ as those of the dimerized chain but they are phase shifted by $\delta(k)=-\operatorname{arctg}\left(\Delta_{k} / \varepsilon_{k}\right)$.

The wave functions of singlc $A$ soliton ( $h=0$ ) read as

$$
\psi_{0}^{\hat{0}}(x)=A_{0}(x)=N_{A} \operatorname{sech}\left(x / \xi_{0}\right) \quad \text { and } \quad \psi_{0}^{B}(x)=B_{0}(x)=0, \quad \text { (14) }
$$

where $N_{A}=\left[\left(1 / 2 \xi_{0}\right) \operatorname{cth}\left(L / 2 \xi_{0}\right)\right]^{1 / 2}$. The amplitudes of continuum states ( $k \neq 0$ ) become
$A_{k}(x)=\frac{B_{k}(x)}{E_{k}-a}\left[-\varepsilon(k)+i \Delta_{k} \operatorname{th}\left(x / \xi_{o}\right)\right] \quad$ and $\quad B_{k}(x)=\left(N_{k} L\right)^{-1 / 2}$, (15)
 soliton and corresponding continuum states are obtained from (14) and (15) by changing $A \rightarrow B$ and $\alpha \rightarrow-\alpha$.

The change in the local electronic density of the dimerized chain induced by the presence of the $A B$ soliton pair is

$$
\begin{equation*}
\Delta \rho(x)=\left(\nu_{+}-2 I_{+}\right) \rho_{0}^{A}(x)+\left(\nu_{-}-2 I_{-}\right) \rho_{0}^{B}(x), \tag{16}
\end{equation*}
$$

where $\nu_{+}$and $\nu_{-}$are occupation numbers of $A$ and $B$ soliton energy levels, respectively. The local deficiency in the continuum states $I_{ \pm}$is given by

$$
\begin{equation*}
I_{ \pm}=\frac{1}{2(1 \pm z)} \mp \frac{1}{n\left(1-z^{2}\right)}\left[n / 2-K(m) E\left(\beta, m^{\prime}\right)-E(m) E\left(\beta, m^{\prime}\right)+K(m) E\left(\beta, m^{\prime}\right)\right] \tag{17}
\end{equation*}
$$

where $F\left(\beta, m^{\prime}\right)$ and $E\left(\beta, m^{\prime}\right)$ are the incomplete elliptic integrals of
the first and second $k$ ind, respectively, $m^{\prime}=\left(1-m^{2}\right)^{1 / 2}$ and $\operatorname{tg} \beta=$ $\Delta\left(1+y^{2}\right)^{1 / 2} / a\left(1-z^{2}\right)^{1 / 2}$. The charge derisities $\rho_{0}^{A}(x)$ and $\rho_{0}^{B}(x)$ read as
$\rho_{0}^{\hat{A}}(x)=\frac{1}{2 \xi_{0}} \operatorname{sech}^{2}\left((x-L / 4) / \xi_{0}\right) \quad$ and $\quad \rho_{0}^{B}(x)=\frac{1}{2 \xi_{0}} \operatorname{sech}^{2}\left((x+L / 4) / \xi_{0}\right)$.
As it follows from (16) the soliton charges of the $A B$ pair are

$$
\begin{equation*}
Q_{A(B)}=-\left(\nu_{ \pm}-2 I_{ \pm}\right)|e| \text {. } \tag{19}
\end{equation*}
$$

Finally, the creation energy of the $A B$ soliton pair becomes
$2 E_{0 x\left(\nu_{+}, \nu_{-}\right)}=\alpha\left(\nu_{+}-\nu_{-}\right)+\frac{4 \Delta}{\pi}\left\{\frac{1}{m^{2}}\left[E(m)-\left(1-m^{2}\right) K(m)\right]+\frac{y^{2}}{\left(1+y^{2}\right)^{2 / 2}} \Pi(\pi / 2, n, m)\right\}$ where $\pi(\pi / 2, n, m)$ is the complete elliptic integral of the third kind and $\mathrm{rl}_{1}=-\left(1-z^{2}\right)$.

