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E17-87-854

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SQUEEZING
IN COLLECTIVE RAMAN SCATTERING

Submitted to "Physics Letters A"

1987

In recent years, a large amount of theoretical[1-14,25] and experimental[15,16] works is concentrated on the problem of generation of squeezed light having noise in one electric-field quadrature being less than that of a coherent state. The first experiment using the squeezed light to reduce the level of fluctuations below the shot-noise limit has been demonstrated in the work[17].

In this letter we discuss the generation of squeezed light in the process of collective Raman scattering in an intense external field. It has been shown the condition for receiving a field with a large (nearly perfect) degree of squeezing that is necessary to increase the signal-to-noise ratio relative to the shot-noise limit[17,18].

The theory of collective Raman scattering has been developed in the works[19]. The N three-level atoms concentrated in a region small compared to the wavelength of all the relevant radiation modes (Dicke model) interact with one mode of monochromatic deriving field of frequency ω_L and with the vacuum of other modes (fig. 1). The states $|1>$ and $|2>$ are the ground and excited states, respectively. The state $|3>$ may be a low-lying vibrational or rotational excitation from the ground state. The transitions $|2> \to |1>$ and $|2> \to |3>$ are electro-dipole transitions and the transition $|3> \to |1>$ is caused by an atomic reservoir and is assumed to be nonradiative[26].

In treating the external field classically and using the Born and Markov approximations to describe the system-reservoir couplings, one obtains a master equation for the reduced density operator ρ of the atomic system alone in the following form[22] ($\hbar \equiv 1$ units are used):

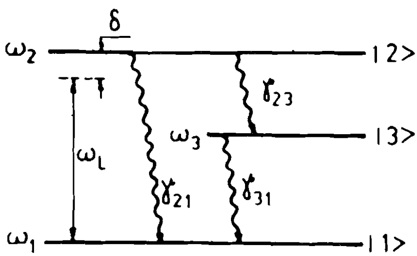


Fig. 1. Level scheme of the atomic system.

$$\frac{\partial \rho}{\partial t} = -i[H_{coh}, \rho] - \frac{\gamma_{21}}{2}(J_{21} J_{12} \rho - J_{12} \rho J_{21} + H.C.) - \tag{1}$$
$$- \frac{\gamma_{23}}{2}(J_{23} J_{32} \rho - J_{32} \rho J_{23} + H.C.) - \frac{\gamma_{31}}{2}(J_{31} J_{13} \rho - J_{13} \rho J_{31} + H.C.) = L\rho,$$

where γ_{21} and γ_{23} are the radiative spontaneous transition probabilities per unit time for a single atom to change from level $|2>$ to $|1>$ and from $|2>$ to $|3>$ respectively; γ_{31} is the nonradiative rate for the atomic transition from $|3>$ to $|1>$.

The coherent part of the Hamiltonian H_{coh} in the interaction picture has the form

$$H_{coh} = \frac{\delta}{2}(J_{22} - J_{11}) + G(J_{21} + J_{12}) - \Omega_3 J_{33}.$$

Here $\Omega_3 = \omega_{23} - \omega_{21}/2$ (where $\omega_{ij} = \omega_i - \omega_j$); $\delta = \omega_{21} - \omega_L$ is the frequency detuning of resonance; $G = -d_{21}E_0$ is the resonance Rabi frequency describing the interaction of the driving field with the atomic system; $J_{k\ell}$ $(k,\ell = 1,2,3)$ are the collective operators of the atomic system having in the Schwinger representation [21] the following form:

$$J_{k\ell} = C_k^+ C_\ell \qquad (k,\ell = 1,2,3),$$

where the operators C_k and C_k^+ obey the boson commutation relation

$$[C_k, C_\ell^+] = \delta_{k\ell}$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level $|k>$.

