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SQUEEZING IN THE NONLINEAR JAYNES - CUMMINGS MODEL

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1. Introduction

The revival of interest in the Jaynes-Cummings model (JCM) [1,2] and its various modifications is due to new experiments in which JCM is realized in practice. These experiments with highly excited Rydberg atoms interacting with mm-wave fields in high-Q cavities [3-5] offer the possibility of studying the quantum dynamics of JCM and investigate the main features of the atom-field interactions.

The light squeezing is one of the problems which are now intensively studied in connection with JCM (for review on squeezed light see [6,7]). It has been shown that the squeezing of light can be observed in the JCM with a coherent cavity field [8]. Besides it has been reported that squeezing can occur in JCM, when at the initial time a two-level atom is prepared in a coherent superposition of excited and ground states and the field is in the vacuum state [9,10]. Recently the multiphoton JCM (the atom has been supposed to be in the ground state at t=0) interacting with a coherent single-mode cavity field has been studied [11] and it has been shown, that the states containing a large degree of squeezing can be obtained. Here the time evolution of the function S_1, which characterizes the level of squeezing (see definition (9)) has been analyzed for various intensities $\bar{n}$ of the initial coherent field and various photon multiples. It has been shown, that this function exhibits some oscillations and that its long time behaviour is characterized by recoveries of squeezing. However, these recoveries are not periodical ones.

The purpose of the present paper is two-fold. First, we will show that in a nonlinear multiphoton JCM (a generalization of the Buck-Sukumar model [12]) with a coherent cavity field, the periodical light squeezing is possible. Here we will suppose the atom to be in the ground state at the initial time. Second, we will study in detail the interaction of a two-level atom, initially prepared in a coherent superposition of excited and ground states, with a single-mode cavity coherent field. Due to the nonlinearity of the model the periodicity of squeezing is preserved in this case too.

2. The model and the equation of motion

We will suppose the electric-dipole hamiltonian in the rotating-wave approximation:

$$\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_3 + \hbar\omega\hat{a}^+\hat{a} + \hbar\lambda(\hat{\sigma}_+\hat{R}^M + \hat{R}^{+M}\hat{\sigma}), \qquad (1)$$

where $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators of the field with the commutation relation $[\hat{a},\hat{a}^+] = 1$; the two-level atom is described by the Pauli raising and lowering operators $\hat{\sigma}^+, \hat{\sigma}$ and the inversion operator $\hat{\sigma}_3$; the coupling constant λ is a real number; ω and ω_0 are the frequencies of the field and the atom, respectively. The operators $\hat{R}^+$, $\hat{R}$ are defined in the following way [12]:

$$\hat{R} = \hat{a}(\hat{a}^+\hat{a})^{1/2} \quad ; \quad \hat{R}^+ = (\hat{a}^+\hat{a})^{1/2}\hat{a}^+ , \qquad (2)$$

with the commutation relation $[\hat{R},\hat{R}^+] = 2\hat{N} + 1$, where $\hat{N} = \hat{a}^+\hat{a}$ is the photon number operator. We can say, that the hamiltonian (1) describes the multiphoton intensity dependent coupling between the atom and the field. Due to the fact, that the excitation number operator $\hat{C} = \hat{N} + \hat{\sigma}^+\hat{\sigma}$ is a constant of motion - $[\hat{H},\hat{C}] = 0$,

the time-dependent Schrödinger equation for the state vector $|\psi(t)\rangle$

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \qquad (3)$$

can be solved easily. Further we will focus on two problems. First, the atom will be supposed to be in the ground state and the single-mode field in the coherent state at $t = 0$. Second, the dynamics of the nonlinear JCM will be studied when at the initial moment the atom is in the coherent superposition of the ground and excited states and the field is in the coherent state. To make the paper more clear the conclusions are drawn in each case separately in the form of several comments.

3. The atom in the ground state at t=0

Let at t=0 the atom is in the ground state $|-\rangle$ and the field in the coherent state $|\alpha\rangle$:

$$|\psi(t=0)\rangle = |-,\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}\frac{\alpha^n}{\sqrt{n!}}|-,n\rangle = \sum_{n=0}Q_n|-,n\rangle , \qquad (4)$$

where $\alpha = \sqrt{\bar{n}}\,e^{i\varphi}$; $\bar{n}$ is a dimensionless intensity of the field and φ is its phase. The initial state has the Poisson's distribution $P_n = |Q_n|^2 = \exp(-\bar{n})\bar{n}^n/n!$.

