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LONG-TIME ASYMPTOTIC BEHAVIOUR OF THE LORENTZ GAS WITH ANISOTROPIC SCATTERING

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1. INTRODUCTION

Recently Piasecki and Wajnryb [1] discussed a model describing the relaxation of a rarefied gas of neutrinos in the stellar matter. In this case the collision integral contains terms related to isotropic and anisotropic scattering. They obtained an exact formal solution from which they inferred the long-time asymptotics of the distribution function.

The purpose of our paper is to generalize results obtained by these authors. Namely, we consider a class of models characterized by a parameter a, which determines the strength of anisotropic scattering. The value $a = 0$ (i.e. isotropic scattering) corresponds to the case of standard Lorentz gas (SLG) discussed by Hauge [2,3]. Therefore, we call the present model the modified Lorentz gas (MLG). We shall show that the long-time asymptotic behaviour of MLG depends on value of a and can be diffusive (for $0 \le a < 1/\tau$) or hydrodynamic (for $a = 1/\tau$). The parameter τ is the relaxation time characterizing the isotropic scattering.

In section 2 we find the formal solution of the Boltzmann equation for the Fourier — Laplace transform (FLT) of the distribution function. In sect. 3 we study the number of the singularities of the FLT of the distribution function and their properties. In section 4 we consider the dependence of the long-time asymptotic behaviour on the parameter $a\tau$.

2. THE FORMAL SOLUTION OF THE BOLTZMANN EQUATION FOR THE MODIFIED LORENTZ GAS

Published in 1872 the Boltzmann equation (considered today to be the fundamental equation of the kinetics of rarefied gases) describes the time and space dependence of the one-particle distribution function $f(\vec{r}, \vec{v}, t)$. We consider the situation when the Boltzmann equation is linear (cf. [2,3]).

$$\frac{\partial f(\vec{r}, \vec{v}, t)}{\partial t} + \vec{v}\vec{\nabla} f(\vec{r}, \vec{v}, t) = \frac{1}{4\pi}\int d\hat{v}'\sigma(\vec{v}, \vec{v}')[f(\vec{r}, \vec{v}, t) - f(\vec{r}, \vec{v}, t)]. \quad (1)$$

For modified Lorentz gas, the probability of transition per unit interval of time, $\sigma(\vec{v}, \vec{v}')$ is equal to

$$\sigma(\vec{v}, \vec{v}') = \sigma(v, \mu) = \frac{1}{\tau} + 3a\mu, \quad (2)$$

where $\mu = \vec{v}\vec{v}'$ is the cosinus of the scattering angle. The case of $a = 0$ corresponds to the standard Lorentz gas (SLG) (cf. /2,3/). Let us introduce the deviation of the distribution function from the equilibrium value

$$\delta f(\vec{r}, \vec{v}, t) = f(\vec{r}, \vec{v}, t) - \frac{1}{4\pi} n_o ,$$

where n_o is the equilibrium isotropic homogeneous density of particles. For the FLT of the deviation function

$$\Phi(\vec{k}, \vec{v}, z) = \int_0^\infty dt\, e^{-zt} \int d^3r\, e^{i\vec{k}\vec{r}}\, \delta f(\vec{r}, \vec{v}, t) \tag{3}$$

one obtains the integral equation

$$r(z - i\vec{k}\vec{v})\,\Phi(\vec{k},\vec{v},z) = (\hat{\mathcal{P}}_0 + a r \hat{\mathcal{P}}_1 - 1)\,\Phi(\vec{k},\vec{v},z) + r h(\vec{k},\vec{v}), \tag{4}$$

where for an arbitrary function of $\vec{v}$, say $g(\vec{v})$,

$$\hat{\mathcal{P}}_0\, g(\vec{v}) = \frac{1}{4\pi} \int d\hat{v}\, g(\vec{v}), \tag{5a}$$

$$\hat{\mathcal{P}}_1\, g(\vec{v}) = \frac{3}{4\pi} \int d\hat{v}'\, \hat{v}\hat{v}'\, g(\vec{v}'), \tag{5b}$$