Further, we investigate the case of an intense external field or of a large detuning δ only so that the following relation is fulfilled:

$$\Omega = (\frac{1}{4}\delta^2 + G^2)^{1/2} \gg N\gamma_{k\ell}. \tag{2}$$

After the canonical (dressing) transformation

$$C_1 = \cos\phi Q_1 + \sin\phi Q_2, \quad C_2 = -\sin\phi Q_1 + \cos\phi Q_2, \quad C_3 = Q_3, \tag{3}$$

where $tg2\phi = 2G/\delta$ and after performing the secular approximation [19,23], i.e. ignoring the part of the Liouville operator L containing rapidly oscillating terms with frequencies 2Ω, 4Ω, one can find a stationary solution of the master equation in the form [19]

$$\tilde{\rho} = U\rho U^+ = A^{-1} \sum_{p=0}^{N} x^p \sum_{M=0}^{p} z^M |p,M><M,p|, \tag{4}$$

where U is the unitary operator representing the canonical transformation (3)

$$x = \gamma_{31}/(\gamma_{23}\, ctg^2\phi), \qquad z = ctg^4\phi,$$

$$A = \frac{z}{z-1} \cdot \frac{(xz)^{N+1} + 1}{xz - 1} - \frac{1}{z-1} \cdot \frac{x^{N+1} - 1}{x - 1}, \tag{5}$$

$|p,M>$ is an eigenstate of the operators $R = R_{11} + R_{22}$, R_{11} and the operator of a number of atoms $\hat{N} = R_{11} + R_{22} + R_{33}$, here $R_{k\ell} = Q_k^+ Q_\ell$ $(k,\ell = 1,2,3)$ are the collective operators of "dressed" atoms. The operator Q_k, Q_k^+ satisfies the boson commutation relation (the transformation (3) is canonical)

$$[Q_k, Q_k^+] = \delta_{k\ell}. \tag{6}$$

The solution (4) allows one to calculate all stationary expectation values of the atomic observables [19].

We proceed with the study of the reduction of the quantum fluctuation in the fluorescent field according to the atomic transition $|2> \to |1>$ (Rayleigh line). In the radiation zone, the positive frequency part of the electric field has the form [7,8]

$$E^{(+)}(\vec{x}, t) = E^{(+)}_{free}(\vec{x}, t) + \psi(\vec{x}) J_{12}(t - \frac{r}{c}) e^{-i\omega_L (t - \frac{r}{c})} \tag{7}$$

where $\psi(\vec{x})$ is a geometrical factor, $r = |\vec{x}|$.

With the use of the canonical transformation (3), the atomic collective operator $J_{12}(t)$ has the structure

$$J_{12}(t) = S_0(t) + S_{-1}(t) + S_{+1}(t), \tag{8}$$

where

$$S_0(t) = \sin\phi\cos\phi(R_{22}(t) - R_{11}(t)),$$

$$S_{+1}(t) = \cos^2\phi R_{12}(t) = \cos^2\phi \tilde{R}_{12}(t) e^{-2i\Omega t},$$

$$S_{-1}(t) = -\sin^2\phi R_{21}(t) = -\sin^2\phi \tilde{R}_{21}(t) e^{2i\Omega t}.$$

In the secular approximation $R_{11}(t)$, $R_{22}(t)$, $\tilde{R}_{12}(t)$ and $\tilde{R}_{21}(t)$ are the slowly varying "dressed" atomic operators. The operators $S_{-1}(t)$, $S_0(t)$ and $S_{+1}(t)$ can be considered as operator-sources of the spectrum components centered at frequencies $\omega_L-2\Omega$, ω_L and $\omega_L+2\Omega$, respectively.

In the following calculations we omit the free part $E^{(+)}_{free}$ in relation (7) that does not affect the normally ordered variance of the fluorescence field. The delayed time contribution has been ignored too [24] in the stationary limit. By using the steady-state solution (4) and eqs. (7-8) one can show that squeezing is absent for the whole stationary fluorescence field $E^+(\vec{x},t)$ and for any spectrum components $S_{-1}(t)$, $S_0(t)$ and $S_{+1}(t)$ taken separately.

