

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА

S-56

E17-88-26

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**COLLECTIVE POPULATION TRAPPING  
AND COLLECTIVE JUMPS IN A SYSTEM  
OF THREE-LEVEL ATOMS**

Submitted to "Physica A"

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**1988**

## I. INTRODUCTION

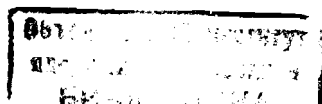
In the last years the problems of bistability (<sup>1</sup> and refs. cited therein) and jumps in the collective limit (<sup>2-8</sup>) have widely been discussed. There have recently been a number of works that has emerged on the novel problem of observing quantum jumps in a single atomic system and applying these jumps to measure linewidths of weak transitions in spectroscopy (<sup>9-16</sup>). Since the weak transition linewidth may be exceptionally narrow, this scheme has been proposed for an ultimate laser frequency standard (<sup>13-14</sup>).

In this paper we investigate the effect of collective population trapping in a system of three-level atoms interacting with an intense external field and collective jump-like behaviour in the intensity and statistical characteristics of the fluorescence field. The potential applications of the collective jumps to measure narrow linewidths of weak transitions have been discussed.

## II. THE BASIC EQUATIONS

We consider a system of  $N$  three-level atoms interacting with one mode of monochromatic driving field of frequency  $\omega_L$  and with the vacuum of other modes. For simplicity the atoms are assumed to be concentrated in a region small compared to the wavelength of all the relevant radiation modes (Dicke model). A schematic diagram of atomic energy level is shown in fig.1. The ground state  $|1\rangle$  is coupled to the excited state  $|2\rangle$  by the strong driving field. The state  $|3\rangle$  may be a low-lying vibrational or rotational excitation accessible from the ground state in the Raman scattering (<sup>6-7</sup>) or it may be a metastable state (<sup>14</sup>). In order to keep the discussion general, we will not specify these states and will return to this question later on.

On treating the exciting laser field classically and making standard (Born and Markov) approximations to describe the system-reservoir couplings, one obtains a master equation for the reduced density operator of the atomic system alone in the



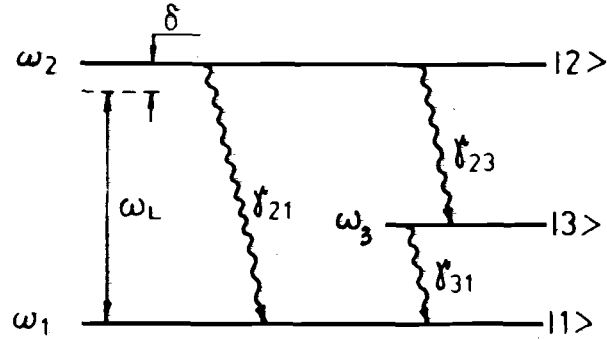


Fig.1. Level scheme of the atomic system.

following form '17' ( $\hbar = 1$  units are used)

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i \left[ \frac{\delta}{2} (J_{22} - J_{11}) + G(J_{21} + J_{12}) - \Omega_3 J_{33}, \rho \right] - \\ & - \gamma_{21} (J_{21} J_{12} \rho - J_{12} \rho J_{21} + \text{H.C.}) - \\ & - \gamma_{23} (J_{23} J_{32} \rho - J_{32} \rho J_{23} + \text{H.C.}) - \\ & - \gamma_{31} (J_{31} J_{13} \rho - J_{13} \rho J_{31} + \text{H.C.}) \equiv L \rho, \end{aligned} \quad (1)$$

where  $2\gamma_{kl}$  ( $k, l = 1, 2, 3$ ) are the transition rates from level  $|k\rangle$  to  $|l\rangle$  due to the atomic interaction with the reservoir;  $\Omega_3 = \omega_{23} - \omega_{21}/2$  (where  $\omega_{kl} = \omega_k - \omega_l$ ),  $\delta = \omega_{21} - \omega_L$  is the detuning of laser frequency from the atomic resonance frequency  $\omega_{21}$ ;  $G = -d_{21} E_0$  is the resonance Rabi frequency describing the interaction of the driving field with the atomic system;  $J_{kl}$  ( $k, l = 1, 2, 3$ ) are the collective angular moment of the atomic system having in the Schwinger representation '18' the following form:

$$J_{kl} = C_k^+ C_l,$$

where the operators  $C_k$  and  $C_k^+$  obey the boson communication relation

$$[C_k, C_l^+] = \delta_{kl}$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level  $|k\rangle$ .