$h(\vec{k}, \vec{v})$ is FT of the initial deviation function

$$h(\vec{k},\vec{v}) = \int d^3r\, e^{i\vec{k}\vec{r}}\, \delta f(\vec{r}, \vec{v}, t=0). \tag{6}$$

Equation (4) is soluble. The solution (for $a = 1/(3r)$) was found by Piasecki and Wajnryb /1/ . We shall extend their solution to the case of the parameter ar belonging to the interval $[0,1]$. Let us introduce the two-dimensionless variables

$$k = kvr, \qquad \zeta = zr.$$

The solution of eq. (4) has the form

$$\Phi(\vec{k}, \vec{v}, \zeta) = \frac{1}{ik(\lambda - \mu)} \Big\{ \frac{rA}{ik} \Big[\hat{\mathcal{P}}_0 \frac{h}{\lambda - \mu} - \frac{3ar}{ik} Q_1 \hat{\mathcal{P}}_0 h\Big] \times \times [1 + \mu B Q_1] + \frac{ar^2}{ik} B \hat{\mathcal{P}}_1 \frac{h}{\lambda - \mu} + rh \Big\}, \tag{7}$$

where $\lambda = (1 + \zeta)/ik$. The functions $A(k,\lambda)$, $B(k,\lambda)$ play an important role in our considerations

$$A(k,\lambda) = [1 - \frac{1}{ik} Q_o(\lambda) + \frac{3ar}{(ik)^2}(ik\lambda - 1)\, Q_1(\lambda)]^{-1}, \tag{8a}$$

$$B(k,\lambda) = [1 - \frac{3ar}{ik} \lambda Q_1(\lambda)]^{-1}. \tag{8b}$$

The functions $Q_\ell(\lambda)$ ($\ell = 0, 1, 2, \ldots$) are the Legendre functions of the second kind, which can be represented by the Cauchy integral (cf. /4/)

$$Q_\ell(\lambda) = \frac{1}{2} \int_{-1}^{1} d\mu \frac{P_\ell(\mu)}{\lambda - \mu} = \hat{\mathcal{P}}_o \frac{P_\ell(\mu)}{\lambda - \mu}, \tag{8c}$$

where P_ℓ ($\ell = 0, 1, 2, \ldots$) is the Legendre polynomial.

The standard methods of complex analysis yield for the inverse Laplace transform of $\Phi(\vec{k}, \vec{v}, \zeta)$, which we denote F, the following expression

$$F(\vec{k},\vec{v},t) = \frac{1}{2\pi i r} \int_C d\zeta\, e^{\zeta t/r}, \tag{9}$$

where the contour C encircles all singularities of $\Phi(\vec{k}, \vec{v}, \zeta)$, i.e.:

i) the cut from $-1 - ik$ to $-1 + ik$. (10a)

ii) the poles corresponding to zeros of the functions

$$A^{-1}(k,\lambda), \qquad B^{-1}(k,\lambda) \qquad (8a, b). \tag{10b}$$

As it is seen from expressions (7), (8a,b,c) the cut is related to the Legendre function of the second kind. Let us mention that in fact it is due to the two branching points, because the formula (8c) can be written in the following form (cf. /4/)

$$Q_n(\lambda) = \frac{1}{2} P_n(\lambda) \ln\Big(\frac{\lambda + 1}{\lambda - 1}\Big) - W_{n-1}(\lambda), \tag{11}$$

where W_{n-1} is a polynomial of the order $n - 1$. For $\lambda \in (-1,1)$ we have

$$\lim_{\epsilon \to 0} Q_n(\lambda \pm i\epsilon) = \frac{1}{2} P_n(\lambda) \ln\Big(\frac{1 + \lambda}{1 - \lambda}\Big) - W_{n-1}(\lambda) \mp \frac{i\pi}{2} P_n(\lambda). \tag{12}$$

