

Объединенный институт ядерных исследований дубна

E17-88-260

1988

Fam Le Kien^{*}, Ho Trung Dung^{*}, A.S.Shumovsky

MULTIPHOTON SPONTANEOUS EMISSION OF AN ATOM IN A DETUNED DAMPED CAVITY

Submitted to "Physica A"

Moscow State University, USSR

Cavity electrodynamics has been one of the central topics of quantum optics in recent years. Interesting new aspects of single-atom single-mode behaviour have come to light. These effects include, for example, the enhancement $^{/1'}$ and suppression $^{/2}$. $^{3'}$ of spontaneous emission. The exact solution for the one-photon-transition spontaneous emission from a two-level atom in a detuned damped cavity has been derived $^{/4.6'}$. In the present paper we treat an analogous situation with multiphoton resonance.

In the rotating-wave approximation the Liouville operator for the two-level atom interacting through the multiphotontransition mechanism with a resonant single-mode radiation field in a detuned damped cavity reads as

$$\mathbf{L} = [\mathbf{H}/\mathbf{\hat{b}},] + \mathbf{i}\Lambda_{\mathbf{p}}. \tag{1}$$

The corresponding Hamiltonian H and the field-damping Liouvillian Λ_r are given by

$$H = \hbar\omega_0 S^{\pi} + \hbar\omega a^{+}a + \hbar g(S^{+}a^{m} + S^{-}a^{+m}), \qquad (2)$$

$$\Lambda_{\mu}(....) = \kappa \{ [a(....), a^{+}] + [a, (...) a^{+}] \}.$$

Here $S^{\mathbf{x}}$ and $S^{\mathbf{t}}$ are the population-inversion and transition operators of the atom, $\omega_0 = m\omega + \Delta$ is the frequency of the atomic transition, ω is the frequency of the cavity field mode which is represented by annihilation and creation operators a and \mathbf{a}^+ , \mathbf{g} is the atom-field coupling constant, \mathbf{m} is the photon multiple of transitions, $\kappa = \omega/2\mathbf{Q}$ is the cavity field damping factor, \mathbf{Q} is the cavity quality factor, and Δ is the detuning parameter.

The statistical density operator p for the combined atomfield system obeys the Liouville equation

$$\frac{d\rho(t)}{dt} = -iL\rho(t).$$
(3)

We consider the special initial case in which the atom is initially inverted and the radiation field is in its vacuum state

)

iI

$$\rho(0) = |+; 0 > <+; 0|.$$
(4)

The equations relevant for calculating atomic population inversion are found to be $^{\prime 7\prime }$

$$<8^{z} > = <+; 0 | \rho | +; 0 > -\frac{1}{2},$$

$$\begin{bmatrix}
0 & 0 & -ig\sqrt{m!} & ig\sqrt{m!} \\
0 & 2\kappa m & ig\sqrt{m!} & -ig\sqrt{m!} \\
-ig\sqrt{m!} & ig\sqrt{m!} & \kappa m + i\Delta & 0 \\
ig\sqrt{m!} & -ig\sqrt{m!} & 0 & \kappa m - i\Delta
\end{bmatrix} \begin{bmatrix} <+; 0 | \rho | +; 0 > \\
<-; m | \rho | +; 0 > \\
<+; 0 | \rho | -; m > \\
<+; 0 | \rho (0) | +; 0 > = 1, <-; m | \rho (0) | -; m > = <-; m | \rho (0) | +; 0 > = <+, 0 | \rho (0) | -; m > = 0.$$
(5a)
$$(5a)$$

(5c)

It is seen from eq. (5b) that the effects of the photon multiplicity of transitions (the multiple of resonance m) on the spontaneous emission of the atom in the cavity can be taken into account by introducing new effective coupling $(g\sqrt{m})$ and damping (κ m) constants.