Further, we investigate the case of an intense external field of large detuning  $\delta$  only so that the following relation is fulfilled:

$$\Omega = \left( \frac{1}{4} \delta^2 + G^2 \right)^{1/2} \gg N \gamma_{kl}. \quad (2)$$

After performing the canonical (dressing) transformation

$$C_1 = Q_1 \cos \phi + Q_2 \sin \phi, \quad C_2 = -Q_1 \sin \phi + Q_2 \cos \phi, \quad C_3 = Q_3, \quad (3)$$

where  $\text{tg} 2\phi = 2G/\delta$ , one can split the Liouville operator  $L$ , appearing in equation (1), into the slowly varying part and the terms oscillating at frequencies  $2\Omega$  and  $\Omega$ . In the case when the Rabi frequency  $\Omega$  is sufficiently large and satisfies the relation (2), the secular approximation '19' is justified and we retain only the slowly varying part of the Liouville operator  $L$ . After using the secular approximation one can find a stationary solution of the master equation in the form

$$\tilde{\rho} = U \rho U^+ = A^{-1} \sum_{p=0}^N x^p \sum_{M=0}^p z^M |p, M\rangle \langle M, p|, \quad (4)$$

where  $U$  is the unitary operator representing the canonical transformation (3),

$$x = \gamma_{31} (\gamma_{32} \text{ctg}^2 \phi), \quad z = \text{ctg}^4 \phi,$$

$$A = \frac{z}{z-1} \cdot \frac{(xz)^{N+1} - 1}{xz-1} - \frac{1}{z-1} \frac{x^{N+1} - 1}{x-1}, \quad (5)$$

$|p, M\rangle$  is an eigenstate of the operators  $R = R_{11} + R_{22}, R_{11}$  and the operator of a number of atoms  $N = R_{11} + R_{22} + R_{33}$ , here  $R_{kl} = Q_k^+ Q_l$  ( $k, l = 1, 2, 3$ ) are the collective angular momenta of "dressed" atoms. The operators  $Q_k, Q_k^+$  satisfy the boson commutation relation

$$[Q_k, Q_l^+] = \delta_{kl} \quad (6)$$

so

$$[R_{kl}, R_{k'l'}] = R_{kl} \delta_{k'l'} - R_{k'l} \delta_{kl'}. \quad (7)$$

As in ref. '19', for a later use we introduce the characteristic function

$$\chi_{R_1 R_1 R}(\eta, \xi) = \langle e^{i\eta R_{11} + i\xi R} \rangle =$$

$$= A^{-1} \left[ \frac{y_2}{y_2 - 1} \cdot \frac{(y_1 y_2)^{N+1} - 1}{y_1 y_2 - 1} - \frac{1}{y_2 - 1} \frac{y_1^{N+1} - 1}{y_1 - 1} \right],$$

where  $y_1 = x e^{i\xi}$ ,  $y_2 = z e^{i\eta}$  and  $\langle \dots \rangle$  denotes the expectation value in the steady state described by the density matrix (4). Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R^n R_{11}^m \rangle = \frac{\partial^n}{\partial (i\xi)^n} \frac{\partial^m}{\partial (i\eta)^m} \chi_{R_1 R_1 R}(\eta, \xi) \Big|_{\substack{i\eta = 0 \\ i\xi = 0}} \quad (8)$$

### III. EFFECT OF COLLECTIVE POPULATION TRAPPING

In this section we discuss the stationary population of the atomic level  $|3\rangle$  that has the following form:

$$N_3 = \langle J_{33} \rangle = N - \langle R \rangle, \quad (9)$$

where the value  $\langle R \rangle$  is found according to eq. (8).

