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SPECTRAL AND STATISTICAL PROPERTIES OF FLUORESCENCE FROM N-ATOMS IN A CAVITY

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Cavity electrodynamics has been one of the central topics of quantum optics in last years (for reviews see[1,2]). A number of interesting effects such as vacuum-field Rabi oscillations, collapse and revival has been predicted and then observed experimentally[7-10]. Recently, the spectrum for the fluorescence photons in the Jaynes-Cummings model has been calculated[3-6,11].

In this letter we discuss the intensities and photon statistics of the spectrum components and the cross-correlations between them in the fluorescence field from N two-level atoms interacting with an intense resonant cavity mode at frequency ω_L. Essential differences in spectral and statistical properties from collective resonance fluorescence in a free space have been shown. We consider a system of N two-level atoms interacting with a single cavity mode. For simplicity, we discuss only the case when the cavity mode is in the exact resonance with atomic resonance frequency. Atoms are assumed to be placed in a cell of dimensions smaller than the wavelength of the field.

In the case when the cavity mode is intense so that it can be treated classically, the Hamiltonian of the system in the interaction picture has the following form[17]

$$H = \Omega (J_{21} + J_{12}).\qquad(1)$$

where Ω is the resonance Rabi frequency describing the interaction of the intense cavity mode with the atomic system; $J_{ij} = \sum_{k=1}^{N} |i \rangle_k {}_k\langle j|$ (i,j = 1,2) are the collective operators for the atomic system. They satisfy the communication relation:

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{ji'} - J_{i'j}\delta_{ij'}.\qquad(2)$$

By using relations (1)-(2) one can write the operators in the following form:

$$J_{21}(t) = \ell^{2\Omega t} S_1^- + \ell^{-2\Omega t} S_{-1}^+ + S_0.\qquad(3)$$

$$J_{12}(t) = \ell^{-2\Omega t} S_1 + \ell^{2\Omega t} S_{-1} + S_0.\qquad(4)$$

where

$$S_0 = \frac{1}{2}(J_{12} + J_{21}),\qquad(5)$$

$$S_1 = \frac{1}{4}(J_{12} - J_{21} - J_3),\qquad S_1^+ = (S_1)^+,\qquad(6)$$

$$S_{-1} = \frac{1}{4}(J_{12} - J_{21} + J_3),\qquad S_{-1}^+ = (S_{-1})^+,\qquad(7)$$

here

$$J_3 = J_{22} - J_{11}.$$

Following refs.[3-5] we define the transient spectrum of the fluorescence in a cavity as

$$S_F(\nu,T) = 2\Gamma \int_0^T dt_1 \int_0^T dt_2 \ell^{-(\Gamma-i\nu)(T-t_1)-(\Gamma+i\nu)(T-t_2)} \times$$
$$\times \langle J_{21}(t_1) J_{12}(t_2)\rangle \ell^{-i\omega_L (t_2-t_1)},\qquad(8)$$

where Γ is the handwidth of the detector, T is the time at which the spectrum is evaluated. The $\langle ...\rangle$ indicates the average over the initial state of the atomic system. By using the definition of fluorescence (8) and relations (3-4) one shows that for the case of $\Gamma T \gg 1$ and for the case of intense cavity mode the members in spectrum (8), which are proportional to Ω^{-1}, are ignored and the operators S_{-1}, S_0 and S_1 can be considered as the operator-sources of the spectrum components at frequencies $\omega_L - 2\Omega$, ω_L and $\omega_L + 2\Omega$, respectively[13,15-16].

Let the atoms be initially in the ground state

$$\rho_a = |0,N\rangle\langle N,0|.\qquad(9)$$

Then, the intensities of the spectrum components are proportional

$$I_{-1} \sim \langle S_{-1}^+ S_{-1}\rangle = \frac{1}{16}(N^2 + N),\qquad(10)$$

$$I_0 \sim \langle S_0^2\rangle = \frac{1}{4}N,\qquad(11)$$

$$I_1 \sim <S_1^+ S_1> = \frac{1}{16}(N^2 + N). \tag{12}$$

Thus, the intensities of the sidebands $S_{\pm 1}$ are proportional to N^2 while the intensity of the center component is proportional to N. This result is quite different from the collective resonance fluorescence in the free space[12,13], where for the case of exact resonance the intensities of all three Molow's triplets are proportional to the square of the number of atoms.

Further, we discuss the photon statistics of spectrum components S_0, $S_{\pm 1}$ and cross-correlations between them.

Following refs.[14,13], we define the degree of second-order coherence of fluorescence light to be

$$G_{i,j}^{(2)} = \frac{<S_i^+ S_j^+ S_j S_i>}{<S_i^+ S_i><S_j^+ S_j>} \quad (i,j = 0, \pm 1). \tag{13}$$

The quantities $G_{i,i}^{(2)}$ (i = 0, ±1) describe the photon statistics of the spectrum components S_i and the quantities $G_{i,j}^{(2)}$ (i ≠ j) describe the cross-correlations between the spectrum components S_i and S_j (i,j = 0, ±1).

By using the relations (9) and (13) one can find

$$G_{0,0}^{(2)} = 1 + 2\frac{N-1}{N}, \tag{14}$$

$$G_{-1,-1}^{(2)} = G_{1,1}^{(2)} = 1 - 2\frac{3N-1}{N(N+1)^2}, \tag{15}$$

For the case of N = 1 the relations (14)-(15) reduce to $G_{0,0}^{(2)}=1$; $G_{\pm 1,\pm 1}^{(2)} = 0$; thus, the center component has a Poissonian statistics while the sidebands $S_{\pm 1}$ have the sub-Poissonian statistics. Analogous conclusions were made[15-16] for resonance fluorescence in a free space.