Let us now discuss briefly the case of $t_{2 A} \neq t_{2 A}$. As relevant derivations within the finite band scheme are lengthy and do not result in close analytical formulas, we restrict our analysis in this Letter only to the linearized version of equations (11) and (12). Approximating in (11) sinc $\propto x$ and $\cos x \propto 1$ and introducing new energy variable $\hat{E}_{k}=E_{k}-\left(t_{2 A}+t_{2 B}\right)$ we arrive at the modified Rice and Mele (1982) equations

$$
\begin{align*}
& \left(\hat{E}_{k}-(a-\Delta t)\right) \psi_{A}(x)=\left[-i v_{F} \nabla_{x}+i \Delta(x)\right] \psi_{B}(x) \\
& \left(\hat{E}_{k}+(a-\Delta t)\right) \psi_{B}(x)=\left[-i v_{F} \nabla_{x}-i \Delta(x)\right] \psi_{A}(x), \tag{21}
\end{align*}
$$

where $\Delta t=t_{2 B}-t_{2 A}$. Hence, changing simply $a \rightarrow a-\Delta t$ one can readily apply all the formulas of the Rice and Mele paper (1982). For example, the creation energy of a neutral $A B$ soliton pair reads as
$2 \mathrm{E}_{\mathrm{exc}}\left(\nu_{+}, \nu_{-}\right)=(a-\Delta t)\left(2+\nu_{+}-\nu_{-}\right)+\frac{4}{\pi}\left[\Delta-(a-\Delta t) \operatorname{arctg} \frac{\Delta}{a-\Delta t}\right]$.
From (22) it follows that the second-neighbour interactions can
significantly modify an energy required to excite an $A B$ eoliton pair. Similarly, changing $a \rightarrow \alpha-\Delta t$ in polaron solutions of (21) (see Campbell 1986), we find that the polaron becomes unstable at ( $a-\Delta t$ ) $>\Delta_{o} / \sqrt{2}$ and decays into an $A B$ soliton pair. Thus, the second neighbour interactions affect also the stability of the polaron states.

To investigate effects caused by discreteness of the lattice we have carried out numerical calculations on periodic chains (i.e. rings with fixed lengths) using the self-consistent discrete method proposed by Shastry (1983) and applied by several authors to polyacetylene and polyyne chains and rings (Stafstrom and Chao 1984; Drechsler et al 1987 , and Malek et al 1988). A modification of the discrete method to a diatomic polymer case is straightforward. The eigenvalue problem for a periodic chain described by the Hawiltonian (3) reads as

$$
\begin{equation*}
H|k\rangle=E_{k}|K\rangle \quad \text { and } \quad|K\rangle=\sum_{n=1}^{N} \psi_{k}(n) c_{n, \infty}^{1}|0\rangle \tag{23}
\end{equation*}
$$

where the eigenvalues $E_{k}$ and the eigenvectors $\{k\rangle$ (the index $k$ includes the spin index now) depend, of course, on the bond length changes $\left\{v_{n}\right\}$. Minimizing the total energy of the periodic chain

$$
\begin{equation*}
E_{i o t}=\sum_{k=1}^{\infty c c} E_{k}+\frac{1}{2 \beta} \sum_{n=1}^{N} v_{n}^{2} \tag{24}
\end{equation*}
$$

under the constraint condition expressing periodic boundary conditions

$$
\begin{equation*}
\sum_{n=1}^{N} v_{n}=0 \tag{25}
\end{equation*}
$$

with respect to $v_{n}$, we arrive at the self-consistent equations $v_{n}=\beta\left(\lambda-\sum_{k=1}^{o c c}\left[\psi_{k}^{*}(n+1) \psi_{k}(n)+\hat{\gamma}_{n} \psi_{k}^{*}(n+2) \psi_{k}(n)+\tilde{\gamma}_{n-1} \psi_{k}^{*}(n+1) \psi_{k}(n-1)+c . c\right]\right)$,
where the Lagrange multiplier $\lambda$ reads as