The cut runs along the segment of length $2k$. On the other hand k is the Knudsen number, i.e. the quotient of the mean free path $\ell = vr$ and the length $\lambda \sim 1/k$, which defines the space inhomogeneity of the system. Small Knudsen numbers mean that there are many collisions on the distance of the length λ: $\lambda \gg \ell$. So we say that condition $k \ll 1$ defines the collision-dominated regime. The opposite inequality $k \gg 1$ defines the collisionless regime. In the collision-dominated regime there exists a local equilibrium and the approach to the complete equilibrium is described with the help of the macroscopic equations of diffusion or hydrodynamics. One can show that for $k \ll 1$ and $t \gg \tau$ the cut contribution to the integral (9) is negligible. Thus, the long-time asymptotics of the FT of $\delta f(\vec{r}, \vec{v}, t)$ is described by zeros of the functions A^{-1}, B^{-1} (8a,b) only.

3. PROPERTIES OF SINGULARITIES OF THE FLT OF THE DEVIATION FUNCTION

Let us study how many zeros have the functions $A^{-1}(k, \lambda)$ and $B^{-1}(k, \lambda)$. According to familiar theorem, the difference of number of zeros N_D and poles P_D of a meromorphic function $f(\lambda)$ in the region D encircled by the contour C is related to the following contour integral

$$N_D - P_D = \int_C d\lambda \frac{\frac{df(\lambda)}{d\lambda}}{f(\lambda)} . \tag{13}$$

The function $f(\lambda)$ is analytic on C and in the region D, with exception of contingent poles at internal points. Additionally, it does not vanish on C. We shall apply the above theorem to our functions A^{-1} and B^{-1}. The only singularity in the whole plane of complex λ of both functions is the cut (9a). Thus, the contour shown on fig. 1 contains (with $R \to \infty$) all the zeros of these functions. Denote by N_σ (σ = A, B) the number of zeros of A^{-1} and B^{-1}.

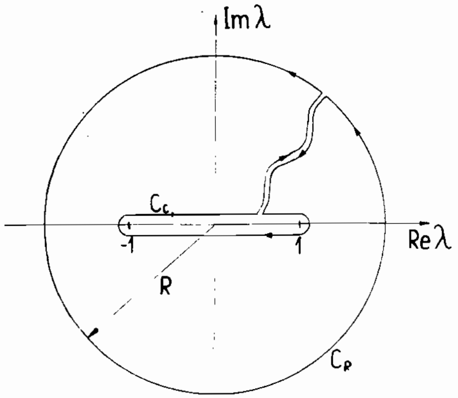


Fig. 1. Contour in the complex plane contains (with $R \to \infty$) all the zeros of functions A^{-1}, B^{-1} (8a, b).

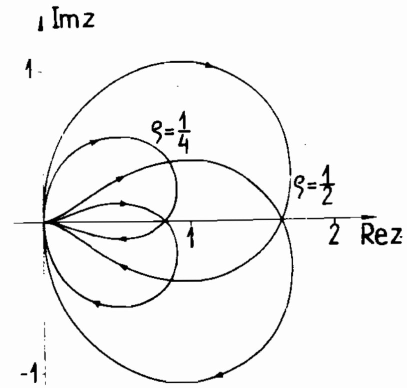


Fig. 2. Contour C_B (cf. eq. 15) depends on $\rho = a\tau/k$ only. Number of cycles made by this contour around the point z = (1,0) is equal to the number of zeros of the function B^{-1} (cf. figs. 5, 9, 10).

We have

$$N_\sigma(k, a\tau) = \frac{1}{2\pi i} \int_{C_c} d\lambda \frac{\frac{df_\sigma}{d\lambda}}{f_\sigma} + \lim_{R\to\infty} \int_{C_R} d\lambda \frac{\frac{df_\sigma}{d\lambda}}{f_\sigma} , \tag{14}$$

where $f_A = A^{-1}$, $f_B = B^{-1}$. Let us notice that the second of these functions depends only on one parameter $\rho = a\tau/k$. For both values of σ the second term of (14) vanishes. With the help of eqs. (8a,b) and (12), we can transform the first of integrals (14) to the form

$$N_\sigma = \frac{1}{\pi i} \int_{C_\sigma} dz \frac{1}{1 - z^2} , \tag{15}$$

where C_σ depends on the parameters k, $a\cdot\tau$ and is given by the following parametrization of the segment (-1, 1)