Analogously to the resonance fluorescence in a system of two-level atoms [13] the squeezing may occur only in the mixture of two sidebands $S_{-1}(t)$ and $S_{+1}(t)$ and the spectral analysis of squeezing will be given in the following calculations.

Applying eqs. (7-8), one finds the Fourier transform of the field at two frequencies ν_1 and ν_2 located on the two sidebands S_{+1} and S_{-1} in the form

$$\begin{aligned} \tilde{E}^{(+)}(\vec{x}, \omega_L + 2\Omega - \epsilon_1) &= \psi(\vec{x}) \cos^2\phi\, \tilde{R}_{12}(\epsilon_1), \\ \tilde{E}^{(+)}(\vec{x}, \omega_L - 2\Omega + \epsilon_2) &= -\psi(\vec{x}) \sin^2\phi\, \tilde{R}_{21}(\epsilon_2), \end{aligned} \tag{9}$$

where

$$\epsilon_1 = \omega_L + 2\Omega - \nu_1, \quad \epsilon_2 = -(\omega_L - 2\Omega) + \nu_2 .$$

The quadrature phase components for the mixture of two frequency components on the two sidebands are defined as

$$\tilde{E}_\theta(\vec{x}, \epsilon_1, \epsilon_2) = \frac{1}{2}[\tilde{E}^{(+)}_M(\vec{x}, \epsilon_1, \epsilon_2) e^{i\theta} + \tilde{E}^{(-)}_M(\vec{x}, \epsilon_1, \epsilon_2) e^{-i\theta}], \tag{10}$$

where

$$\tilde{E}^{(+)}_M(\vec{x}, \epsilon_1, \epsilon_2) = \frac{1}{\sqrt{2}}[\tilde{E}^{(+)}(\vec{x}, \omega_L + 2\Omega - \epsilon_1) + \tilde{E}^{(+)}_M(\vec{x}, \omega_L - 2\Omega + \epsilon_2)],$$

$$\tilde{E}^{(-)}_M(\vec{x}, \epsilon_1, \epsilon_2) = \frac{1}{\sqrt{2}}[\tilde{E}^{(-)}(\vec{x}, \omega_L + 2\Omega - \epsilon_2) + \tilde{E}^{(-)}_M(\vec{x}, \omega_L - 2\Omega + \epsilon_1)].$$

For the cases $\theta = 0$ and $\theta = \pi/2$ the quadrature phase components $\tilde{E}_\theta(\vec{x}, \epsilon_1, \epsilon_2)$ coincide with the in-phase ($\tilde{E}_1(\vec{x}, \epsilon_1, \epsilon_2)$) and out-phase ($\tilde{E}_2(\vec{x}, \epsilon_1, \epsilon_2)$) components, respectively.

In the case of a large number of atoms [19] one finds

$$\langle \tilde{R}_{21}(\epsilon_1) \tilde{R}_{12}(\epsilon_2) \rangle = \delta(\epsilon_1 - \epsilon_2) \langle R_{21} R_{12} \rangle \cdot \frac{2\Gamma_{+1}}{\epsilon_1^2 + \Gamma_{+1}^2}, \tag{11}$$

$$\langle \tilde{R}_{12}(\epsilon_2) \tilde{R}_{21}(\epsilon_1) \rangle = \delta(\epsilon_1 - \epsilon_2) \cdot \frac{2\langle R_{12} R_{21} \rangle \cdot \Gamma_{-1}}{\epsilon_1^2 + \Gamma_{-1}^2}, \tag{12}$$

where

$$\langle R_{12} R_{21} \rangle = \langle R_{11} \rangle + \langle R R_{11} \rangle - \langle R_{11}^2 \rangle, \tag{13}$$

$$\langle R_{21} R_{12} \rangle = \langle R R_{11} \rangle + \langle R \rangle - \langle R_{11}^2 \rangle - \langle R_{11} \rangle, \tag{14}$$