In the resonant case, when $M\hbar\omega = E_+ - E_- = \hbar\omega_0$, the state vector $|\psi(t)\rangle$ for $t>0$ is found to be

$$|\psi(t)\rangle = \sum_{n=0} Q_n e^{-i(E_- + n\hbar\omega)t/\hbar}\left\{C_n^{(M)}(t)|+,n-M\rangle + D_n^{(M)}(t)|-,n\rangle\right\} , \qquad (5)$$

where

$$C_n^{(M)}(t) = -i\sin\chi_n^{(M)}\tau \quad ; \qquad (6a)$$

$$D_n^{(M)}(t) = \cos\chi_n^{(M)}\tau \qquad ; \quad \tau = \lambda t \qquad (6b)$$

and $\chi_n^{(M)} = n!/(n-M)!$ for $n \geqslant M$ and 0 for $n < M$.

Having found the state vector $|\Phi(t)\rangle$ we can calculate the mean photon number $\langle \hat{a}^+\hat{a}\rangle$, the mean photon amplitude $\langle \hat{a}\rangle$ as well as the mean squared photon amplitude $\langle \hat{a}^2\rangle$:

$$\langle \hat{a}^+\hat{a}\rangle = \bar{n} - M\sum_{n=0} P_n |C_n^{(M)}(t)|^2 \equiv A_0 \; ; \tag{7a}$$

$$e^{i(\omega t-\varphi)}\langle \hat{a}\rangle = \sqrt{\bar{n}}\sum_{n=0} P_n\left\{ \overset{*}{D}{}_n^{(M)} D_{n+1}^{(M)} + \sqrt{1-\frac{M}{n+1}}\, \overset{*}{C}{}_n^{(M)} C_{n+1}^{(M)} \right\} \equiv \sqrt{\bar{n}}\, A_1 \; ; \tag{7b}$$

$$e^{2i(\omega t-\varphi)}\langle \hat{a}^2\rangle = \bar{n}\sum_{n=0} P_n\left\{ \overset{*}{D}{}_n^{(M)} D_{n+2}^{(M)} + \sqrt{\left(1-\frac{M}{n+2}\right)\left(1-\frac{M}{n+1}\right)}\, \overset{*}{C}{}_n^{(M)} C_{n+2}^{(M)} \right\} \equiv \bar{n} A_2 \,. \tag{7c}$$

To analyze the squeezing of the field we introduce two slowly varying hermitian quadrature operators $\hat{a}_1$, $\hat{a}_2$ of the field

$$\hat{a}_1 = (\hat{a}e^{i(\omega t-\delta)} + \hat{a}^+e^{-i(\omega t-\delta)})/2 \; ;$$
$$a_2 = (\hat{a}e^{i(\omega t-\delta)} - \hat{a}^+e^{-i(\omega t-\delta)})/2i \; , \tag{8}$$

where δ is an arbitrary phase, which is chosen to be equal to the phase φ of the coherent field.

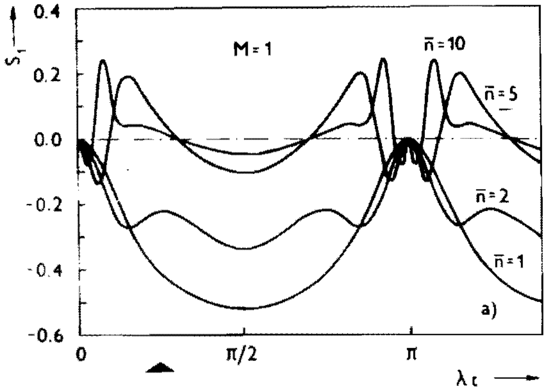


Fig.1. The time evolution of the function S_1. (a) Photon multiple M is equal to 1. The curves correspond to various values of the intensity of the coherent field $\bar{n}$. (b) Field intensity is fixed ($\bar{n}$ = 1). The curves correspond to two values of the multiple M.