$$(-1, 1) \ni x \to \frac{\beta_\sigma(x)}{1 - a_\sigma(x)} , \tag{16}$$

where

$$a_A(\lambda) = -\frac{3a\tau}{(ik)^2}(ik\lambda - 1) + \frac{1}{2ik} \ln\left(\frac{1+\lambda}{1-\lambda}\right)\left[1 + \frac{3a\tau}{ik}(ik\lambda - 1)\lambda\right],$$

$$a_B(\lambda) = -\frac{3a\tau\lambda}{ik}\left[1 - \frac{1}{2}\lambda \ln\left(\frac{1+\lambda}{1-\lambda}\right)\right],$$

$$\beta_A(\lambda) = \frac{\pi}{2k}\left[1 + \frac{3a\tau}{ik}(ik\lambda - 1)\lambda\right],$$

$$\beta_B(\lambda) = \frac{3a\tau\pi}{2k}\lambda^2 .$$

Calculating the residua of $(1 - z^2)^{-1}$ at $z = \pm 1$ we obtain

$$N_\sigma = \ell_{-1} - \ell_{+1} , \tag{17}$$

where ℓ_ϕ ($\phi = \pm 1$) is the number of cycles made by the contour C_σ around the points $z = \phi$.

The contours C_B of integral (15) for different values of ρ are depicted in fig. 2. It is seen that $N_B(\rho = 1/2) = 2$ and $N_B(\rho = 1/4) = 0$. There exists a critical value of ρ, namely

$$\rho_C \approx 0{,}31 ,$$

such, that for $\rho \geq \rho_C$ the number N_B equals two and for $\rho < \rho_C$ it vanishes,

$$N_B(\rho) = \begin{cases} 2, & \rho \geq \rho_C \\ 0, & \rho < \rho_C . \end{cases}$$

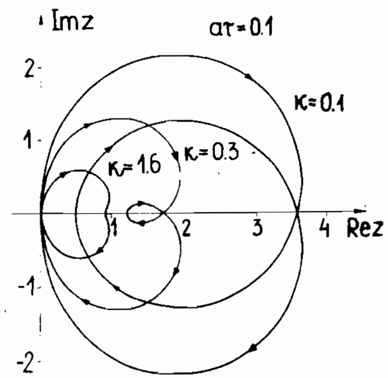


Fig. 3. Contour C_A *(cf. eq. 15) depends on both* $a\tau$ *and* κ *. For* $a\tau = 0.1$ N_A *may be equal to 0.1 or 2 dependent on* κ *(cf. fig. 9).*

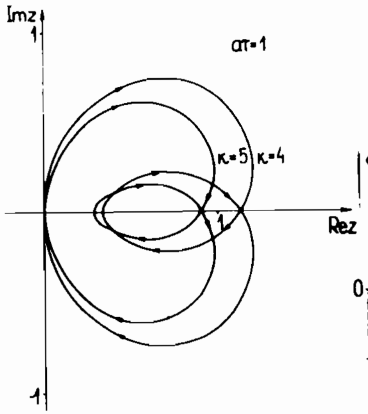


Fig. 4. The same as on fig. 3 for $a\tau = 1$*. For* $\kappa < 4.55$ *function* A^{-1} *has two zeros, for* $\kappa >$ > 4.55 *this function has no zeros (cf. figs. 10, 11).*