First, let us consider the case  $\gamma_{31}/\gamma_{23} < 1$ . By using the relations (8-9) one can show that

$$(i) \quad \frac{\gamma_{31}}{\gamma_{23}} < 1, \quad \frac{\gamma_{31}}{\gamma_{21}} < \text{ctg}^2 \phi < \frac{\gamma_{23}}{\gamma_{31}}$$

(i.e. when  $x < 1$ ,  $xz < 1$ ) and  $N \gg 1$  so that  $x^N$ ,  $(xz)^N < N^{-1}$  almost all the atoms are populated on the level  $|3\rangle$ ,  $N_3/N \rightarrow 1$ , that is the atomic level  $|3\rangle$  plays a role of a "trap" for the atoms. We note that in the one-atom case the level  $|3\rangle$  is the "trap" of an atom, i.e.  $N_3/N \rightarrow 1$ , only in the case  $\gamma_{31}/\gamma_{23} \rightarrow 0$  thus the above described phenomenon can be treated as an effect to collective population trapping<sup>7/</sup>.

(ii) For  $\text{ctg}^2 \phi < \gamma_{31}/\gamma_{23} < 1$  (i.e.  $x > 1$ ,  $xz < 1$  and  $N \gg 1$  so that  $x^N \gg 1$ ,  $(xz)^N \ll N^{-1}$  and in the case of  $\text{ctg}^2 \phi > \gamma_{23}/\gamma_{31} > 1$  (i.e.  $x < 1$ ,  $xz > 1$ ) and  $N \gg 1$  so that  $x^N < N^{-1}$ ,  $(xz)^N \gg 1$  the population of the level  $|3\rangle$  is small in comparison with  $N$ .

(iii) At the points  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23}$  or  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31}$  the nearly half of the atoms ( $N_3 \approx N/2$ ) is populated on the level  $|3\rangle$ .

For the collective limit  $N \rightarrow \infty$  and when  $\gamma_{31}/\gamma_{21} < 1$  the population on the atomic level  $|3\rangle$  takes the form

$$N_3/N \Big|_{N \rightarrow \infty} \begin{cases} 0 & \text{if } \text{ctg}^2 \phi < \gamma_{31}/\gamma_{23} \\ 1/2 & \text{if } \text{ctg}^2 \phi = \gamma_{31}/\gamma_{23} \\ 1 & \text{if } \gamma_{31}/\gamma_{21} < \text{ctg}^2 \phi < \gamma_{23}/\gamma_{31} \\ 1/2 & \text{if } \text{ctg}^2 \phi = \gamma_{23}/\gamma_{31} \\ 0 & \text{if } \text{ctg}^2 \phi > \gamma_{23}/\gamma_{31} \end{cases}$$

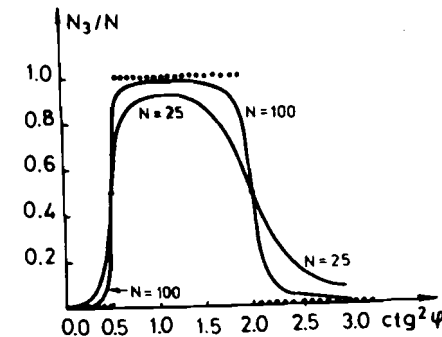
Thus in the collective limit  $N \rightarrow \infty$  the function  $N_3/N$  shows the jump-like behaviour at critical points  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23}$  and  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31}$ . The jump-like behaviour of the atomic population on the level  $|3\rangle$  (per atom), i.e. the quantity  $N_3/N$ , is plotted in fig.2 as a function of the parameter  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ .

In a similar manner one can show that in the case  $\gamma_{31}/\gamma_{23} > 1$  and for  $N \gg 1$  the population of the level  $|3\rangle$  is small compared to  $N$  for all values of the parameter  $\text{ctg}^2 \phi$ ; in this case the collective population trapping is absent and collective jump in the behaviour of the function  $N_3/N$  is absent too.