$$\begin{aligned} \Gamma_{11} &= 2\gamma_{21} \sin^2\phi \cos^2\phi + \gamma_{21} \cos^2\phi + \frac{1}{2}\gamma_{23} + \\ &+ \frac{1}{2}\gamma_{21} (\sin^2\phi - \cos^2\phi)(\langle R \rangle - 2\langle R_{11} \rangle) + \frac{1}{2}(\gamma_{23} - \gamma_{31}) \cdot (N - \langle R \rangle), \end{aligned} \tag{15}$$

$$\begin{aligned} \Gamma_{-1} &= 2\gamma_{21} \sin^2\phi \cos^2\phi + \gamma_{21} \sin^4\phi + \frac{1}{2}\gamma_{23} + \frac{1}{2}\gamma_{21} \times \\ &\times (\sin^2\phi - \cos 2\phi)(\langle R \rangle - 2\langle R_{11} \rangle) + \frac{1}{2}(\gamma_{23} - \gamma_{31})(N - \langle R \rangle). \end{aligned} \tag{16}$$

In eqs. (13-16) stationary expectation values $\langle R^n R_{11}^m \rangle$ are calculated over the atomic steady-state (4) [19]. The quantities Γ_{+1} and Γ_{-1} are the spectral widths of two sidebands S_{+1} and S_{-1}, respectively.

The normally ordered variance of the quadrature phase component $\tilde{E}_\theta(\epsilon_1, \epsilon_2)$ can be found using the relations (9-12) and takes the form

$$\langle : (\Delta \tilde{E}_\theta(\epsilon_1, \epsilon_2))^2 : \rangle = \delta(\epsilon_1 - \epsilon_2) \tag{17}$$

with

$$\begin{aligned} S_\theta(\epsilon_1) &= \frac{1}{2}\gamma_{21} \{(\cos^2\phi - \cos 2\theta \sin^2\phi) \cdot \cos^2\phi \langle R_{21} R_{12} \rangle \cdot \\ &\times \frac{\Gamma_{+1}}{\epsilon_1^2 + \Gamma_{+1}^2} + (\sin^2\phi - \cos 2\theta \cos^2\phi) \cdot \sin^2\phi \cdot \langle R_{12} R_{21} \rangle \cdot \frac{\Gamma_{-1}}{\epsilon_1^2 + \Gamma_{-1}^2}\}, \end{aligned} \tag{18}$$

where we identify $S_\theta(\epsilon_1)$ with the spectrum of squeezing in the mixture of two sidebands S_{+1} and S_{-1}.

In relations (17-18) we have dropped the argument $\vec{x}$ (the position of the detector) and have followed the usual convention and have renormalized the correlation functions to the total flux [7,12].

We speak of squeezing for the quadrature phase component $\tilde{E}_\theta(\epsilon_1, \epsilon_2)$ if

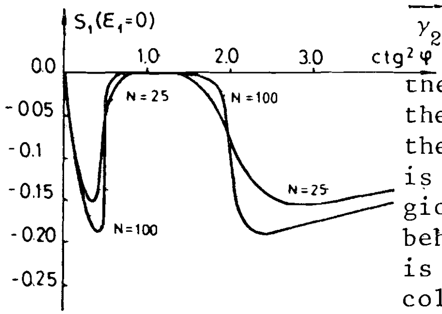
$$S_\theta(\epsilon_1) < 0. \tag{19}$$

From relation (18) one can see that the spectrum of squeezing $S_\theta(\epsilon_2)$ has the negative Lorentzian form with the peak degree of squeezing occuring at the point $\epsilon_1 = 0$, i.e. when the two frequency components $\tilde{E}^{(+)}(\nu_1)$ and $\tilde{E}^{(+)}(\nu_2)$ are located at the centered frequencies of the two sidebands S_{+1} and S_{-1}, respectively (i.e. $\nu_1 = \omega_L + 2\Omega$; $\nu_2 = \omega_L - 2\Omega$). The peak degree of squeezing $S_1(\epsilon_1 = 0)$ as a function of the parameter $\mathrm{ctg}^2\phi$ for the case $\gamma_{21} = \gamma_{23} = 2\gamma_{31}$ and for the case $\gamma_{21} = \gamma_{23} = \frac{1}{2}\gamma_{31}$ is plotted on fig. 2a and fig. 2b, respectively.