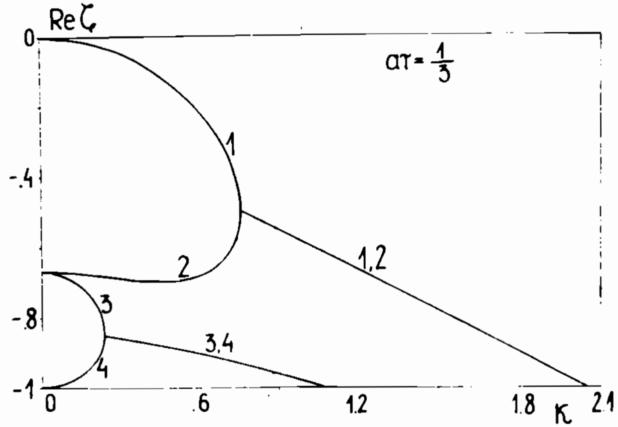


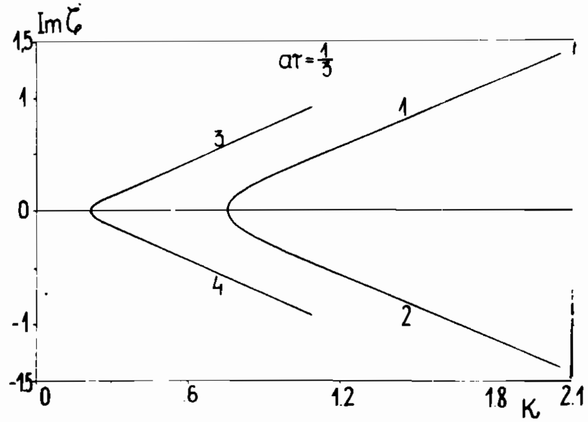
Fig. 5. The dependence of the real part of pole ζ_i *(i = 1, ... , 4) on* κ *for* $a\tau = 1/3$.

The number of zeros of the function A^{-1} depends on both parameters κ and $a\tau$. For $a\tau = 0.1$ the number N_A is equal to 0 ($\kappa = 1.6$), 1 ($\kappa = 0.3$) or 2 ($\kappa = 0.1$) (cf. fig. 3), whereas for $a\tau = 1$ the number N_A takes two values, namely 0 ($\kappa = 5$) or 2 ($\kappa = 4$) (fig. 4).

The above considerations made the starting point to the numerical calculations. The dependence of real parts of poles of $A(\kappa, \lambda, a\tau)$, which we denote ζ_1, ζ_2, and of $B(\kappa, \lambda, a\tau)$ - ζ_3, ζ_4, on κ for $a\tau = 1/3$ is shown in fig. 5. The dependence of imaginary parts of these poles on κ is presented in fig. 6. It is seen that for small κ each of denominators A^{-1} and B^{-1} has two real zeros. With growing κ they approach each other and for some critical value of κ

$$\kappa_C^j = \kappa_C^j(a\tau)$$

Fig. 6. The dependence of the imaginary part of pole ζ_i *(i = 1, ..., 4) on* κ *for* $a\tau = 1/3$.



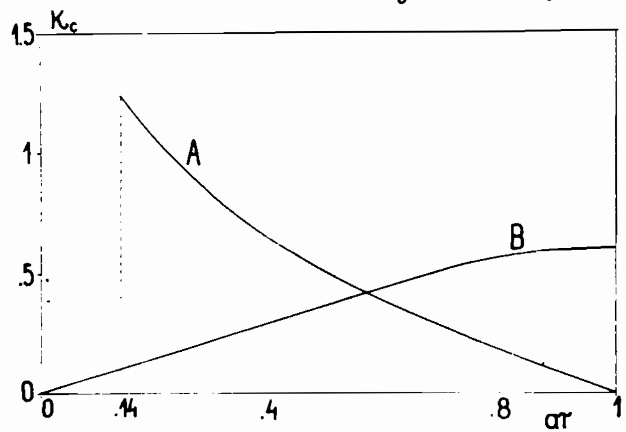


Fig. 7. The dependence of κ_C^j *(j = A, B) on* $a\tau$ *.*

(j = A, B) they leave the real axis. The dependence of κ_C^j on $a\tau$ is shown in fig. 7. For κ greater than κ_C^j the poles become complex conjugated to each other. In fact this property follows from the relation

$$Q_m\left(\frac{1+\zeta^*}{i\kappa}\right) = Q_m\left(-\left(\frac{1+\zeta^*}{i\kappa}\right)^*\right) = (-1)^{m+1} Q_m^*\left(\frac{1+\zeta}{i\kappa}\right).$$

According to our results concerning the number of poles, there exist regions of values of κ and $a\tau$, where some of poles or all poles disappear. Let us consider this problem in detail. For a given value of $a\tau$ and growing κ the complex poles approach the cut and starting from some limiting value of κ

$$\kappa_{lim}^j = \kappa_{lim}^j(a\tau)$$

(j = A, B) they vanish. According to eq. (9) for κ greater than κ_{lim}^j only the cut (10a) contributes to the FT of δf . The dependence of κ_{lim}^j on $a\tau$ is shown in fig. 8.

