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CONDENSATE STABILITY IN THE MODEL OF BOSE-GAS WITH TWO- AND THREE-PARTICLE INTERACTIONS

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1. The wide range of physical problems can be studied on the basis of the nonideal Bose-gas models. On quantum-mechanical levels such models of N particles is described by the Shrödinger equation $H\Phi=(T+U)\Phi=E\Phi$ with, for example, δ-function interaction potential U. Three-particle repulsive and two-particle attractive forces were taken into account, e.g., in[1] through the following potential

$$U=2U_c\sum_{i\neq j}^{N}\delta(x_i-x_j)-3W_c\sum_{i\neq j\neq k}^{N}\delta(x_i-x_j)\,\delta(x_i-x_k)\ ,\ (U_c,W_c>0).$$

In this concern for the one-particle wave function ψ in a self-consistent fields we have

$$i\psi_t+\psi_{xx}-\alpha\psi+|\psi|^2\psi-|\psi|^4\psi=0\ ,\quad(\alpha>0). \qquad (1)$$

Eq. (1) is also used to describe a great number of other physycal phenomena. For example, it describes propagation of light beams in nonlinear media[2], nuclear hydrodynamics with Skyrme's forces[3], it also arises in studying some problems of the theory of magnetism and molecular crystalls. The phenomenological description of HeII superfluidity with inhomogeneous and nonstationary order parameter ψ is also based on Eq.(1)[4].

Various soliton-like solutions to this equation under trivial boundary conditions $(\psi(x,t)\to 0,\ x\to\pm\infty)$, which correspond to "drops" of quasiparticles, have been obtained in[1,5]. The authors of Ref.[5] have also considered the "condensate" version of Eqs. (1)

$$i\varphi_t+\varphi_{xx}-\rho_c(2A+\rho_c)\varphi+2(A+2\rho_c)|\varphi|^2\varphi-3|\varphi|^4\varphi=0 \qquad (2)$$

with nonvanishing boundary conditions $(|\varphi(x,t)|\to\sqrt{\rho_0}\ ,\ \varphi_x(x,t)\to 0,$

$x\to\pm\infty$), which correspond to the finite density gas $\rho_c=\lim\limits_{\substack{N\to\infty\\ L\to\infty}}\frac{N}{L}$ (thermodynamical limit). They also obtained the following soliton-like solution:

$$\varphi(x,t)=\sqrt{2\rho_c}\,\mathrm{ch}(\tilde{x}-i\mu)\left[(2\rho_c-A)\cdot(A^2+V^2)^{-1/2}+\mathrm{ch}\,2\tilde{x}\right]^{-1/2}, \quad (3)$$

where $\tilde{x}=\frac{\sqrt{c^2-V^2}}{2}(x-Vt-x_0)$, $\cos 2\mu=\frac{A\rho_0+V^2/2}{\rho_0\sqrt{A^2+V^2}}$ and $c=2\sqrt{\rho_c(\rho_c-A)}$ is the sound velocity in the condensate of finite density ρ_c. This condensate is stable under small perturbations (Bogolubov mode) of the frequency $\omega_b=|\vec{k}|\sqrt{\vec{k}^2+4\rho_c(\rho_c-A)}$.

Two integrals of motion of the Eqs. (2) look like:

1) energy of system

$$E=\int_{-\infty}^{\infty}\{|\varphi_x|^2+(|\varphi|^2-\rho_c)^2\,(|\varphi|^2-A)\}\,dx$$

2) number of particles

$$N=\int_{-\infty}^{\infty}(|\varphi|^2-\rho_c)\,dx .$$

Solution (3) describes a solitary wave of rarefaction-bubble, propagating in a condensate with the velocity $V<c$ at $0<\mu<\pi/2$ (or $0<A<\rho_c$). The question of stability of such bubbles was investigated in works/6,7/, where the connection between the bubble stability and condensate stability was established; namely static bubble (3) (V=0) at $0<A<\rho_0$ is infinitesimally unstable/6/, so that one of the bubble instability modes creates a cavity expanding unlimitedly, which gives rise to condensate decay/7/. As was numerically shown in /7/, for a moving bubble there is a critical velocity $V_{1cr}(A)$ - so bubble (3) is stable at $V_{1cr}<V\le c$ and it decays through the very modes at $V\le V_{1cr}(A)$. Stability region for moving bubbles (3) lies above the line 1 in the Figure.

