объединенный
институт
ядерных
исследований
дубна

E17-88-260

Fam Le Kien*, Ho Trung Dung*,
A.S.Shumovsky

MULTIPHOTON SPONTANEOUS EMISSION OF AN ATOM IN A DETUNED DAMPED CAVITY

Submitted to "Physica A"

* Moscow State University, USSR

1988

Cavity electrodynamics has been one of the central topics of quantum optics in recent years. Interesting new aspects of single-atom single-mode behaviour have come to light. These effects include, for example, the enhancement [1] and suppression [2,3] of spontaneous emission. The exact solution for the one-photon-transition spontaneous emission from a two-level atom in a detuned damped cavity has been derived [4,6]. In the present paper we treat an analogous situation with multiphoton resonance.

In the rotating-wave approximation the Liouville operator for the two-level atom interacting through the multiphoton-transition mechanism with a resonant single-mode radiation field in a detuned damped cavity reads as

$$L=[H/\hbar,]+i\Lambda_F. \qquad (1)$$

The corresponding Hamiltonian H and the field-damping Liouvillian Λ_F are given by

$$H=\hbar\omega_0 S^z+\hbar\omega a^+a+\hbar g(S^+a^m+S^-a^{+m}),$$
$$\Lambda_F(\ldots.)=\kappa\{[a(\ldots.),a^+]+[a,(\ldots)a^+]\}. \qquad (2)$$

Here S^z and $S^\pm$ are the population-inversion and transition operators of the atom, $\omega_0=m\omega+\Delta$ is the frequency of the atomic transition, ω is the frequency of the cavity field mode which is represented by annihilation and creation operators a and a^+, g is the atom-field coupling constant, m is the photon multiple of transitions, $\kappa=\omega/2Q$ is the cavity field damping factor, Q is the cavity quality factor, and Δ is the detuning parameter.

The statistical density operator ρ for the combined atom-field system obeys the Liouville equation

$$\frac{d\rho(t)}{dt}=-iL\rho(t). \qquad (3)$$

We consider the special initial case in which the atom is initially inverted and the radiation field is in its vacuum state

$$\rho(0) = |+;0\rangle\langle +;0| . \tag{4}$$

The equations relevant for calculating atomic population inversion are found to be[7]

$$\langle S^z\rangle = \langle +;0|\rho|+;0\rangle - \frac{1}{2}, \tag{5a}$$

$$\left[\frac{d}{dt} + \begin{bmatrix} 0 & 0 & -ig\sqrt{m!} & ig\sqrt{m!} \\ 0 & 2\kappa m & ig\sqrt{m!} & -ig\sqrt{m!} \\ -ig\sqrt{m!} & ig\sqrt{m!} & \kappa m + i\Delta & 0 \\ ig\sqrt{m!} & -ig\sqrt{m!} & 0 & \kappa m - i\Delta \end{bmatrix}\right] \begin{bmatrix} \langle +;0|\rho|+;0\rangle \\ \langle -;m|\rho|-;m\rangle \\ \langle -;m|\rho|+;0\rangle \\ \langle +;0|\rho|-;m\rangle \end{bmatrix} = 0, \tag{5b}$$

$$\langle +;0|\rho(0)|+;0\rangle = 1, \langle -;m|\rho(0)|-;m\rangle = \langle -;m|\rho(0)|+;0\rangle = \langle +,0|\rho(0)|-;m\rangle = 0. \tag{5c}$$

It is seen from eq. (5b) that the effects of the photon multiplicity of transitions (the multiple of resonance m) on the spontaneous emission of the atom in the cavity can be taken into account by introducing new effective coupling ($g\sqrt{m!}$) and damping (κm) constants.