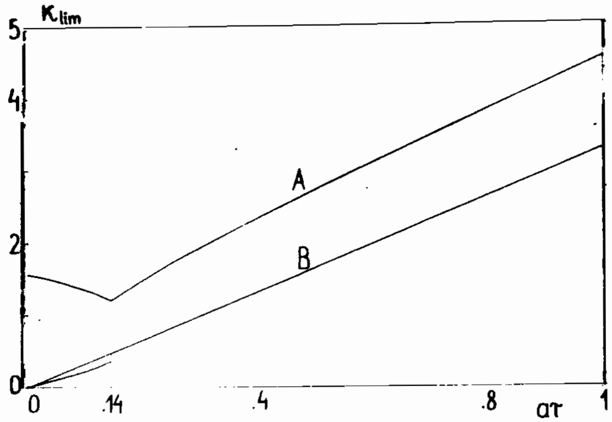


Fig. 8. The dependence of k^{j}_{lim} *(j = A, B) on* $a\tau$ *.*

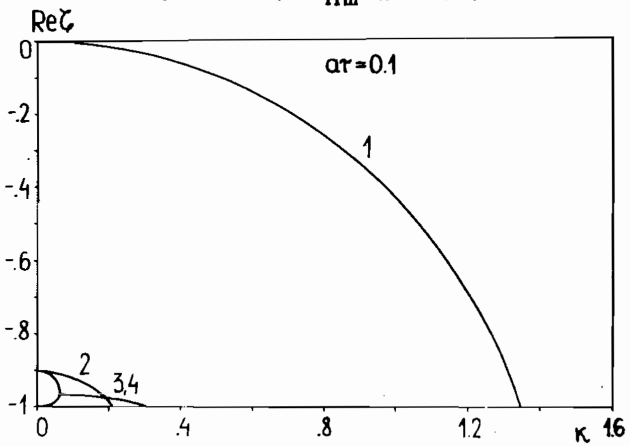


Fig. 9. The same as on fig. 5 for $a\tau = 0.1$*. The zeros of* A^{-1} *are real in the whole range of* $k \in (0,\ k^{A}_{lim})$*.*

Generally, the poles lie in the complex plane and only for small k they move along the real axis. There exists a critical value of $a\tau$

$$(a\tau)_C \approx 0{,}14,$$

such, that for $a < (a\tau)_C$ the poles of A are real for all $k \in (0,\ k^{A}_{lim})$ (cf. fig. 5, 9). Thus, the curve corresponding to the dependence of k^{A}_{C} on $a\tau$ starts with a finite value at the point $a\tau = (a\tau)_C$ and then diminishes almost lineary (cf. fig. 7). The dependence of k^{A}_{lim} on $a\tau$ for $a\tau < (a\tau)_C$ is represented by two line charts corresponding to the two zeros of A^{-1} (see fig. 8).

4. THE INFLUENCE OF THE ANISOTROPIC SCATTERING ON THE LONG-TIME ASYMPTOTIC BEHAVIOUR OF FT OF $\delta f(\vec{r}, \vec{v}, t)$

Following [1] for $0 \le a\tau < 1$ we get

$$\zeta_1 \approx \frac{k^2}{3(a\tau - 1)}\,, \tag{18a}$$

$$\zeta_2 \approx (a\tau - 1) + dk^2, \tag{18b}$$

$$\zeta_3 \approx (a\tau - 1) - \frac{3k^2}{5a\tau}, \tag{19a}$$

$$\zeta_4 \approx -1, \tag{19b}$$

where

$$d = -\frac{\frac{1}{3} + \frac{3}{5}a\tau + \frac{1}{3(a\tau - 1)}}{(a\tau)^2}\,.$$