For the case $\frac{\gamma_{31}}{\gamma_{23}} < 1$ (see fig. 2a), the function $S_1(\epsilon_1 = 0)$ shows the behaviour

(i) In the region of the parameter $\mathrm{ctg}^2\phi$:

$$\frac{\gamma_{31}}{\gamma_{23}} < \mathrm{ctg}^2\phi < \frac{\gamma_{23}}{\gamma_{31}}$$

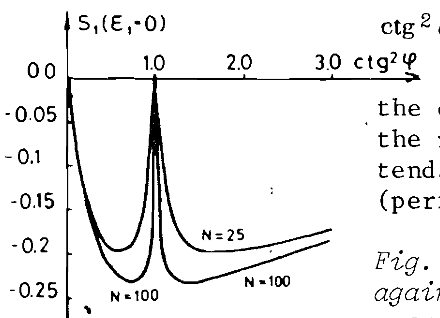


(a)

the degree of squeezing reduces as the number of atoms increases. For the limiting case $N \to \infty$, squeezing is absent ($S_1(\epsilon_1 = 0) = 0$) in this region of the parameter $\mathrm{ctg}^2\phi$. Such a behaviour of the degree of squeezing is connected with the phenomenon of collective population trapping/20/ when almost all atoms are trapped on the atomic level $|3\rangle$.

(ii) In the other regions

$$\mathrm{ctg}^2\phi < \frac{\gamma_{31}}{\gamma_{23}} \quad \text{or} \quad \mathrm{ctg}^2\phi > \frac{\gamma_{23}}{\gamma_{31}}$$



(b)

the degree of squeezing increases as the number of atoms increases. and tends to the limiting value $S_1 \approx -0.25$ (perfect squeezing) when $N \to \infty$.

Fig. 2. Function $S_1(\epsilon_1 = 0)$ *is plotted against the parameter* $\mathrm{ctg}^2\phi$ *for the case (a):* $\gamma_{21} = \gamma_{23} = 2\cdot\gamma_{31}$ *and for the case (b):* $\gamma_{21} = \gamma_{23} = \frac{1}{2}\cdot\gamma_{31}$.

For the case $\frac{\gamma_{31}}{\gamma_{23}} > 1$ (see fig. 2b) the collective population trapping on the level $|3\rangle$ is absent for any value of the parameter $\mathrm{ctg}^2\phi$ /20/ and in this case the degree of squeezing increases as the number of atoms increases. In the limiting case $N \to \infty$ the degree of squeezing tends to the limiting value $S_1 \approx -0.25$ in the region of $\mathrm{ctg}^2\phi \approx 1$ ($\mathrm{ctg}^2\phi \neq 1$) (see fig. 2b).

By using the above procedure one can show that squeezing is absent in the stokes line according to the atomic transition $|2\rangle \to |3\rangle$.

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Received by Publishing Department
on December 2, 1987.

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Сжатие в коллективном рассеянии

Обсуждена генерация сжатого состояния для коллективного рассеяния Римана в интенсивном внешнем поле. Показано, что сжатие существует только в смеси двух крайних спектральных компонент рэлеевской линии и отсутствует для стоксовой линии. Даны спектральный анализ сжатия и условия для получения почти предельной степени сжатия.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

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Squeezing in Collective Raman Scattering

The generation of a squeezed state via the collective Raman scattering in an intense external field is discussed. It is shown that squeezing is present only in the mixture of two sidebands of the Rayleigh line while it is absent for the Stokes line. The spectral analysis of the squeezing and the condition for receiving prefect squeezing is given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987