For the collective case N > 1 we have $G_{0,0}^{(2)} > 1$ and the center component S_0 has a super-Poissonian statistics while $G_{\pm 1,\pm 1}^{(2)} < 1$ i.e., the sidebands as for the single-atom case, also have the sub-Poissonian statistics for the collective case. This property is different from collective resonance fluorescence in a free space[13] where the sidebands have sub-Poissonian statistics only for the case of several atoms. We note that though the sidebands $S_{\pm 1}$ have the sub-Poissonian statistics for the collective case N > 1, the measure of this

phenomenon, the value $G_{\pm 1,\pm 1}^{(2)} - 1 = \frac{-2(3N-1)}{N(N+1)^2}$ is proportional

to N^{-2}, i.e., to reverse number of photons of the sidebands. For the case $N \gg 1$ the sidebands are Poissonian light ($G_{\pm 1,\pm 1}^{(2)} \cong 1$).

Finally, we discuss the cross-correlations between the spectrum components. By using the relations (9) and (13) one finds

$$G_{1,-1}^{(2)} = G_{-1,1}^{(2)} = 1 + \frac{2}{N(N+1)}, \tag{16}$$

$$G_{1,0}^{(2)} = G_{-1,0}^{(2)} = 1 + 2\frac{N-1}{N(N+1)}, \tag{17}$$

$$G_{0,1}^{(2)} = G_{0,-1}^{(2)} = 1 - 2\frac{N-1}{N(N+1)}. \tag{18}$$

From the relations (16)-(18) one can make conclusions:

(i) The correlation between the sidebands comes into existence for the single and collective case ($G_{1,-1}^{(2)} = G_{-1,1}^{(2)} > 1$), i.e., the photons of the sidebands have a tendency to be emitted in pairs.

(ii) For the single-atom case $G_{1,0}^{(2)} = G_{-1,0}^{(2)} = G_{0,1}^{(2)} = G_{0,-1}^{(2)} = 1$. It means that for the single-atom case there is no correlation between the side components and the central ones[15-16].

(iii) For the collective case we have $G_{1,0}^{(2)} = G_{-1,0}^{(2)} > 1$ and $G_{0,-1}^{(2)} = G_{0,1}^{(2)} < 1$, i.e., the atoms have a tendency to emit in pairs the photons of the sideband and center component only in the time order: the first photon of the sideband and the second photon of the center component. This property in cross-correlation is quite different from collective resonance fluorescence in a free space[13] where the anti-correlation between the sidebands $S_{\pm 1}$ and the central component S_0 occurs.

In conclusion we note that for the case of a large number of atoms ($N \gg 1$) $G_{i,j}^{(2)}$ (i ≠ j) ≃ 1; thus, the photons of the spectrum components have the tendency to be emitted independently. For the one case (N = 1) our results are in agreement with the previous works[3,5].

REFERENCES

1. Haroche S., Raimond J.M. - Advances in Atomic and Molecular Physics, edited by D.R.Bates and B.Bederson (New York: Academic). 1985, 20, p.347.
2. Yoo H.I., Eberly J.H. - Phys.Rep., 1985, 118, p.239.
3. Sancher.Mondragon J.J., Narozhny N.B., Eberly J.H. . Phys. Rev.Lett., 1983, 51, p.550.

4. Agarwal G.S., Puri R.R. - Phys.Rev., 1986, A33, p.1757.
5. Agarwal G.S. - JOSA, 1985, B2, p.480.
6. Aliskenderov E.I., Rustamov K.A., Shumovsky A.S.,Tran Quang. - J.Phys., 1987, A20, p.6265.
7. Kaluzny Y., Goy P., Gross M., Raimond J.M., Haroche S. - Phys.Rev.Lett., 1983, 51, p.1175.
8. Meschede D., Walther H., Mueller G. - Phys.Rev.Lett., 1985, 54, p.551.
9. Gallas J.A.C., Leuchs G., Walther H., Figger H. - Advances in Atomic and Molecular Physics, edited by D.R.Bates and B.Bederson (New York: Academic). 1985, 20, p.413.
10. Rempe G., Walther H., Klein N. - Phys.Rev.Lett, 1987, 58, p.353.
11. Fam Le Kien, Shumovsky A.S., Tran Quang - J.Phys.A, 1988, 21A, p.119.
12. Agarwal G.S., Narducci L.M., Da Hsuan Feng and Gilmole R. - Phys.Rev.Lett., 1978, 42, p.1260.
13. Bogolubov N.N.(Jr.), Aliskenderov E.I., Shumovsky A.S., Tran Quang. - J.Phys., 1987, B20, p.1885.
14. Loudon R. - Rep. Prog. Phys., 1980, 43, p.913.
15. Apanasevich P.A., Kilin J.Ja. - J.Phys., 1979, B12, p.L83.
16. Aspect A., Roger G., Reynaud S., Dalibard J., Cohen-Tannoudji C. - Phys.Rev.Lett., 1980, 45, p.617.
17. Tavis M., Cummings F.W. - Phys.Rev., 1968, 170, p.379.

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Спектральные и статистические свойства
флуоресценции атомов в резонаторе

Исследованы спектральные и статистические свойства флуоресценции N двухуровневых атомов, взаимодействующих с интенсивной резонансной модой. Получены результаты, сильно отличающиеся от коллективной резонансной флуоресценции в свободном пространстве.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Shumovsky A.S., Tran Quang   E17-88-222
Spectral and Statistical Properties
of Fluorescence from N-Atoms in a Cavity

Spectral and statistical properties of fluorescence from N two-level atoms, interacting with an intense cavity mode, are discussed. Quite different results from collective resonance fluorescence in a free space are shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.