The eigenvalues of the matrix in eq. (5b) are

$$\lambda_{1,2,3,4} = -\kappa m \pm \sqrt{\frac{1}{2} \left[\left(\kappa^2 m^2 - \Delta^2 - 4g^2 m! \right) \pm \sqrt{\left(\kappa^2 m^2 - \Delta^2 - 4g^2 m! \right)^2 + 4\Delta^2 \kappa^2 m^2} \right]}.$$
(6)

By using the notation

$$D(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \det \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{\mathbf{a} + \kappa \mathbf{m}} & \frac{1}{\mathbf{b} + \kappa \mathbf{m}} & \frac{1}{\mathbf{c} + \kappa \mathbf{m}} \\ \frac{1}{\mathbf{a} + 2\kappa \mathbf{m}} & \frac{1}{\mathbf{b} + 2\kappa \mathbf{m}} & \frac{1}{\mathbf{c} + 2\kappa \mathbf{m}} \end{vmatrix}$$

$$\mathbf{A} = -\frac{D(\lambda_{2}, \lambda_{3}, \lambda_{4})}{\lambda_{1}} + \frac{D(\lambda_{3}, \lambda_{4}, \lambda_{1})}{\lambda_{2}} - \frac{D(\lambda_{4}, \lambda_{1}, \lambda_{2})}{\lambda_{3}} + \frac{D(\lambda_{1}, \lambda_{2}, \lambda_{3})}{\lambda_{4}} (7)$$

we can represent the exact solution for the atomic population inversion in the form

$$\langle S^{\mathbf{z}} \rangle = -\frac{1}{2} + \frac{1}{A} \left[-\frac{D(\lambda_{\mathbf{z}}, \lambda_{\mathbf{3}}, \lambda_{\mathbf{4}})}{\lambda_{\mathbf{1}}} e^{\lambda_{\mathbf{1}} t} + \frac{D(\lambda_{\mathbf{3}}, \lambda_{\mathbf{4}}, \lambda_{\mathbf{1}})}{\lambda_{\mathbf{g}}} e^{\lambda_{\mathbf{g}} t} - \frac{D(\lambda_{\mathbf{4}}, \lambda_{\mathbf{1}}, \lambda_{\mathbf{g}})}{\lambda_{\mathbf{3}}} e^{\lambda_{\mathbf{3}} t} + \frac{D(\lambda_{\mathbf{1}}, \lambda_{\mathbf{g}}, \lambda_{\mathbf{3}})}{\lambda_{\mathbf{4}}} e^{\lambda_{\mathbf{4}} t} \right].$$

$$(8)$$

In the case of an ideal (undamped) cavity ($\kappa = 0$), the expression (8) reduces to

$$\langle S^{z} \rangle = \frac{1}{2} - \frac{g^{2} m!}{g^{2} m! + \Delta^{2}/4} \sin^{2}(t\sqrt{g^{2} m! + \Delta^{2}/4}).$$
 (9)

Under the condition of exact resonance ($\Delta = 0$), the expression (8) reads as follows: (a) For $\kappa m < 2g\sqrt{m}i$:

$$\langle S^{2} \rangle = -\frac{1}{2} + \frac{e}{4g^{2}m! - \kappa^{2}m^{2}} \left\{ 4g^{2}m! \cos^{2}(\frac{t}{2}\sqrt{4g^{2}m! - \kappa^{2}m^{2}}) - \frac{1}{2} + \frac{e}{4g^{2}m! - \kappa^{2}m^{2}} + \frac{1}{2} \sqrt{4g^{2}m! - \kappa^{2}m^{2}} + \frac{1}{2} \sqrt{$$

(b) For
$$\kappa m = 2g\sqrt{m!}$$
:
 $\langle S^{2} \rangle = -\frac{1}{2} + \frac{-2g\sqrt{m!t}}{2} (1 + 2g\sqrt{m!t} + g^{2}m!t^{2})$ (11)