The eigenvalues of the matrix in eq. (5b) are

$$\lambda_{1,2,3,4} = -\kappa m \pm \sqrt{\frac{1}{2}\left[(\kappa^2 m^2 - \Delta^2 - 4g^2 m!) \pm \sqrt{(\kappa^2 m^2 - \Delta^2 - 4g^2 m!)^2 + 4\Delta^2\kappa^2 m^2}\right]} . \tag{6}$$

By using the notation

$$D(a,b,c) = \det \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{a+\kappa m} & \frac{1}{b+\kappa m} & \frac{1}{c+\kappa m} \\ \frac{1}{a+2\kappa m} & \frac{1}{b+2\kappa m} & \frac{1}{c+2\kappa m} \end{vmatrix}$$

$$A = -\frac{D(\lambda_2,\lambda_3,\lambda_4)}{\lambda_1} + \frac{D(\lambda_3,\lambda_4,\lambda_1)}{\lambda_2} - \frac{D(\lambda_4,\lambda_1,\lambda_2)}{\lambda_3} + \frac{D(\lambda_1,\lambda_2,\lambda_3)}{\lambda_4} \tag{7}$$

we can represent the exact solution for the atomic population inversion in the form

$$\langle S^z\rangle = -\frac{1}{2} + \frac{1}{A}\left[-\frac{D(\lambda_2,\lambda_3,\lambda_4)}{\lambda_1} e^{\lambda_1 t} + \frac{D(\lambda_3,\lambda_4,\lambda_1)}{\lambda_2} e^{\lambda_2 t} - \frac{D(\lambda_4,\lambda_1,\lambda_2)}{\lambda_3} e^{\lambda_3 t} + \frac{D(\lambda_1,\lambda_2,\lambda_3)}{\lambda_4} e^{\lambda_4 t}\right]. \tag{8}$$

In the case of an ideal (undamped) cavity ($\kappa = 0$), the expression (8) reduces to

$$\langle S^z\rangle = \frac{1}{2} - \frac{g^2 m!}{g^2 m! + \Delta^2/4} \sin^2\left(t\sqrt{g^2 m! + \Delta^2/4}\right). \tag{9}$$

Under the condition of exact resonance ($\Delta = 0$), the expression (8) reads as follows:

(a) For $\kappa m < 2g\sqrt{m!}$:

$$\langle S^z\rangle = -\frac{1}{2} + \frac{e^{-\kappa m t}}{4g^2 m! - \kappa^2 m^2}\Big\{4g^2 m! \cos^2\left(\frac{t}{2}\sqrt{4g^2 m! - \kappa^2 m^2}\right) - \kappa^2 m^2 \cos\left(t\sqrt{4g^2 m! - \kappa^2 m^2}\right) + \kappa m\sqrt{4g^2 m! - \kappa^2 m^2}\sin\left(t\sqrt{4g^2 m! - \kappa^2 m^2}\right)\Big\}. \tag{10}$$

(b) For $\kappa m = 2g\sqrt{m!}$:

$$\langle S^z\rangle = -\frac{1}{2} + e^{-2g\sqrt{m!}\,t}\left(1 + 2g\sqrt{m!}\,t + g^2 m!\, t^2\right) \tag{11}$$

(c) For $\kappa m > 2g\sqrt{m!}$:

$$\langle S^z\rangle = -\frac{1}{2} + \frac{e^{-\kappa m t}}{\kappa^2 m^2 - 4g^2 m!}\Big\{-4g^2 m!\,\mathrm{ch}^2\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) + \kappa^2 m^2\,\mathrm{ch}\left(t\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) + \kappa m\sqrt{\kappa^2 m^2 - 4g^2 m!}\,\mathrm{sh}\left(t\sqrt{\kappa^2 m^2 - 4g^2 m!}\right)\Big\}. \tag{12}$$

The expression (10) describes the oscillatory regime of evolution. In this regime the atomic population inversion $\langle S^z\rangle$ contains the terms oscillating at the frequency $\sqrt{4g^2 m! - \kappa^2 m^2}$

and damped at the rate κm. The condition $\kappa m < 2g\sqrt{m!}$ for the oscillatory regime has a very clear physical meaning: the photons emitted spontaneously by the excited atom are stored in the cavity long enough for the atom to reabsorb them.

Expressions (11) and (12) describe the overdamped regime of evolution which occurs if $\kappa m \geq 2g\sqrt{m!}$ and $\Delta = 0$. In this regime the atomic population inversion $\langle S^z \rangle$ exhibits an irreversible damping with the characteristics time τ_c,

$$\tau_c^{-1} = [\kappa m - \sqrt{\kappa^2 m^2 - 4g^2 m!}] \quad (\kappa m \geq 2g\sqrt{m!},\ \Delta = 0). \qquad (13)$$