In the case when  $\gamma_{31}/\gamma_{23} = 1$ , we have

$$N_3/N \Big|_{N \rightarrow \infty} \begin{cases} 0 & \text{if } \text{ctg}^2 \phi > 1 \\ 1/3 & \text{if } \text{ctg}^2 \phi = 1 \\ 0 & \text{if } \text{ctg}^2 \phi < 1 \end{cases}$$

thus in the collective limit



$N \rightarrow \infty$  the function  $N_3/N$  shows a discontinuous behaviour at the critical point  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23} = 1$  but as for the

Fig.2. Population (per atom) of the level  $|3\rangle$  as a function of  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

case of  $\gamma_{31}/\gamma_{23} > 1$ , the effect of collective population trapping is absent.

#### IV. COLLECTIVE JUMP IN THE STEADY-STATE INTENSITY OF RESONANCE FLUORESCENCE

In this section we discuss the influence of the collective population trapping on the behaviour of the stationary intensity  $I$  of the fluorescent field due to the atomic transition  $|2\rangle \rightarrow |1\rangle$ . The explicit form for the intensity  $I$  can be found by applying the canonical transformation (3) and the stationary density matrix solution (4)

$$I \sim \langle J_{21} J_{12} \rangle = \sin^2 \phi \cos^2 \phi \langle (R - 2R_{11})^2 \rangle + \cos^4 \phi \langle R_{21} R_{12} \rangle + \sin^4 \phi \langle R_{12} R_{21} \rangle, \quad (10)$$

where

$$\langle (R - 2R_{11})^2 \rangle = \langle R^2 \rangle + 4 \langle R_{11}^2 \rangle - 4 \langle RR_{11} \rangle, \quad (11)$$

$$\langle R_{12} R_{21} \rangle = \langle R_{11} \rangle + \langle RR_{11} \rangle - \langle R_{11}^2 \rangle, \quad (12)$$

$$\langle R_{21} R_{12} \rangle = \langle R \rangle - \langle R_{11} \rangle + \langle RR_{11} \rangle - \langle R_{11}^2 \rangle, \quad (13)$$

In eqs. (11-13), the statistical moments  $\langle R \rangle$ ,  $\langle R^2 \rangle$ ,  $\langle R_{11} \rangle$ ,  $\langle R_{11}^2 \rangle$  and  $\langle RR_{11} \rangle$  can be found according to relation (8).

By using eqs. (10-13) one can show that in the case of  $\gamma_{31}/\gamma_{23} < 1$ ,  $\gamma_{31}/\gamma_{23} < \text{ctg}^2 \phi < \gamma_{23}/\gamma_{31}$  and  $N \gg 1$  (the condition (1)), i.e., when atoms are "trapped" on the level  $|3\rangle$  the intensity  $I$  is independent of the numbers of atoms. For all other values of the parameters  $\gamma_{31}/\gamma_{23}$  and  $\text{ctg}^2 \phi$  the intensity  $I$  is proportional to  $N^2$ . Thus, for the case  $\gamma_{31}/\gamma_{23} < 1$  the jump-like behaviour of the quantity  $I/N^2$  as a function of the parameter  $\text{ctg}^2 \phi$  is presented (see fig.3). In the case of  $\gamma_{31}/\gamma_{23} < 1$  and  $N \rightarrow \infty$  the function  $I/N^2$  shows the jump-like behaviour at the critical points  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23}$  and  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31}$  (see the dotted curve in fig.3). Further we shall discuss potential applications of collective jumps to measure weak transition linewidths. Let the level  $|3\rangle$  be a metastable state, the transition  $|3\rangle \rightarrow |1\rangle$  be a forbidden and other transitions  $|2\rangle \rightarrow |1\rangle$  and  $|2\rangle \rightarrow |3\rangle$  be allowed transitions<sup>14</sup>. It has argued that the weak transition  $|3\rangle \rightarrow |1\rangle$ , which is difficult to detect, could be monitored by the scattered light

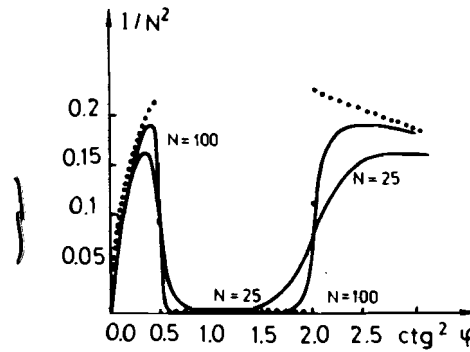


Fig.3. Scaled intensity of fluorescent light  $I/N^2$  as a function of  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

of the strong transition  $|2\rangle \rightarrow |1\rangle$ . Changing the parameter  $\text{ctg}^2 \phi$ , i.e., changing the detuning  $\delta$  or intensity of the external field, one can observe

the jump (see fig.3) in the intensity of the fluorescence corresponding to the strong transition  $|2\rangle \rightarrow |1\rangle$  at the critical point  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23}$  or  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31}$ , and this allows us in principle to measure the quantity  $\gamma_{31}$ .