(c) For
$$\kappa m > 2g\sqrt{m!}$$
:
 $\langle S^{2} \rangle = -\frac{1}{2} + \frac{e}{\kappa^{2}m^{2} - 4g^{2}m!} \left\{ -4g^{2}m! \cosh^{2}(\frac{t}{2}\sqrt{\kappa^{2}m^{2} - 4g^{2}m!}) + \frac{e}{\kappa^{2}m^{2} - 4g^{2}m!} + \frac{e}{\kappa^{2}m^{2} - 4g^{2$

The expression (10) describes the oscillatory regime of evolution. In this regime the atomic population inversion $\langle S^z \rangle$ contains the terms oscillating at the frequency $\sqrt{4g^2m_1^2 - \kappa^2m^2}$

and damped at the rate κm . The condition $\kappa m < 2g\sqrt{m}$ for the oscillatory regime has a very clear physical meaning: the photons emitted spontaneously by the excited atom are stored in the cavity long enough for the atom to reabsorb them.

Expressions (11) and (12) describe the overdamped regime of evolution which occurs if $\kappa m \ge 2g\sqrt{m!}$ and $\Delta = 0$. In this regime the atomic population inversion $\langle S^Z \rangle$ exhibits an irreversible damping with the characteristics time r_c ,

$$r_{\rm c}^{-1} = [\kappa m - \sqrt{\kappa^2 m^2 - 4g^2 m!}] \quad (\kappa m \ge 2g\sqrt{m!}, \ \Delta = 0). \tag{13}$$

As soon as $\kappa m \gg 2g\sqrt{m!}$ the decay of the population inversion is quasi-exponential with the characteristic time

$$r_{\rm c} \approx \frac{\kappa m}{2g^2 m!}$$
 (14)

We take into account the relation $\kappa = \omega/2 \, Q = \omega_0/2 \, Qm$ between the cavity field damping rate κ and the quality factor Q of the cavity in the case of exact resonance ($\Delta = 0$). Then, one can easily show from eq.(13) that the increase (decrease) of the Q-factor in the region $Q \leq \omega_0/4g\sqrt{m!}$ leads to the enhancement (suppression) of the spontaneous emission of the atom in the cavity in the overdamped regime of evolution. For some experiments^(1,8) the cavity spontaneous emission time τ_c may be smaller than the free-space spontaneous emission time τ_c . This is the so-called effect of spontaneous emission enhancement due to the presence of the cavity^(8,9). The subsequent increase of Q in the region $Q > \omega_0/4g\sqrt{m!}$ leads to the oscillatory regime of evolution. The self-induced Rabi oscillations eventually decay away with the characteristic time $\tau_c = 2Q/\omega_0$.

For the case of an arbitrary detuning Δ and an arbitrary field damping κ , the characteristic time r_c of the spontaneous emission process can be determined by the smallest value among the real parts of the eigenvalues (6) to read

$$\tau_{\rm c}^{-1} = \kappa {\rm m} - \sqrt{\frac{1}{2}} \left[\left(\kappa^2 {\rm m}^2 - \Delta^2 - 4 {\rm g}^2 {\rm m}^2\right) + \sqrt{\left(\kappa^2 {\rm m}^2 - \Delta^2 - 4 {\rm g}^2 {\rm m}^2\right)^2 + 4 \,\Delta^2 \,\kappa^2 \,{\rm m}^2} \right]. \tag{15}$$

It can be shown from eq.(15) that

$$\frac{\mathrm{d}r_{\mathrm{c}}}{\mathrm{d}|\Delta|} > 0. \tag{16}$$

Therefore, the larger the detuning is, the larger the spontaneous emission characteristic time r_c becomes. Evaluating eq.(15) in the limit of large detuning, one finds

$$r_{c} \approx \frac{\Delta^{2}}{2g^{2}m!\kappa m} \quad (\Delta^{2} \gg \kappa^{2}m^{2} + 4g^{2}m!). \qquad (17)$$