As soon as $\kappa m >> 2g\sqrt{m!}$ the decay of the population inversion is quasi-exponential with the characteristic time

$$\tau_c \approx \frac{\kappa m}{2g^2 m!}. \qquad (14)$$

We take into account the relation $\kappa = \omega/2Q = \omega_0/2Qm$ between the cavity field damping rate κ and the quality factor Q of the cavity in the case of exact resonance ($\Delta = 0$). Then, one can easily show from eq.(13) that the increase (decrease) of the Q-factor in the region $Q \leq \omega_0/4g\sqrt{m!}$ leads to the enhancement (suppression) of the spontaneous emission of the atom in the cavity in the overdamped regime of evolution. For some experiments[1,8] the cavity spontaneous emission time τ_c may be smaller than the free-space spontaneous emission time τ_0. This is the so-called effect of spontaneous emission enhancement due to the presence of the cavity[8,9]. The subsequent increase of Q in the region $Q > \omega_0/4g\sqrt{m!}$ leads to the oscillatory regime of evolution. The self-induced Rabi oscillations eventually decay away with the characteristic time $\tau_c = 2Q/\omega_0$.

For the case of an arbitrary detuning Δ and an arbitrary field damping κ, the characteristic time τ_c of the spontaneous emission process can be determined by the smallest value among the real parts of the eigenvalues (6) to read

$$\tau_c^{-1} = \kappa m - \sqrt{\frac{1}{2}[(\kappa^2 m^2 - \Delta^2 - 4g^2 m!) + \sqrt{(\kappa^2 m^2 - \Delta^2 - 4g^2 m!)^2 + 4\Delta^2\kappa^2 m^2}]}. \qquad (15)$$

It can be shown from eq.(15) that

$$\frac{d\tau_c}{d|\Delta|} > 0. \qquad (16)$$

Therefore, the larger the detuning is, the larger the spontaneous emission characteristic time τ_c becomes. Evaluating eq.(15) in the limit of large detuning, one finds

$$\tau_c \approx \frac{\Delta^2}{2g^2 m!\,\kappa m} \quad (\Delta^2 >> \kappa^2 m^2 + 4g^2 m!). \qquad (17)$$

Hence, we can see that if the cavity is enough off-resonant for the atomic transition, spontaneous emission can even be altogether suppressed[2,3]. Expanding the right-hand side part of eq. (15) in terms of $g^2 m!$, one obtains the evaluation

$$\tau_c = \frac{\kappa^2 m^2 + \Delta^2}{2g^2 m!\,\kappa m} \quad (g^2 m! << \kappa^2 m^2 + \Delta^2). \qquad (18)$$

In fig. 1 we plot the time dependence of the atomic population inversion $\langle S^z \rangle$ calculated for different values of $\Delta_{eff} = \Delta/g\sqrt{m!}$ and $\kappa_{eff} = \kappa m/g\sqrt{m!}$. It is seen from the figure that the detuning and enough large damping of the cavity lead to the suppression of the spontaneous emission process. The oscillatory (overdamped) behaviour of $\langle S^z \rangle$ occurs in the case of

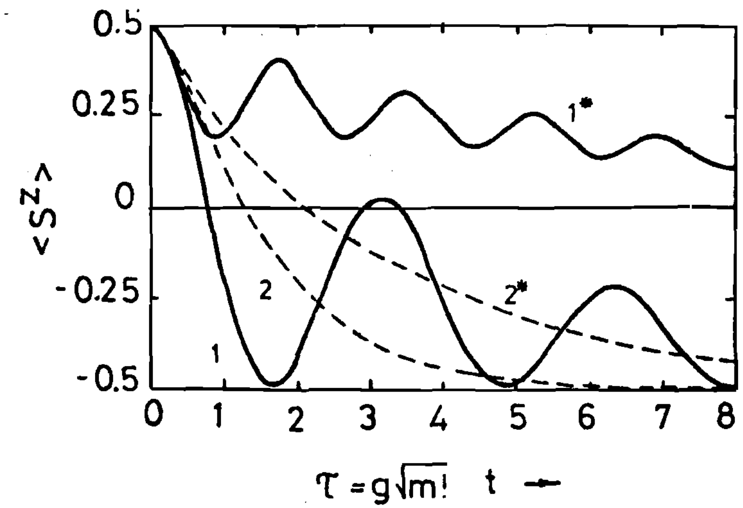

Fig.1. Time evolution of the atomic population inversion (1): $\Delta_{eff} = 0.1$, $\kappa_{eff} = 0.2$, (1): $\Delta_{eff} = 3$, $\kappa_{eff} = 0.2$; (2): $\Delta_{eff} = 0.1$, $\kappa_{eff} = 2.5$; (2*): $\Delta_{eff} = 3$, $\kappa_{eff} = 2.5$.*