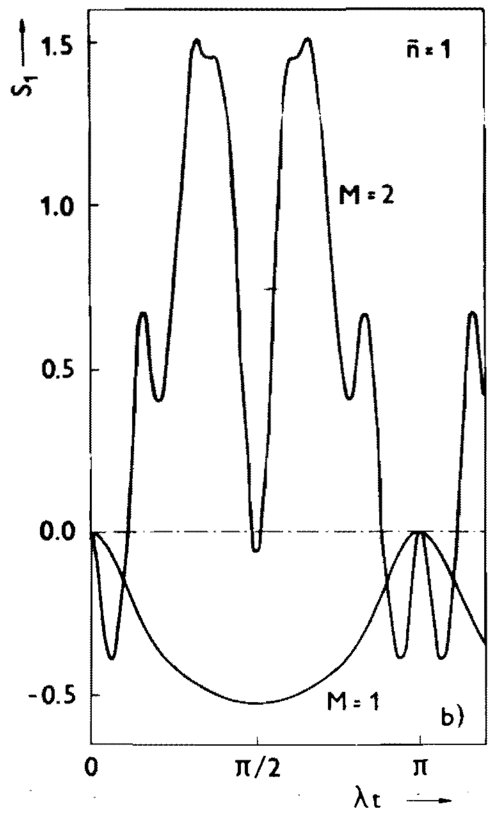
Since the squeezed states are defined as the states with a smaller uncertainty (variance) in one quadrature of the field than that associated with the coherent field, it is convenient to define two functions S_i , $i = 1,2$:

$$S_i = \frac{\langle(\Delta\hat{a}_i)^2\rangle - \langle(\Delta\hat{a}_i)^2\rangle_{coh}}{\langle(\Delta\hat{a}_i)^2\rangle_{coh}} = 4\langle(\Delta\hat{a}_i)^2\rangle - 1 \quad , \tag{9}$$

where $\langle(\Delta\hat{a}_i)^2\rangle = \langle\hat{a}_i^2\rangle - \langle\hat{a}_i\rangle^2$ and $\langle(\Delta\hat{a}_i)^2\rangle_{coh} = 1/4$.

The squeezing condition now looks very simply

$$S_i < 0 \,. \tag{10}$$



The variances of the quadrature operators can be expressed through the mean values of the photon operators (7) $\langle\hat{a}^+\hat{a}\rangle$, $\langle\hat{a}\rangle$ and $\langle\hat{a}^2\rangle$, so the functions S_i can be written as

$$S_1 = 2A_0 + 2\bar{n}(\mathrm{Re}A_2) - 4\bar{n}(\mathrm{Re}A_1)^2 , \tag{11a}$$

$$S_2 = 2A_0 - 2\bar{n}(\mathrm{Re}A_2) - 4\bar{n}(\mathrm{Im}A_1)^2 . \tag{11b}$$

The time evolution of S_1 for various values of $\bar{n}$ and M are given in Fig.1. It is seen for instance that the large magnitude of squeezing (over 52%) can be obtained for M=1 and $\bar{n}$=1 (see Fig.1a). From this figure it is also visible, that for constant M the magnitude of squeezing decreases by increasing of the field intensity $\bar{n}$. Furthermore, squeezing disappears when M is too large.

Here several comments shoud be made:

i) The functions S_i and A_i are periodical - squeezing recovers periodically with the period $T=\pi/\lambda$. This is the main difference between the nonlinear and linear multiphoton JCM.

ii) The results of the linear JCM [11] can be obtained from ours by a simple substitution $\chi_n^{(M)} \longrightarrow (\chi_n^{(M)})^{1/2}$ in (6).

iii) It can be proved by direct calculations that a "dual relation" exists by means of which one can immediately find the solution for the problem with the initial state $|+,\alpha\rangle$ (the atom is in the excited state $|+\rangle$ and the field in the coherent state $|\alpha\rangle$ at t=0):

$$\chi_n^{(M)} \longrightarrow (\chi_n^{(-M)})^{-1} \equiv \xi_n^{(M)}$$

and $M\to -M$ in all expressions, so that:

$$S_i^{|+\rangle} = S_i^{|-\rangle}\Big|_{\substack{M\to -M\\ \chi_n^{(M)}\to \xi_n^{(M)}}} ,$$

here the indeces $|\pm\rangle$ in the functions S_i reflect the initial state of the atom. The functions $S_i^{|\pm\rangle}$ are periodical too.