Hence, we can see that if the cavity is enough off-resonant for the atomic transition, spontaneous emission can even be altogether suppressed 2,3 . Expanding the right-hand side part of eq. (15) in terms of $g^2m!$, one obtains the evaluation

$$r_{\rm c} \approx \frac{\kappa^2 {\rm m}^2 + \Delta^2}{2 {\rm g}^2 {\rm m}! \, \kappa {\rm m}} \left({\rm g}^2 {\rm m}! \ll \kappa^2 {\rm m}^2 + \Delta^2 \right). \tag{18}$$

In fig. 1 we plot the time dependence of the atomic population inversion $\langle S^z \rangle$ calculated for different values of $\Delta_{eff} = \Delta/g\sqrt{m!}$ and $\kappa_{eff} = \kappa m/g\sqrt{m!}$. It is seen from the figure that the detuning and enough large damping of the cavity lead to the suppression of the spontaneous emission process. The oscillatory (overdamped) behaviour of $\langle S^z \rangle$ occurs in the case of





4

5



Fig.2. The dependence on $\kappa_{\rm eff}$ of the characteristic time $r_{\rm c}$ calculated for different values of $\Delta_{\rm eff}$.

high-Q (low-Q) cavity. These results have been obtained for the one-photon-transition $case^{/\$}$. Therefore, they are not surprising in the multiphoton case.

In fig.2 we show the dependence upon κ_{eff} of the characteristic time r_{c} calculated for different values of Δ_{eff} . Now we consider the radiation (energy change) rate of the atom

$$I(t) = -\frac{d}{dt} \langle S^{z} \rangle.$$
 (19)

In the general case of detuning and damping, the expression for I is found from eq.(8) to be

$$I(t) = \frac{1}{A} [D(\lambda_{2}, \lambda_{3}, \lambda_{4})e^{\lambda_{1}t} - D(\lambda_{3}, \lambda_{4}, \lambda_{1})e^{\lambda_{2}t} + D(\lambda_{4}, \lambda_{1}, \lambda_{2})e^{\lambda_{3}t} - D(\lambda_{1}, \lambda_{2}, \lambda_{3})e^{\lambda_{4}t}].$$

$$(20)$$

In the case of an ideal cavity, this expression reduces to

$$I(t) = \frac{g^2 m!}{\sqrt{g^2 m! + \Delta^2/4}} \sin(2t\sqrt{g^2 m! + \Delta^2/4}).$$
(21)

Oscillating in times, expression (21) can take both positive and negative values. It describes the periodical change of emission and absorption processes. The corresponding maximal value of I(t) is equal to

$$I_{max} = \frac{g^2 m!}{\sqrt{g^2 m! + \Delta^2 / 4}}$$
(22)

and periodically occurs at the times

$$t_{max} = \frac{\pi(1+4k)}{4\sqrt{g^2m! + \Delta^2/4}} \qquad (k = 0, 1, 2, ...).$$
(23)

In the case of exact resonance $(\Delta = 0)$, one finds from eqs.(8), (9) and (10) the following:

(a) For $\kappa m < 2g\sqrt{m!}$

$$I(t) = \frac{4g^{2}m!}{4g^{2}m! - \kappa^{2}m^{2}} e^{-\kappa mt} \sin\left(\frac{t}{2}\sqrt{4g^{2}m! - \kappa^{2}m^{2}}\right) \times$$

$$\times \left\{ \kappa m \sin\left(\frac{t}{2}\sqrt{4g^{2}m! - \kappa^{2}m^{2}}\right) + \sqrt{4g^{2}m! - \kappa^{2}m^{2}} \cos\left(\frac{t}{2}\sqrt{4g^{2}m! - \kappa^{2}m^{2}}\right) \right\}.$$
(24)

This expression describes the radiation rate of the atom in the oscillatory regime. The maximal value of I(t) is equal to

$$I_{max} = g\sqrt{m!} \exp\left[-\frac{\kappa m}{\sqrt{4g^2 m! - \kappa^2 m^2}} \arccos\left(\frac{\kappa^2 m^2}{4g^2 m!}\right)\right] \times \left(\frac{\kappa m}{2g\sqrt{m!}} + \sqrt{\frac{\kappa^2 m^2}{4g^2 m!} + 1}\right).$$
(25)

and occurs at

$$t_{max} = \frac{1}{\sqrt{4g^2 m! - \kappa^2 m^2}} \arccos\left(\frac{\kappa^2 m^2}{4g^2 m!}\right)$$
 (26)