The contribution of a pole ζ_j (j = 1,...,4) to the integral (9) is proportional to the exponential function $e^{\zeta_j t/\tau}$. So, according to eqs. (18a,b), for $0 \le a\tau < 1$, $t \gg \tau$ and $k \ll 1$ the contribution of ζ_1 dominates. We can write it in the following form

$$\exp(-Dk^2 t)\,, \tag{20}$$

where

$$D = \frac{v^2\tau}{3(1 - a\tau)}\,. \tag{21}$$

One can check that, for $0 \le a\tau < 1$, the deviation of the density of particles from equilibrium

$$\delta n(\vec{r}, t) = \hat{\mathcal{P}}_0\, \delta f(\vec{r}, \vec{v}, t) \tag{22}$$

obeys the diffusion equation

$$\frac{\partial}{\partial t}\, \delta n(\vec{r}, t) = D\Delta\delta n(\vec{r}, t)\,. \tag{23}$$

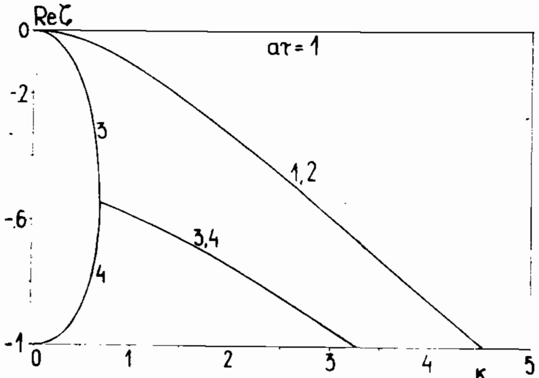


Fig. 10. The same as on fig. 5 for $a\tau = 1$.

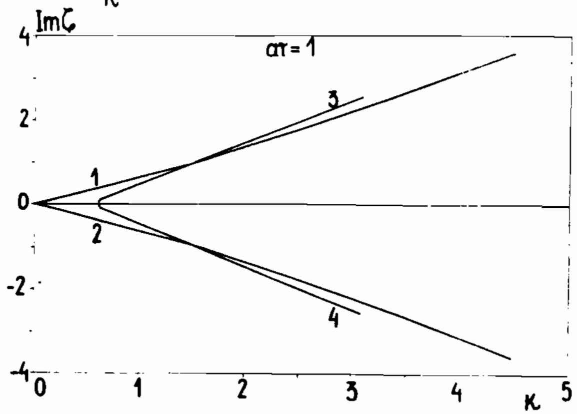


Fig. 11. The same as on fig. 6 for $a\tau = 1$.

Hence, for $0 \leq a\tau < 1$ the long-time behaviour of the MLG in the collision-dominated regime is purely diffusive. The diffusion constant is defined by eq. (21). It is seen also, that the diffusion constant for SLG [2,3] is modified by the factor $(1 - a\tau)^{-1}$.

Let us consider the behaviour of poles for $a\tau$ going to unity. The pole ζ_3 (19a) becomes diffusive and the expressions (18a,b) are not valid now. In this limit the diffusion constant (21) is divergent, which signals the change of behaviour from diffusive to hydrodynamic. Figs. 10 and 11 show the dependence of the real and imaginary parts of the poles respectively for $a\tau = 1$. As we see, for small k there exists two real zeros of B^{-1} and two complex conjugated to each other zeros of A^{-1}. Instead of (18), (19) we have now

$$\zeta_1 \approx -\frac{2}{15}k^2 + i\frac{k}{\sqrt{3}}, \tag{24a}$$

$$\zeta_2 = \zeta^* \approx -\frac{2}{15}k^2 - i\frac{k}{\sqrt{3}}, \tag{24b}$$

$$\zeta_3 \approx -\frac{3}{5}k^2, \tag{25a}$$

$$\zeta_4 \approx -1. \tag{25b}$$

The long time asymptotics is determined by eqs. (24a,b) and (25a). The contribution of (24a,b) to the integral (9) obeys the following set of hydrodynamic equations