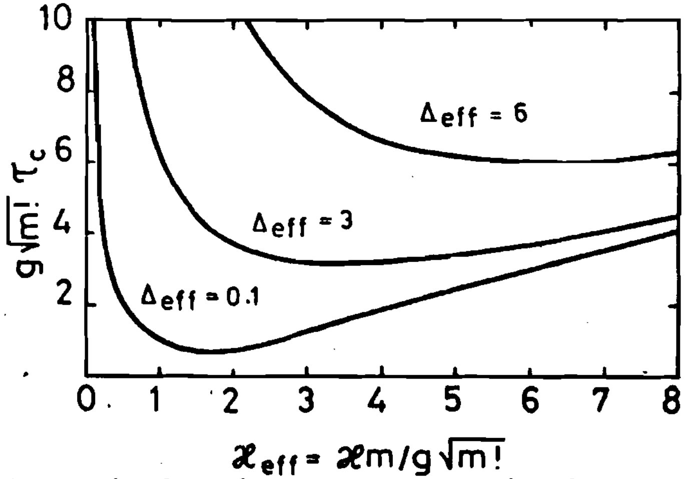

Fig. 2. The dependence on κ_{eff} of the characteristic time τ_c calculated for different values of Δ_{eff} .

high-Q (low-Q) cavity. These results have been obtained for the one-photon-transition case/8/ . Therefore, they are not surprising in the multiphoton case.

In fig.2 we show the dependence upon κ_{eff} of the characteristic time τ_c calculated for different values of Δ_{eff} . Now we consider the radiation (energy change) rate of the atom

$$I(t) = -\frac{d}{dt}\langle S^z \rangle . \tag{19}$$

In the general case of detuning and damping, the expression for I is found from eq.(8) to be

$$I(t) = \frac{1}{A}[D(\lambda_2,\lambda_3,\lambda_4)e^{\lambda_1 t} - D(\lambda_3,\lambda_4,\lambda_1)e^{\lambda_2 t} + D(\lambda_4,\lambda_1,\lambda_2)e^{\lambda_3 t} - D(\lambda_1,\lambda_2,\lambda_3)e^{\lambda_4 t}]. \tag{20}$$

In the case of an ideal cavity, this expression reduces to

$$I(t) = \frac{g^2 m!}{\sqrt{g^2 m! + \Delta^2/4}} \sin(2t\sqrt{g^2 m! + \Delta^2/4}). \tag{21}$$

Oscillating in times, expression (21) can take both positive and negative values. It describes the periodical change of emission and absorption processes. The corresponding maximal value of I(t) is equal to

$$I_{max} = \frac{g^2 m!}{\sqrt{g^2 m! + \Delta^2/4}} \tag{22}$$

and periodically occurs at the times

$$t_{max} = \frac{\pi(1+4k)}{4\sqrt{g^2 m! + \Delta^2/4}} \qquad (k = 0,1,2,\dots). \tag{23}$$

In the case of exact resonance ($\Delta = 0$), one finds from eqs.(8), (9) and (10) the following:

(a) For $\kappa m < 2g\sqrt{m!}$

$$I(t) = \frac{4g^2 m!}{4g^2 m! - \kappa^2 m^2} e^{-\kappa m t} \sin\left(\frac{t}{2}\sqrt{4g^2 m! - \kappa^2 m^2}\right) \times \left\{\kappa m \sin\left(\frac{t}{2}\sqrt{4g^2 m! - \kappa^2 m^2}\right) + \sqrt{4g^2 m! - \kappa^2 m^2}\cos\left(\frac{t}{2}\sqrt{4g^2 m! - \kappa^2 m^2}\right)\right\}. \tag{24}$$