#### V. COLLECTIVE JUMPS IN PHOTON STATISTICS

In this section we discuss the influence of the collective population trapping on the behaviour of the photon statistics and cross-correlations between spectrum components of the fluorescence field due to the atomic transition  $|2\rangle \rightarrow |1\rangle$ .

With the use of the dressing transformation (3), the atomic collective angular momentum  $J_{21}$  has the structure

$$J_{21} = \sin \phi \cos \phi (R - 2R_{11}) + \cos^2 \phi R_{21} - \sin^2 \phi R_{12}. \quad (14)$$

Following the work<sup>6,19</sup> we can consider the operators  $\cos^2 \phi R_{21}$ ,  $\sin \phi \cos \phi (R - 2R_{11})$  and  $-\sin^2 \phi R_{12}$  as the amplitude-operators for the sources of Mollow's triplet centered at the frequencies  $\omega_L + 2\Omega$ ,  $\omega_L$  and  $\omega_L - 2\Omega$  and for simplicity we denote these operators by  $S_1^+$ ,  $S_0^+$  and  $S_{-1}^+$ , respectively.

As London<sup>20</sup>, we define a degree of second-order coherence between the spectrum components  $S_i$  and  $S_j$  in the form

$$G_{i,j}^{(2)} = \frac{\langle S_i^+ S_j^+ S_j S_i \rangle}{\langle S_i^+ S_i \rangle \langle S_j^+ S_j \rangle} \quad (i, j = 0, \pm 1). \quad (15)$$

Since the operator  $S_i$  does not commute with the operator in the general case, we have

$$G_{i,j}^{(2)} \neq G_{j,i}^{(2)} \quad (i \neq j).$$

In particular the correlation functions  $G_{i,i}^{(2)}$  ( $i = 0, \pm 1$ ) describe the photon statistics of the spectrum components  $S_i$  have the form

$$G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = \frac{\langle R_{12} R_{12} R_{21} R_{21} \rangle}{\langle R_{12} R_{21} \rangle^2}, \quad (16)$$

$$G_{0,0}^{(2)} = \frac{\langle (R - 2R_{11})^4 \rangle}{\langle (R - 2R_{11})^2 \rangle^2}, \quad (17)$$

where

$$\begin{aligned} \langle R_{12} R_{12} R_{21} R_{21} \rangle &= \langle R_{11}^4 \rangle - 2\langle RR_{11}^3 \rangle - 4\langle R_{11}^3 \rangle + \langle R^2 R_{11}^2 \rangle + 5\langle RR_{11}^2 \rangle + \\ &+ 5\langle R_{11}^2 \rangle - \langle R^2 R_{11} \rangle - 3\langle RR_{11} \rangle - 2\langle R_{11} \rangle, \end{aligned} \quad (18)$$

$$\langle (R - 2R_{11})^4 \rangle = 16\langle R_{11}^4 \rangle - 32\langle RR_{11}^3 \rangle + 24\langle R^2 R_{11}^2 \rangle - 8\langle R^3 R_{11} \rangle + \langle R^4 \rangle. \quad (19)$$

In eqs. (16-19) the statistical moments  $\langle (R - 2R_{11})^2 \rangle$ ,  $\langle R_{12} R_{21} \rangle$  and  $\langle R^n R_{11}^m \rangle$  can be found according to eqs. (11-12) and (8), respectively.