iv) For M=1, the atom inversion $W(t)=\langle\hat{\sigma}_3\rangle$ can be expressed in an analytical form. For the initial state $|\psi(t=0)\rangle = |-,\alpha\rangle$ we obtain the well known result of Buck and Sukumar [12]:

$$W^{|-\rangle}(t) = -\exp(-2\bar{n}\cdot\sin^2\tau)\cos(\bar{n}\cdot\sin 2\tau).$$

When the atom is in the excited state at t=0, then for the atomic inversion $W^{|+\rangle}(t)$ we find [13]:

$$W^{|+\rangle}(t) = \exp(-2\bar{n}\cdot\sin^2\tau)\cos(2\tau+\bar{n}\sin 2\tau).$$

In the nonlinear JCM the atomic inversion is a periodical function for any M and for any initial conditions.

4. The atom in the coherent state at t=0

Now we will consider the atom to be in a coherent superposition of the excited and ground states and the field to be in a coherent state at t=0:

$$|\psi(t=0)\rangle = C|+,\alpha\rangle + D|-,\alpha\rangle , \tag{12}$$

where the coefficients C and D are supposed to be normalized to one - $|C|^2+|D|^2=1$, and the relative phase between them is φ. Following [9] we choose C and D to be:

$$C = \cos\theta/2 \quad ; \quad D = e^{i\varphi}\cdot\sin\theta/2 . \tag{13}$$

Here, for simplicity, we suppose just one-photon hamiltonian (1) with M=1 (the generalization for M>1 is trivial).

The solution of the time-dependent Schrödinger equation (3) with the initial condition (12) in the resonant case is:

$$|\psi(t)\rangle = \sum_{n=0} e^{-i(n\hbar\omega+E_-)t/\hbar}\left\{ C_n(t)e^{-i\omega t}|+,n\rangle + D_n(t)|-,n\rangle\right\} , \tag{14}$$

where

$$C_n(t) = -iQ_{n+1}D\sin(n+1)\tau + Q_nC\cos(n+1)\tau ; \tag{15a}$$

$$D_n(t) = -iQ_{n-1}C\sin n\tau + Q_nD\cos n\tau \tag{15b}$$

and Q_n is defined by (4) for $n \geq 0$ and equal to zero for $n < 0$.

If we want to analyze the squeezing effects it is necessary to calculate the variance of the quadrature operators (8), or, what is the same, the functions S_i (9). To do that, the mean values $\langle \hat{a}^+\hat{a}\rangle = A_0$, $e^{i(\omega t-\varphi)}\langle \hat{a}\rangle = \sqrt{\bar{n}}A_1$ and $e^{2i(\omega t-\varphi)}\langle \hat{a}^2\rangle = \bar{n}A_2$ should be found. For instance, the mean value of the number operator A_0 is:

$$A_0 = \bar{n} + \sum_{n=0} P_n \Big\{ \cos^2\theta/2 \cdot \sin^2(n+1)\tau - \sin^2\theta/2 \cdot \sin^2 n\tau - \frac{1}{2}\sin\theta\,\sin\phi' \sqrt{\frac{n}{n+1}}\,\sin 2(n+1)\tau \Big\} . \tag{16}$$

Other mean values can be found easily too, but the expressions for them are lengthy and not transparent, therefore we do not write them explicitly here. Nevertheless it should be pointed out, that all measurable quantities do not depend on the phases ϕ (the phase of the atomic dipole) and φ (the phase of the coherent field) independently, but only on their sum $(\phi+\varphi) \equiv \phi'$. Using formulae (11) we will obtain the explicit expressions for S_i^{coh} (subscript coh indicates the coherent state of the atom at t=0).