and the index $k$ runs over all occupied spinorbitals.
Equations (23)-(27) may be solved by an iteration procedure for the ground, $A B$ soliton pair and polaron states, respectively, utilizing continum theory expreseions for displacements $u_{n}$ as starting ansatz. The creation energy $2 E_{o x c}$ of an $A B$ soliton pair and the formation energy $E_{\text {for }}^{P}$ of $a$ polaron $\varepsilon$ tate are $2 E_{\text {exc }}=E_{t_{\text {ot }}}^{A B}$ -$-E_{\text {tot }}^{D I M}$ and $E_{\text {for }}^{P}=E_{\text {tot }}^{P}-E_{\text {tot }}^{D I M}$, where $E_{\text {tot }}^{A B}, E_{\text {tot }}^{P}$ and $E_{\text {LOL }}^{D I M}$ are the relevant total energies given by (24). The ground state charge structure along the periodic chain (in ZDO approximation) reads

$$
\begin{equation*}
q^{D I M}(n)=\left(-\sum_{k=1}^{o c c}\left|\psi_{k}^{D I M}(n)\right|^{2}+1\right)|e|, \tag{28}
\end{equation*}
$$

where the second term in (28) describes the charge of atomic
 induced by the $A B$ soliton pair, $\Delta \rho^{A B}(n)$, or by the polaron, $\Delta \rho^{P}(n)$, becomes

$$
\begin{equation*}
\Delta \rho^{A B(P)}(n)=-\sum_{k=1}^{\infty C C}\left(\left|\psi_{k}^{A B(P)}(n)\right|^{2}-\left|\psi_{k}^{D I M}(n)\right|^{2}\right)|e| \tag{29}
\end{equation*}
$$

where $\left\{\psi_{k}^{\hat{A}^{\mathbf{B}}}(n)\right\},\left\{\psi_{k}^{\mathbf{P}}(n)\right\}$ and $\left\{\psi_{k}^{\mathrm{Dr}}(n)\right\}$ are the relevant eigenvectors of (3). Typical shape of the charge distribution $\Delta D^{A B}(n)$ is shown in Fig. 1.

The stable arrangement of single $A B$ soliton pair on the ring is such that ring regions with positive and negative values of the order parameter $(-1)^{n+1} v_{n}$, respectively, have the same number of sites. Thus excited periodic chains of $N=4 M+2$ sites can be divided into two "open" chains of $2 M+1$ sites with soliton centres on

their middle sites. For even values of $M$, the $A$ soliton with energy +a is centered on $A$ atom and $B$ soliton with energy $-a$, on $B$ atom. The soliton charges $Q_{A}$ and $Q_{B}$ result from summing up the charge distribution $\Delta \rho^{A B}(n)$ over the corresponding regions (i.e. over A- or B-"oper" chains) of the ring.

We have performed numerical calculations on the ringe of $N=82$, 162 and 202 atoms. With the employment of the representative values $t_{o}=3 \mathrm{ev}, \mathrm{K}=68.6 \mathrm{ev} / \AA^{2}, \gamma=8 \mathrm{ev} / \AA$ and $a=0.3 \mathrm{ev}$, which yield the characteristic length $\xi_{0} \propto 9.1 a$, we have calculated the Peierls gap $2 \Delta_{0}$, the dimerization coordinate $u_{0}$, the soliton charges $Q_{A}$ and $Q_{B}$, and the creation and formation enereies of $A B$ soliton pairs and polarons, respectively. Neglecting second neighbour transfers we have obtained resulte given in Tables 1 and 2. One can immediately observe the perfect agreement between the results of the finite band version of the continuum theory and the discrete model. From Tables 1 and 2 it follows also that for $a / \xi_{\text {, }} 1$ the Rice-Mele scheme is sufficicntly accurate to describe rell all the characteristics of the ground and exciived or polaron states of $A B$ polymers. We note there is practically no difference between the numerical results derived from the ringe of 162 and 202 sites.

To verify limits of the continum theory we have changed the spring constant to $K=34.3 \mathrm{ev} / \AA^{2}$ which yields $\xi_{0} \propto 2.0 a$. The results can be found in Table 3. In this case, of course, the linearized scheme doesn't work so well but the finite band scheme still conforms well to the results of the discrete model calculation. This is an important fact because in $A B$ analogs of the polyyne chains one can expect solitons and polarons with very short characteristic lengths.