Further we shall investigate the photon statistics of spectrum components  $S_i$  ( $i = 0, \pm 1$ ) for the case of  $\gamma_{31}/\gamma_{23} < 1$ , i.e., when the effect of collective population trapping can occur. The behaviour of the degree of second-order coherence  $G_{0,0}^{(2)}$  and  $G_{\pm 1, \pm 1}^{(2)}$  as functions against the parameter  $\text{ctg}^2 \phi$  for the case of  $\gamma_{31}/\gamma_{23} = 0.5$  is plotted in figs. 4 and 5, respectively. As is seen from figs. 4,5, the photon statistics of the spectrum components  $S_i$  ( $i = 0, \pm 1$ ) in the region of the parameters

$$\frac{\gamma_{31}}{\gamma_{23}} < \text{ctg}^2 \phi < \frac{\gamma_{23}}{\gamma_{31}}; \quad \frac{\gamma_{31}}{\gamma_{23}} < 1,$$

i.e., when atoms are trapped on the level  $|3\rangle$ , is quite different from the photon statistics in the other regions of the parameters

$$\text{ctg}^2 \phi < \frac{\gamma_{31}}{\gamma_{23}} < 1 \quad \text{or} \quad \text{ctg}^2 \phi > \frac{\gamma_{23}}{\gamma_{31}} > 1,$$

when the effect of collective population trapping is absent.

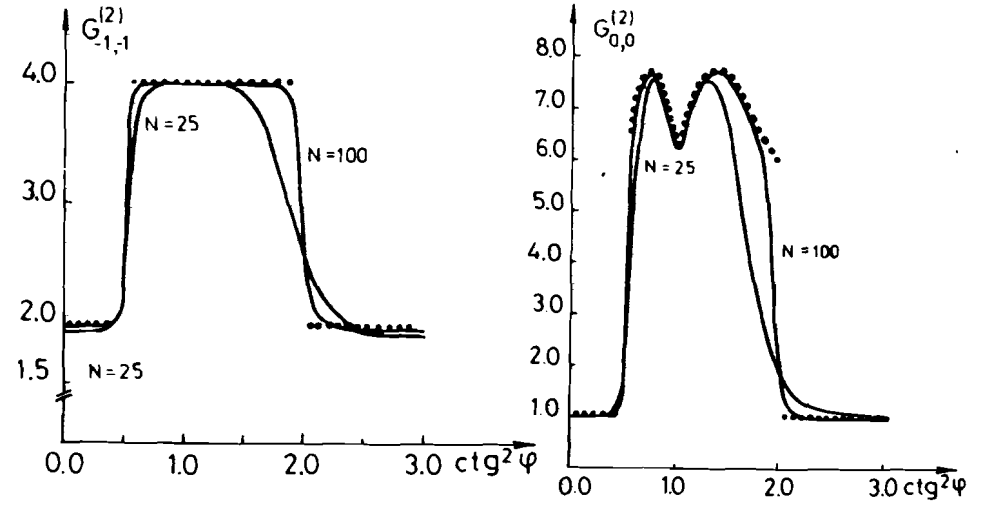


Fig. 4. Function  $G_{-1,-1}^{(2)}$  against the parameter for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

Fig. 5. Function  $G_{0,0}^{(2)}$  against the parameter  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

For the case of  $N \gg 1$  the central spectrum component  $S_0$  has superpoissonian statistics in the region  $\gamma_{31}/\gamma_{23} < \text{ctg}^2 \phi < \gamma_{23}/\gamma_{31}$  and has the poissonian statistics in other regions  $\text{ctg}^2 \phi < \gamma_{31}/\gamma_{23}$  or  $\text{ctg}^2 \phi > \gamma_{23}/\gamma_{31}$ .

In the collective limit  $N \rightarrow \infty$  (dotted curves in figs. 4,5), the functions  $G_{0,0}^{(2)}$  and  $G_{\pm 1, \pm 1}^{(2)}$  show the jump-like behaviour at the critical points  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23}$  and  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31}$ .