The time evolution of the function S_1^{coh} is seen from Fig.2. and Fig.3. First of all we should stress the periodicity (with period $T=\pi/\lambda$) of the function S_1^{coh}. Second, for $\bar{n}=1$ the maximum of squeezing for $|\phi(t=0)\rangle$ defined by (12) does not exceed the value of squeezing in the case when the atom is initially in the ground state. This is not the case for weak-intensity fields (see below). Third, when the relative phase between the field and the atomic dipole ϕ' is from the interval $(0,\pi)$, then maximum squeezing is reached for times $\pi(k+1/2) < \lambda t < \pi(k+1)$, where k=0,1,2 .. (see line 3 in Fig.2a. and lines 2,3 in Fig.2b.). When $\phi' \in (\pi, 2\pi)$, then the maximum squeezing can be observed for $\pi k < \lambda t < \pi(k+1/2)$ (see line 4 in Fig.2a. and line 1 in Fig.2b.). If $\phi' = \pi$, then $S_1^{coh} = \cos^2\theta/2\, S_1^{|+\rangle} + \sin^2\theta/2\, S_1^{|-\rangle}$ and the function S_1^{coh} reaches its minima for $\lambda t = \pi k + \pi/2$ (see line 5 in Fig.2a.).

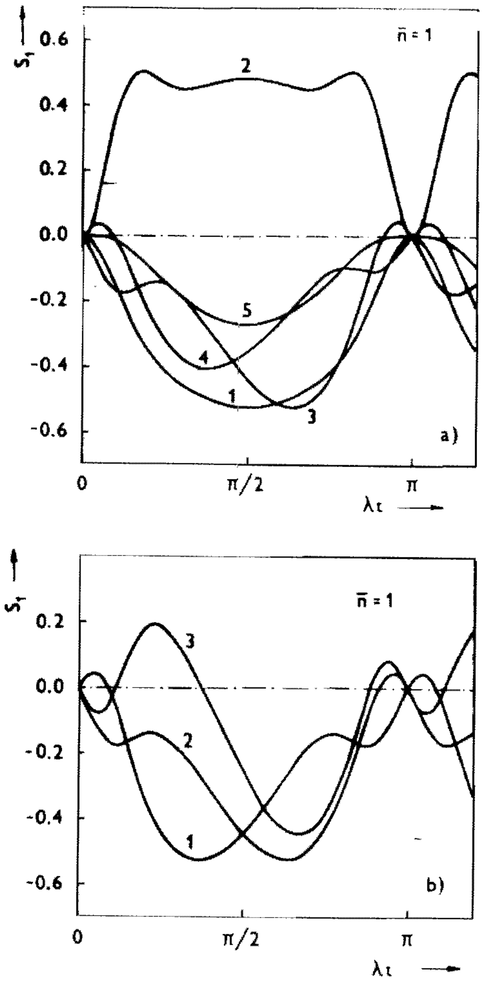


Fig.2. The time evolution of the function S_1 for $\bar{n} = 1$ and various values of the parameters θ and ϕ'. (a) line 1 - $\theta=\pi$, $\phi'=0$ (the atom in the ground state); line 2 - $\theta = 0$, $\phi' = 0$ (the atom in the excited state); line 3 - $\theta = 2\pi/3$, $\phi' = \pi/2$; line 4 - $\theta = 2\pi/3$, $\phi' = 5\pi/4$; line 5 - $\theta = 2\pi/3$, $\phi' = \pi$. (b) line 1 - $\theta = 2\pi/3$, $\phi' = 3\pi/2$ line 2 - $\theta = 2\pi/3$, $\phi' = \pi/2$; line 3 - $\theta = \pi/2$, $\phi' = \pi/2$.

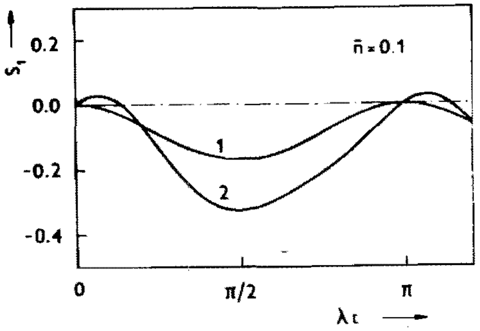


Fig.3. The time evolution of the function S_1 for $\bar{n}$ = 0.1. Line 1- the atom is in the ground state at t =0 ($\theta = \pi$, $\phi' = 0$).; line 2- the atom is in the coherent atomic state at t =0 ($\theta = 2\pi/3$; $\phi' = 3\pi/2$).