This expression describes the radiation rate of the atom in the oscillatory regime. The maximal value of I(t) is equal to

$$I_{max} = g\sqrt{m!}\exp\left[-\frac{\kappa m}{\sqrt{4g^2 m! - \kappa^2 m^2}}\arccos\left(\frac{\kappa^2 m^2}{4g^2 m!}\right)\right] \times \left(\frac{\kappa m}{2g\sqrt{m!}} + \sqrt{\frac{\kappa^2 m^2}{4g^2 m!} + 1}\right). \tag{25}$$

and occurs at

$$t_{max} = \frac{1}{\sqrt{4g^2 m! - \kappa^2 m^2}}\arccos\left(\frac{\kappa^2 m^2}{4g^2 m!}\right). \tag{26}$$

(b) For $\kappa m = 2g\sqrt{m!}$

$$I(t) = 2g^2 m!\, t\,(g\sqrt{m!}\, t + 1)\, e^{-2g\sqrt{m!}\, t}. \tag{27}$$

This expression has a peak equal to

$$I_{max} = g\sqrt{m!}\,(1 + \sqrt{2})\, e^{-\sqrt{2}}, \tag{28}$$

at

$$t_{max} = \frac{1}{\sqrt{2}}\,\frac{1}{g\sqrt{m!}} : \tag{29}$$

(c) For $\kappa m > 2g\sqrt{m!}$

$$I(t) = \frac{4g^2 m!}{\kappa^2 m^2 - 4g^2 m!}\, e^{-\kappa m t}\, \mathrm{sh}\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) \times$$
$$\times \left\{\kappa m\, \mathrm{sh}\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right) + \sqrt{\kappa^2 m^2 - 4g^2 m!}\ \mathrm{ch}\left(\frac{t}{2}\sqrt{\kappa^2 m^2 - 4g^2 m!}\right)\right\}. \tag{30}$$

This expression describes the radiation rate of the atom in the overdamped regime of evolution. Its maximum value is equal to

$$I_{max} = g\sqrt{m!}\left(\frac{\kappa^2 m^2}{4g^2 m!} + \sqrt{\frac{\kappa^4 m^4}{16 g^4 (m!)^2} - 1}\right)^{-\kappa m/\sqrt{\kappa^2 m^2 - 4g^2 m!}} \times$$
$$\times \left(\frac{\kappa m}{2g\sqrt{m!}} + \sqrt{\frac{\kappa^2 m^2}{4g^2 m!} + 1}\right) \tag{31}$$

and occurs at the time

$$t_{max} = \frac{1}{\sqrt{\kappa^2 m^2 - 4g^2 m!}}\, \ln\left(\frac{\kappa^2 m^2}{4g^2 m!} + \sqrt{\frac{\kappa^4 m^4}{16 g^4 (m!)^2} - 1}\right). \tag{32}$$

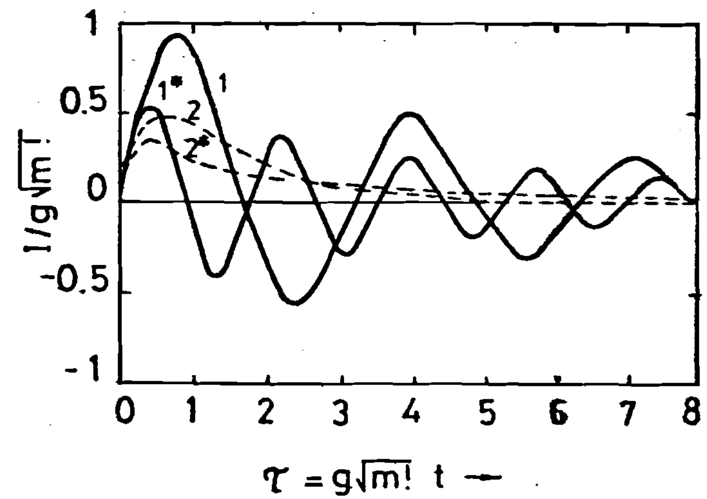

Fig.3. Time evolution of the radiation rate I(t) *. Curves are marked as in fig.1.*