The effect of collective population trapping also strongly affects the behaviour of the cross-correlation functions between spectrum components as well. For example, we investigate degrees of cross-correlation between sidebands  $S_{\pm 1}$  and central component  $S_0$  that have the following form

$$G_{0,1}^{(2)} = G_{-1,0}^{(2)} = \frac{\langle (R - 2R_{11}) R_{21} R_{12} (R - 2R_{11}) \rangle}{\langle (R - 2R_{11})^2 \rangle \langle R_{21} R_{12} \rangle}, \quad (20)$$

$$G_{1,0}^{(2)} = G_{0,-1}^{(2)} = \frac{\langle (R - 2R_{11}) R_{12} R_{21} (R - 2R_{11}) \rangle}{\langle (R - 2R_{11})^2 \rangle \langle R_{12} R_{21} \rangle}. \quad (21)$$

where

$$\langle (R - 2R_{11}) R_{21} R_{12} (R - 2R_{11}) \rangle = -4 \langle R_{11}^4 \rangle + 8 \langle RR_{11}^3 \rangle - \quad (22)$$

$$-4 \langle R_{11}^3 \rangle - 5 \langle R^2 R_{11}^2 \rangle + 8 \langle RR_{11}^2 \rangle + \langle R^3 R_{11} \rangle - 5 \langle R^2 R_{11} \rangle + \langle R^3 \rangle,$$

$$\langle (R - 2R_{11}) R_{12} R_{21} (R - 2R_{11}) \rangle = -4 \langle R_{11}^4 \rangle + 8 \langle RR_{11}^3 \rangle + \quad (23)$$

$$+4 \langle R_{11}^3 \rangle - 5 \langle R^2 R_{11}^2 \rangle - 4 \langle RR_{11}^2 \rangle + \langle R^3 R_{11} \rangle + \langle R^2 R_{11} \rangle.$$

Here the statistical moments  $\langle R^n R_{11}^m \rangle$  can be found according to eq. (8).

Further we shall investigate the functions  $G_{1,0}^{(2)}$  and  $G_{0,1}^{(2)}$  only for the case  $\gamma_{31}/\gamma_{23} < 1$  when the collective population trapping can occur. The behaviour of  $G_{1,0}^{(2)}$  and  $G_{0,1}^{(2)}$  as functions against the parameter  $\text{ctg}^2 \phi$  for the case of  $\gamma_{31}/\gamma_{23} = 0.5$  is plotted in figs. 6 and 7, respectively. As is seen from these figures, for the case  $N \gg 1$ , the correlation between the central component  $S_0$  and sidebands  $S_{\pm 1}$  occurs ( $G_{0,1}^{(2)}, G_{1,0}^{(2)} > 1$ ) in the region of the parameter

$$\frac{\gamma_{31}}{\gamma_{23}} < \text{ctg}^2 \phi < \frac{\gamma_{23}}{\gamma_{31}}; \quad \frac{\gamma_{31}}{\gamma_{23}} < 1$$

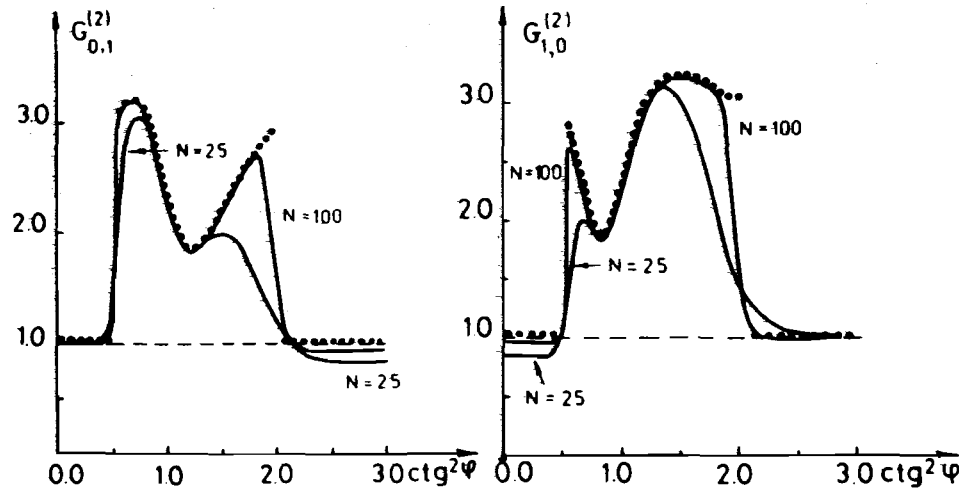


Fig. 6. Function  $G_{0,1}^{(2)}$  against  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