The decay process of the negatively charged polaron state into

Table 1. The dimerization gap parameter $\Delta_{o}$ (see eqs. (8) and (9) for finite band and linearized version of the continuum theory, resp.), the dimerization coordinate $u_{0}$ (determined as $u_{0}=\Delta / 4 \gamma$ in the continuum model and as $u_{0}=-t_{0} v_{1} / 2 \gamma$ in the discrete model, resp), the soliton charges $Q_{A}$ and $Q_{B}$, and the creation energy of the neutral $A B$ soliton pair $2 E_{\text {exc }}$ for $t_{o}=3 \mathrm{eV}, K=68.6 \mathrm{eV} / \AA^{2}, \gamma=8 \mathrm{eV} / \AA, \quad \alpha=0.3 \mathrm{eV}$, and $t_{2 A}=t_{2 B}=\gamma_{2 A}=\gamma_{2 B}=0$.

| model | $N($ sites $)$ | $\Delta_{o}(e v)$ | $u_{o}(\AA)$ | $Q_{A(B}(\|e\|)$ | $2 \mathrm{E}_{\mathrm{on}}(\mathrm{ev})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 82 | 0.718 | 0.0193 | $\pm 0.685$ | 0.390 |
| discrete | 162 | 0.722 | 0.0204 | $\pm 0.722$ | 0.391 |
|  | 202 | 0.722 | 0.0204 | $\pm 0.722$ | 0.391 |
| finite <br> band | - | 0.724 | 0.0206 | $\pm 0.722$ | 0.394 |
| linearized | - | 0.706 | 0.0200 | $\pm 0.721$ | 0.382 |

Table 2. The characteriatics of the negatively charged polaron. For notations and the parameter eet used see Table 1

| model | $N(s i t e s)$ | $\Delta_{o}(e v)$ | $u_{o}(\AA)$ | $\left.Q_{A(B} f\|e\|\right)$ | $\mathbf{E}_{\mathrm{fof}}^{P}(\mathrm{ev})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| discrete | 82 | 0.726 | 0.0203 | -1 | 0.647 |
|  | 162 | 0.722 | 0.0204 | -1 | 0.645 |
|  | - | 0.722 | 0.0204 | -1 | 0.645 |

Table 3. The characteristics of the neutral $A B$ soliton pairs calculated with use of $t_{0}=3 \mathrm{eV}, \gamma=8 \mathrm{eV} / \AA, \mathrm{K}=34.3 \mathrm{eV} / \AA^{2}, a=0.3 \mathrm{eV}$ and $t_{2 A}=t_{2 B}=\gamma_{2 A}=\gamma_{2 B}=0\left(\xi_{o} \propto 2.0 a\right)$.

| model | N(sites) | $\Delta_{0}(\mathrm{ev})$ | $\mathrm{u}_{0}(\mathrm{R})$ | $Q_{A(B)}(\|e\|)$ | $2_{\text {exc }}(\mathrm{ev})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| discrete | 82 | 2.978 | 0.0920 | $\pm 0.923$ | 2.877 |
|  | 162 | 2.978 | 0.0927 | $\pm 0.923$ | 2.875 |
|  | 202 | 2.978 | 0.0927 | $\pm 0.923$ | 2.875 |
| finite band | - | 2.991 | 0.0930 | $\pm 0.923$ | 2.844 |
| linearized | - | 2.497 | 0.0775 | $\pm 0.923$ | 2.603 |

the charged pair of $A$ and $B$ solitons is shown in Fig. 2. The calculation has beeri carried out on the ring of 162 atome ueing the above mentioned parameter set with $a=0$. 6ev and $t_{2 A}=t_{2 B}=0$. The polaron decays into the $B$ soliton with the charge $Q_{B}=-0.367 / \mathrm{e} \mid$ and the A soliton with $Q_{A}=-0.633|e|$. The linearized theory (see Table 1) Eives $\Delta_{o}=0.706 e v$ and $\Delta=\left(\Delta_{o}^{2}-a^{2}\right)^{1 / 2}=0.372 e v$, which in accord with discrete calculation, yields $Q_{B}=-\frac{2}{\pi} \operatorname{arctg}(\Delta / a)|e|=-0.353|e|$ and $Q_{A}=$ $-\left(1+Q_{A}\right)=-0.647 / \mathrm{e} \mid$.