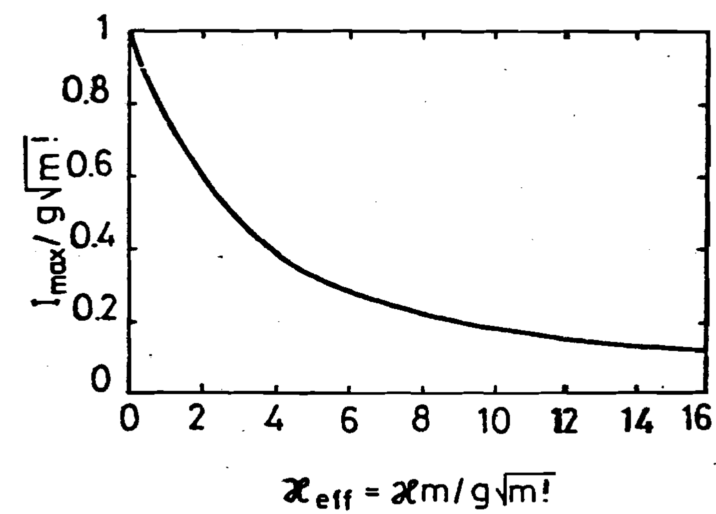

Fig.4. Maximal radiation rate I_{max} *in the case of exact resonance plotted as a function of the effective damping parameter* κ_{eff}.

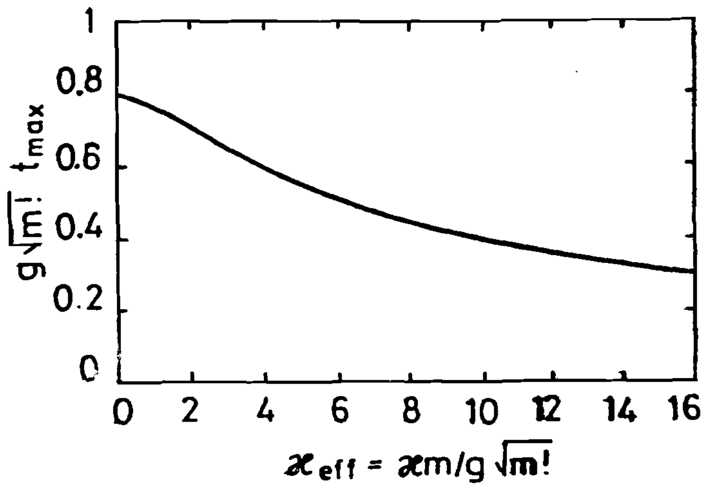

Fig.5. Peak time t_{max} in the case of exact resonance plotted as a function of the effective damping parameter κ_{eff} .

In fig.3 we show the time evolution of the radiation rate I calculated for different values of $\Delta_{eff} = \Delta/g\sqrt{m!}$ and $\kappa_{eff} = \kappa m/g\sqrt{m!}$. The parameters are the same as in fig.1.

In figs. 4 and 5 we plot the maximal radiation rate I and the peak time t_{max} against the effective damping parameter $\kappa_{eff} = \kappa m/g\sqrt{m!}$ for the case of exact resonance ($\Delta = 0$). It is seen from the figures that the κ-dependences of these quantities are described by monotonously decreased functions.

Thus, in this paper we have examined the multiphoton spontaneous emission of a two-level atom in a detuned damped cavity. The exact expressions for the atomic population inversion, radiation rate and characteristic times have been obtained. The influence of the cavity field damping and detuning on the spontaneous emission has been analytically and numerically discussed. It has been shown that the effects of transition-photon multiplicity (multiple of resonance) in the model considered can be taken into account by introducing new effective coupling and damping constants. The enhancement and suppression of multiphoton spontaneous emission and the possibility of various regimes of evolution have also been demonstrated.

REFERENCES

1. Goy P. et al. - Phys.Rev.Lett., 1983, 50, p.1903.
2. Hulet R.G., Hilfer E.S., Kleppner D. - Phys.Rev.Lett., 1985, 55, p.2137.
3. Jhe W. et al. - Phys.Rev.Lett., 1987, 58, p.666.
4. Seke J. - Phys.Rev., 1986, A33, p.4409.
5. Seke J. Rattay F. - J.Opt.Soc.Am., 1987, B4, p.380.
6. Agarwal G.S., Puri R.R. - Phys.Rev., 1986, A33, p.1757.
7. Aliskenderov E.I. et al. - J.Phys., 1987, A20, p.6265.
8. Haroche S., Raimond J.M. In: Adv. Atom. Mol. Phys., v.20, (Eds.: D.R.Bates and B.Bederson, Academic Press, New York, 1985), p.347.
9. Purcell E.M. - Phys.Rev., 1946, 69 p.681.

Received by Publishing Department
on April 21, 1988.