Of course, the magnitude of squeezing depends on the values of cos $\theta/2$ and sin $\theta/2$. If the atom is in the ground state at t=0 ($\theta = \pi$, so D=1 and C=0), then the maximum squeezing is reached for $\lambda t = \pi(k+1/2)$ (see line 1 in Fig.2a.). When the atom is in the excited state ($\theta = 0$, so C=1, D=0) no squeezing occurs (see line 2 in Fig.2a.). In Fig.2b. the evolution of S_1^{coh} is shown for constant $\phi' = \pi/2$ and different values of θ (lines 2 and 3). In this case the maximum squeezing is obtained for $\theta = 2\pi/3$ (line2).

As seen from Fig.2a. the relative phase ϕ' does not only influence the moment when maximum squeezing is obtained, but also the magnitude of squeezing depends on its value. This can be seen, when the lines 3,4 and 5 of Fig.2a. are compared. Here the values of θ are equal, only the relative phases ϕ' are different.

In Fig.3. it is shown, that for weak intensity fields the magnitude of squeezing can be enhanced when the coherent field interacts not with the atom in the ground state (line 1), but with the atom, which is in the coherent state at t=0 (line 2).

In this particular case ($\bar{n}=0.1$, $\theta = 2\pi/3$, $\phi' = 3\pi/2$) squeezing is enlarged approximatelly twice when the atom is in the coherent state at t=0.

We will conclude this section with a few comments:

i) When the relative phase ϕ' cannot be measured, then in the expressions for measurable quantities the average over this phase should be done, and the result for $\bar{S}_1^{coh}$ (bar over S_1^{coh} means average over ϕ') is:

$$\bar{S}_1^{coh} = \cos^2\theta/2 \cdot S_1^{|+\rangle} + \sin^2\theta/2 \cdot S_1^{|-\rangle} .$$

ii) The atomic inversion for the initial state vector (12) can be calculated in a compact analytical form [13]:

$$W^{coh}(t) = \cos^2\theta/2 \cdot W^{|+\rangle}(t) + \sin^2\theta/2 \cdot W^{|-\rangle}(t)$$

$$+\sqrt{\bar{n}}\, \sin\theta \cdot \sin\phi' \cdot \exp(-2\bar{n}\sin^2\tau)\, \sin(\bar{n}\sin 2\tau + 2\tau).$$

This again is a periodical function and it is seen here explicitly, that when the average over ϕ' is done, then:

$$\bar{W}^{coh}(t) = \cos^2\theta/2 \cdot W^{|+\rangle}(t) + \sin^2\theta/2 \cdot W^{|-\rangle}(t).$$

iii) In the weak-intensity limit $\bar{n} \to 0$ for the functions S_i^{coh} we have:

$$\lim_{\bar{n}\to 0} S_1^{coh} = 4\cos^2\theta/2 \cdot \sin^2\tau \left(\tfrac{1}{2} - \sin^2\theta/2 \cdot \sin^2\phi'\right)$$

$$\lim_{\bar{n}\to 0} S_2^{coh} = 4\cos^2\theta/2 \cdot \sin^2\tau \left(\tfrac{1}{2} - \sin^2\theta/2 \cdot \cos^2\phi'\right).$$

This result can be identified with that derived by Knight [9,10] when at t=0 the atom was in a coherent state and the field in the vacuum state. In the weak-intensity limit squeezing is produced just by the establishment of an atomic coherent state. Maximum amplitude of squeezing in this case is 25% (for $\phi' = \pi/2$ and $\theta = 2\pi/3$).

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Бужек В. E17-88-399

Сжатие в нелинейной модели Джейнса-Каммингса

Показано, что если в модель Джейнса-Каммингса ввести нелинейность типа Бака-Сукумара, то сжатие поля излучения периодически возобновляется. Эффект сжатия исследован для различных начальных состояний атома /в частности, исследованы процессы, когда в начальный момент атом находится в основном возбужденном или когерентном состояниях/.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1988

Buzek V. E17-88-399

Squeezing in the Nonlinear Jaynes-Cummings Model

It is shown that if nonlinearity a la Buck and Sukumar is introduced into Jaynes-Cummings model, then squeezing of the radiation field exhibits periodicity. Squeezing is examined for various initial states of the atom (ground, excited and coherent atomic states are taken into account).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1988