6

(b) For
$$\kappa m = 2g\sqrt{m!}$$

 $I(t) = 2g^2 m! t(g\sqrt{m!} t + 1) e^{-2g\sqrt{m!} t}$. (27)

This expression has a peak equal to

$$I_{max} = g\sqrt{m!} (1 + \sqrt{2}) e^{-\sqrt{2}}, \qquad (28)$$

at

$$t_{max} = \frac{1}{\sqrt{2}} \frac{1}{g\sqrt{m!}}$$
 (29)

(c) For
$$\kappa m > 2g\sqrt{m!}$$

$$I(t) = \frac{4g^2 m!}{\kappa^2 m^2 - 4g^2 m!} e^{-\kappa mt} \sinh\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) \times \left\{ \kappa m \sinh\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) + \sqrt{\kappa^2 m^2 - 4g^2 m!} \cosh\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) \right\}.$$
(30)

This expression describes the radiation rate of the atom in the overdamped regime of evolution. Its maximum value is equal to

$$I_{max} = g\sqrt{m!} \left(\frac{\kappa^2 m^2}{4g^2 m!} + \sqrt{\frac{\kappa^4 m^4}{16g^4 (m!)^2} - 1} \right)^{-\kappa m} / \sqrt{\kappa^2 m^2 - 4g^2 m!} \times \left(\frac{\kappa m}{2g\sqrt{m!}} + \sqrt{\frac{\kappa^2 m^2}{4g^2 m!} + 1} \right)$$
(31)

and occurs at the time

$$t_{\max} = \frac{1}{\sqrt{\kappa^2 m^2 - 4g^2 m l}} \ln \left(\frac{\kappa^2 m^2}{4g^2 m l} + \sqrt{\frac{\kappa^4 m^4}{16g^4 (m!)^2}} - 1 \right). \quad (32)$$



1

Í

IJ

Fig.3. Time evolution of the radiation rate I(t). Curves are marked as in fig.1.



Fig.4. Maximal radiation rate I_{max} in the case of exact resonance plotted as a function of the effective damping parameter κ_{eff} .



Fig. 5. Peak time t_{max} in the case of exact resonance plotted as a function of the effective damping parameter κ_{aff} .

In fig.3 we show the time evolution of the radiation rate I calculated for different values of $\Delta_{eff} = \Delta/g\sqrt{m!}$ and $\kappa_{eff} = \kappa m/g\sqrt{m!}$. The parameters are the same as in fig.1.

In figs. 4 and 5 we plot the maximal radiation rate I and the peak time t_{max} against the effective damping parameter $\kappa_{eff} = \kappa m/g \sqrt{m!}$ for the case of exact resonance ($\Delta = 0$). It is seen from the figures that the κ -dependences of these quantities are described by monotonously decreased functions.

Thus, in this paper we have examined the multiphoton spontaneous emission of a two-level atom in a detuned damped cavity. The exact expressions for the atomic population inversion, radiation rate and characteristic times have been obtained. The influence of the cavity field damping and detuning on the spontaneous emission has been analytically and numerically discussed. It has been shown that the effects of transitionphoton multiplicity (multiple of resonance) in the model considered can be taken into account by introducing new effective coupling and damping constants. The enhancement and suppression of multiphoton spontaneous emission and the possibility of various regimes of evolution have also been demonstrated.