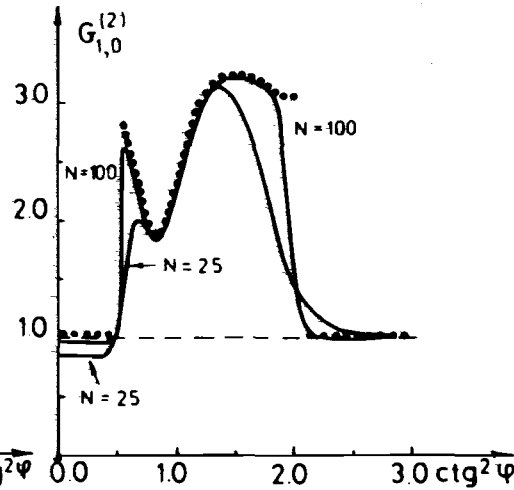


Fig. 7. Function  $G_{1,0}^{(2)}$  against  $\text{ctg}^2 \phi$  for  $\gamma_{31}/\gamma_{23} = 0.5$ . The dotted curve illustrates the case  $N \rightarrow \infty$ .

i.e., in the region where collective population trapping occurs. Physically it means that when the collective population trapping is present the photon of the central components  $S_0$  and sidebands  $S_{\pm 1}$  have a tendency to be emitted in pairs.

In other regions of the parameters  $\text{ctg}^2 \phi < \gamma_{31}/\gamma_{23}$ ;  $\text{ctg}^2 \phi > \gamma_{23}/\gamma_{31}$ , where the collective population trapping is absent, the anticorrelation or no correlation between the central component  $S_0$  and sidebands  $S_{\pm 1}$  occurs (see figs. 6, 7), and in the collective limit  $N \rightarrow \infty$  the photons of the central component  $S_0$  and sidebands  $S_{\pm 1}$  are emitted independently (i.e.  $G_{\pm 1,0}^{(2)} = G_{0,\pm 1}^{(2)} = 1$ ) in these regions of the parameter  $\text{ctg}^2 \phi$ .

In the collective limit  $N \rightarrow \infty$  the cross-correlation function  $G_{0,1}^{(2)}$  and  $G_{1,0}^{(2)}$  show the jump-like behaviour at the critical points  $\text{ctg}^2 \phi = \gamma_{31}/\gamma_{23} < 1$  and  $\text{ctg}^2 \phi = \gamma_{23}/\gamma_{31} > 1$  (see dotted curves in figs. 6, 7).

## V. CONCLUSIONS

We have considered the collective population trapping and collective jumps in a system of three-level atoms interacting with an intense external field.

It is shown that for  $\gamma_{31}/\gamma_{23} < 1$ ,  $N \gg 1$  and  $\gamma_{31}/\gamma_{23} < \text{ctg}^2 \phi < \gamma_{23}/\gamma_{31}$  almost all the atoms are populated on the level  $|3\rangle$  (collective population trapping). The influence of the collective population trapping on the behaviour of steady-state intensity and statistical properties of a resonance fluorescence field has been discussed. For the case  $\gamma_{31}/\gamma_{23} < 1$  the system displays discontinuous behaviour (jumps) in the collective limit  $N \rightarrow \infty$ . Potential applications of collective jumps to measure weak transition linewidths are discussed.

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Received by Publishing Department  
on January 12, 1988.

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Коллективная ловушка и коллективные скачки  
в системе трехуровневых атомов

Рассмотрены коллективная ловушка атомов и коллективные скачки в системе трехуровневых атомов взаимодействующих с сильным внешним полем. Обсуждено возможное применение коллективных скачков для измерения узкой ширины слабых переходов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Shumovsky A.S., Fam Le Kien, Tran Quang E17-88-26  
Collective Population Trapping and Collective  
Jumps in a System of Three-Level Atoms

The collective population trapping and collective jumps in a system of three-level atoms interacting with intense external field are considered. Potential applications of collective jumps to measure weak transition linewidths are shortly discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988