2. Let us look how the situation changes when two-bubble collision takes place. For two bubbles, moving with the velo-

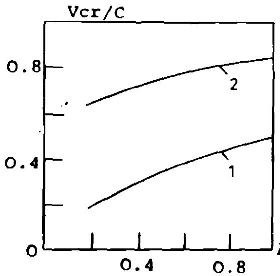

Fig. (1) the threshold bubble velocity V_{1cr} as a function of A; (2) the same for velocity V_{2cr} which separates regions of inelastic and quasi-elastic collisions of two bubbles.

cities $V_1<V_2$, the relation $E_b(V_1)>E_b(V_2)$ is easily shown to hold for all A. First we describe the head-on collisions of two remoted bubbles (3), with velocities V_1 and V_2. The calculations at $\rho_c=1$ (without losing generality) and the several values of A were carried out. The results indicate a critical velocity $V_{2cr}(A)>V_{1cr}(A)$ to exist such that at $V_1,V_2>V_{2cr}(A)$ the "fast" bubbles emerge from collision without essential change. At $V_{1cr}<V_1,V_2<V_{2cr}$ an unlimitedly expanding cavity is formed as a result of the bubble collision (note that gas from the cavity is completely forced out). The behaviour of such a cavity is the same as in the case of decaying a slow $V<V_{1cr}$ (or static) bubble. The quasielastic bubble collision region lies above the line 2 in the Figure.

Collisions of unidirectionally moving bubbles do not influence the condensate stability.

In the Table the energies of system (condensate plus bubble) $E_b(V_{1cr}(A))$ and $2E_b(V_{2cr}(A))$, corresponding to excitations of one and two bubbles, at several values of A and the threshold velocities are listed. From these data the excitation energy $E_b(V_{1cr})$ for one threshold bubble able to destroy condensate is easily seen to be somewhat greater than $2E_b(V_{2cr})$ the energy needed to create two threshold bubbles interaction of which can cause condensate to decay as well. This means in the system under consideration there is an additional "dynamical" channel for the condensate to decay due to bubble gas formation.

Table

A	0.2	0.5	0.8	0.9
V_{1cr}/C	0.2	0.35	0.45	0.475
$E_b(1)$	0.482519	0.133013	0.012715	0.002217
V_{2cr}/C	0.64	0.75	0.8	0.83
$2E_b(2)$	0.470493	0.118053	0.012156	0.001978

This channel is somewhat energetically preferable. However, more important is the following: the input energy density necessary for the gas channel decay is nearly twice less than that for formation of a static (or slowly moving) unstable bubble ("static" channel). Roughly speaking, this implies a decay of condensate through the gas channel to begin at temperature twice less than that needed for the "static" channel to open.

In conclusion one of the authors (V.G.M.) is grateful to Academician N.N.Bogolubov for his valuable comments on the results of the work.

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Холмуродов Х.Т., Маханьков В.Г. E17-87-861
Устойчивость конденсата в рамках модели бозе-газа
с двух- и трехчастичным взаимодействием

Исследована задача об устойчивости бозе-конденсата в рамках модели с двухчастичным притяжением и трехчастичным отталкиванием. Установлены два канала распада конденсата, связанные с возбуждением в нем одного пузыря и газа пузырей. Как показывают расчеты, газовый канал распада энергетически более выгоден, так что он имеет место при более низких, чем односолитонный канал, температурах.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Kholmurodov Kh.T., Makhankov V.G. E17-87-861
Condensate Stability in the Model of Bose-Gas
with Two- and Three-Particle Interactions

The problem of the Bose-condensate stability in the model with two-particle attraction and three-particle repulsion is investigated. Two channels of condensate decay due to excitation of one bubble and a gas of bubbles are observed. Calculations show the bubble gas channel of decay to be energetically preferable so that the decay should proceed at relatively low temperatures than in the case of the one bubble channel.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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