Finally, in Fig. 3 we show the effect of the second neighbour transfers on the creation energy of a neutral $A B$ soliton pair. The calculations have been performed on the ring of 162 sites by using the parameter set with $\alpha=0.3 \mathrm{eV}$ and $\Delta \mathrm{t}$ varied from -0.3 ev to 0.3 ev . The values $t_{2 A}=t_{2 B}=0.05 t_{0}$ have been set as a centre of $\Delta t$ interval. We have supposed the exponential dependence of hopping integrals $t_{2 A}$ and $t_{2 B}$ on the interatomic separation which yields $\tau_{A}=\bar{\gamma}_{A}$ and $\tau_{B}=\gamma_{B}$ i.e. $\Delta \tau=\Delta \gamma \equiv \gamma_{B}-\gamma_{A}$ (see also (4)). In accord with analytical predictions (see (22)) the calculated creation energy $2 E_{\text {exc }}$ strongly depends on the value of $\Delta t$. Since $a / \xi_{o} \ll 1$ the second derivatives $\hat{\gamma}_{n}$ do not play any essential role. We note that for $\Delta t \rightarrow a_{0}$ the localized levels are shifted to the midgap and we arrive at the well-known picture of trans-polyacetylene case. For $\Delta t \rightarrow\left(a-\Delta_{o}\right)$ the localized levels tend to band edges and, therefore, in the polymers with a close to $\Delta_{o}$, the second neighbour transfers can make the creation of $A B$ soliton paire impossible.

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Fig. 2 The decay process of a negatively charged polaron into a singly charged $A B$ soliton pair. The order parameter $\phi_{n}=(-1)^{n+1}$. $v_{n} t_{0} / \gamma$ (in units of $10^{-2} A$ ) is shown. The lines $A, B$ and $C$ display the starting ansatz, the $\phi_{n}$ after 100 iterations and the self-consistent solution, respectively.


Fig. 3 The creation energy of an neutral $A B$ soliton pair as a function of $\Delta t \equiv t_{2 B}-t_{2 A}$ and $\Delta \gamma \equiv \gamma_{2 B}-\gamma_{2 A}$. Analytic continuum form with $\Delta \gamma=0$ (line A); self-consistent discrete form with $\Delta t / t_{0}=\Delta \gamma / \gamma$ (line B).

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Осипов В.А., Малек И., Федянин В.К
E17-88.461 Нелинейные возбуждения в двухатомном полимере

Представлены континуальный и самосогласованньй дискретный аиализы осиов ного состояния, солитонной пары, а также поляронного состояния в протяженных цепочках двухатомных полимеров. Обсуждается роль эффектов, обусловлениых взаимодействием вторых соседей, которая здесь существенно возрастает по сравне нию со случаем полиацетилена. Вычислены пайерлсовская щель $2 \Delta_{0}$ амплитуда диме ризащии $u_{0}$, заряды солитонов $Q_{A}$ и $Q_{\mathrm{B}}$, энергии рождения AB солитонной пары и полярона соответственно. Численный расчет подтверждает, что конечноэонная версия континуальной теории правильно описьвает все характернстики нелинейных состояний, даже при очень коротких характеристических длинах солитонов

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## Osipov V.A., Malek J., Fedyanin V.K.

E17-88-461 Nonlinear Excitations of a Diatomic Polymer

The continuum and self-consistent discrete analysis of the ground-state, the soliton pair and polaron states of long-chain diatomic polymers is presented. The role of electron transfers between second neighbour sites is discussed and shown to be considerably enhanced in comparison with polyacetylene case. The Peierla gap $2 \Delta_{0}$, the dimerization coordinate $u_{0}$, the soliton charges $Q_{A}$ and $Q_{B}$, and the creation and formation energies of $A B$ soliton pairs and polarons, resp., are calculated. It is verified numerically that the finite band version of the continuum theory accurately describes all the characteristics of nonlinear states under study, even when very short characteristic lengths of the inhomogeneities are considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