$$\frac{\partial}{\partial t}\delta n + \nabla_\alpha p_\alpha = 0, \tag{26a}$$

$$\frac{\partial p_\alpha}{\partial t} + \frac{1}{3}v^2\nabla_\alpha \delta n - \frac{1}{5}v\ell\Delta p_\alpha - \frac{1}{15}v\ell\nabla_\alpha\nabla_\beta p_\beta = 0, \tag{26b}$$

where

$$\vec{p}(\vec{r}, t) = \hat{\mathcal{P}}_1 \delta f(\vec{r}, \vec{v}, t)$$

is the density of momentum. Let us mention, that since the energy density is proportional to the density of particles we deal with the set of two hydrodynamic equations only.

We conclude, that changing the strength of anisotropic scattering (i.e. the parameter $a\tau$) one observes the crossover from the relaxation of the diffusive type (described by solution of eq. (23)) to hydrodynamic behaviour described by eqs. (26a,b).

5. CONCLUSIONS

The long-time asymptotics for MLG is either diffusive ($0 \leq a\tau < 1$) or hydrodynamic ($a\tau = 6$). For example, the relaxation of neutrinos in supernovae ($a\tau = 1/3$) [1] is diffusive. In this work we studied the long-time asymptotics only in the collision-dominated regime ($k \ll 1$). It is clear that in order to study the solution in the whole range of t and k we need some assumptions about the FT of the initial deviation function $h(\vec{k}, \vec{v})$ (cf. eqs. 6 and 7). This problem was solved in [5] for the SLG. In the paper [7] we extend the result of [5] to the case of the modified Lorentz gas. We confine ourselves to a certain (quite general) class of initial conditions for which we are able to obtain the exact solution of the Boltzmann equation for all k and $0 < t < \infty$. This allows us to study the whole process of relaxation of initially disturbed system, i.e. the crossover from kinetic (collisionless) to the collision-dominated regime.

Finally, let us comment on the use of some simplified models of the collision integral in the lattice phonon kinetics (cf. [8]). Our relaxation time plays the role of relaxation time for normal processes τ_N. These processes do not change the total quasimomentum of the phonon gas. The parameter $(1 - a\tau)/\tau$ plays the role of the inverse of relaxation time for resistive processes τ_R. The resistive processes change the total quasimomentum. For massive ideal speciments at low temperatures the resistive processes are related solely to intrinsic lattice Umklapp processes and $\tau_R = \tau_U >> \tau_N$. This inequality corresponds to our condition $(1 - a\tau) << 1$. In such situation the total quasimomentum of the phonon gas is treated as an almost conserved quantity and in the collision-dominated regime this gas is described with the help of hydrodynamic equations for the local temperature $T(\vec{r}, t)$ and the local drift velocity $\vec{V}(\vec{r}, t)$. Our results suggest that these equations, which quite accurately describe the phonon phenomena such as the second sound, the Poisseuille flow and the heat conductivity, correspond to an extension of the description to finite values of k, where the difference between diffusive and hydrodynamic behaviour vanishes.

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Долговременное асимптотическое поведение лоренц-газа с анизотропией рассеяния

Рассматривается лоренц-газ с анизотропией рассеяния. Интеграл столкновения содержит члены, отвечающие изотропному и анизотропному рассеянию. Исследована проблема Коши для уравнения Больцмана.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1988

Jasiukiewicz Cz., Paszkiewicz T., Wozny J. **E17-88-171**

Long-Time Asymptotic Behaviour of the Lorentz Gas with Anisotropic Scattering

The rarefied Lorentz gas with anisotropic scattering is considered. The collision integral of the Boltzmann equation contains terms related to isotropic and anisotropic scattering. The Cauchy problem for this equation is studied. When isotopic and anisotropic contributions are equally effective, the long-time asymptotics of the solution is hydrodynamic, whereas for prevaling isotropic scattering the long-time asymptotics is purely diffusive. In particular, the gas of neutrinos in supernovae behaves diffusively.

The investigation has been performed at the Laboratory of Theoretical Physics.

Communication of the Joint Institute for Nuclear Research. Dubna 1988