REFERENCES

- 1. Goy P. et al. Phys.Rev.Lett., 1983, 50, p.1903.
- Hulet R.G., Hilfer E.S., Kleppner D. Phys.Rev.Lett., 1985, 55, p.2137.
- 3. Jhe W. et al. Phys.Rev.Lett., 1987, 58, p.666.
- 4. Seke J. Phys.Rev., 1986, A33, p.4409.
- 5. Seke J. Rattay F. J.Opt.Soc.Am., 1987, B4, p.380.
- 6. Agarwal G.S., Puri R.R. Phys.Rev., 1986, A33, p.1757.
- 7. Aliskenderov E.I. et al. J.Phys., 1987, A20, p.6265.
- 8. Haroche S., Raimond J.M. In: Adv. Atom. Mol. Phys., v.20,
- (Eds.: D.R.Bates and B.Bederson, Academic Press, New York, 1985), p.347.
- Purcell E.M. Phys.Rev., 1946, 69 p.681.

Received by Publishing Department on April 21, 1988.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices — in US , including the packing and registered postage.

D13-84-63	Proceedings of the XI International Symposium on Nuclear Electronics. Bratislava, Czechoslovakia, 1983.	12.00
E1,2-84-160	Proceedings of the 1983 JINR-CERN School of Physics. Tabor, Czechoslovakia, 1983.	6.50
D2-84-366	Proceedings of the VII International Conference on the Problems of Quantum Field Theory. Alushta, 1984.	11.00
D1,2-84-599	Proceedings of the VII International Seminar on High Energy Physics Problems. Dubna, 1984.	12.00
D17-84-850	Proceedings of the III International Symposium on Selected ¹ }Topics in Statistical Mechanics. Dubna, 1984 (2 volumes).	22.00
	Proceedings of the IX All-Union Conference on Charged Particle Accelerators. Dubna, 1984. (2 volumes)	25.00
D11-85-791	Proceedings of the International Conference on Computer Algebra and Its Applications in Theoretical Physics. Dubna, 1985.	12.00
D13-85-793	Proceedings of the XII International Symposium on Nuclear Electronics, Dubna, 1985.	14.00
D4-85-851	Proceedings of the International School on Nuclear Structure Alushta, 1985.	11.00
D1,2-86-668	Proceedings of the VIII International Seminar on High Energy Physics Problems, Dubna, 1986 (2 volumes)	23.00
D3,4,17-86-747	Proceedings of the V International School on Neutron Physics. Alushta, 1986.	25.00
D9-87-105	Proceedings of the X All-Union Conference on Charged Particle Accelerators. Dubna, 1986 (2 volumes)	25.00
D7-87-68	Proceedings of the International School-Seminar on Heavy Ion Physics. Dubna, 1986.	25.00
D2-87-123	Proceedings of the Conference "Renormalization Group-86". Dubna, 1986.	12.00
D4-87-692	Proceedings of the International Conference on the Theory of Few Body and Quark-Hadronic Systems. Dubna, 1987.	12.00
D2-87-798	Proceedings of the VIII International Conference on the Problems of Quantum Field Theory. Alushta, 1987.	10.00
D14-87-799	Proceedings of the International Symposium on Muon and Pion Interactions with Matter. Dubna, 1987.	13.00
D17-88-95	Proceedings of the IV International Symposium on Selected Topics in Statistical Mechanics. Dubna, 1987.	14.00
C	orders for the above-mentioned books can be sent at the address: Publishing Department, JINR Head Post Office, P.O.Box 79 101000 Moscow, USSR	

Фам Ле Киен, Хо Чунг Зунг, Шумовский А.С. E17-88-260 Многофотонное спонтанное излучение атома в резонаторе с расстройкой и конечной добротностью

Исследовано многофотонное спонтанное излучение двухуровневого атома в резонаторе с расстройкой и конечной добротностью. Получены точные выражения для атомной инверсии заселенности и скорости излучения. Вычислены также различные характерные времена.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Fam Le Kien, Ho Trung Dung, Shumovsky A.S.E17-88-260Multiphoton Spontaneous Emission of an Atomin a Detuned Damped Cavity

The multiphoton spontaneous emission of a two-level atom in a detuned damped cavity is investigated. The exact expressions for the atomic population inversion and the radiation rate are obtained. The various characteristics times are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988

o