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A PSEUDO-GOLDSTONE MODE IN THE EXACTLY SOLUBLE MODEL OF PHASE TRANSITION

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There are only a few exactly soluble models of phase transitions /1/ and all of them play an important role in statistical physics. In this note we shall discuss an interesting property of the model with an anharmonic interaction of an infinite range proposed by Schneider, Stoll and Beck /2/

$$H = \sum_{l} \frac{P_l^2}{2M} - \frac{A}{2}\sum_{l} Q_l^2 + \frac{B}{4N}\Big(\sum_{l} Q_l^2\Big)^2 + \frac{1}{4}\sum_{ll_1} \varphi_{ll_1}(Q_l - Q_{l_1})^2 \tag{1}$$

$$= E_k + V + U \qquad , \tag{1a}$$

where E_k and U describe the kinetic energy and harmonic interaction of N atoms in d-dimensional lattice, $l = (l_1 \dots l_d)$, V is the rest of the potential energy; P_l and Q_l are canonically conjugated momentum and coordinate, respectively. A detailed analysis of the thermodynamical properties of this model in classical and quantum limits, has been given by Plakida and Tonchev /3/. Schneider et al./2/ found that in the ordered phase ($d \geq 3$, classical limit, see also /3/) there appears a gapless mode. Plakida and Tonchev /3/, by using the method of approximating Hamiltonian, showed how the free energy may be estimated exactly in the thermodynamical limit ($N \to \infty$) and also noticed the appearance of this acoustic-type mode below T_c (in fact, the term of an interaction with an external field has been added to (1) and only then free energy was found in the whole range of temperatures. Nextly, the limit of vanishing external field was taken). The natural question arises: In this gapless mode a Goldstone-type excitation arises: Is this gapless mode a Goldstone-type excitation or, in the other words, is there any continuous symmetry in the model (1) broken by the phase transition?

Let us notice that a part of potential energy, V, may be written in the symmetrical form

$$V = \frac{NB}{4}\Big(\frac{A}{B} - X^T \circ X\Big) - N\frac{A^2}{4B} \qquad , \tag{2}$$

where X is N-component column-vector with coordinates $\{Q_l\}$,

$$X = \frac{1}{\sqrt{N}}\big(\{Q_l\}\big) \qquad . \tag{3}$$

Since both the kinetic energy term E_k and the potential energy term, V, have a spherical symmetry in the N-dimensional space, we easily characterize small oscillations around any of the minima of V determined by the sphere equation

$$X^T \circ X = \frac{A}{B} \qquad . \tag{4}$$

The spectrum of such oscillations consists of N-1-times degenerated zeroth frequency mode corresponding to the free motion and oscillations with frequency

$$\omega_0^2 = \frac{2A}{M} \qquad , \tag{5}$$

perpendicular to the sphere (4). The presence in the Hamiltonian (1) of the term U "breaks" this spherical symmetry of $E_K + V$ term, but it is not obvious if some continuous symmetry is left. Using notation (3) for the Fourier coordinates $\{Q_q\}$

$$\tilde{X} = \frac{1}{\sqrt{N}}\begin{pmatrix} Q_0 \\ \{Q_q\} \end{pmatrix} , \qquad \tilde{X}^+ = \frac{1}{\sqrt{N}}\big(Q_0 , \{Q_{-q}\}\big) , \tag{6}$$

and assuming that

$$\varphi_0 - \varphi_q > 0 \qquad (q \neq 0) \qquad , \tag{7}$$

where φ_q is a Fourier transform of $\{\varphi_{ll_1}\}$, we can write down (1) in the following form:

$$H = N\Big\{\frac{1}{2M}\tilde{P}^+ \circ \tilde{P} + \frac{B}{4}\Big(\frac{A}{B} - \tilde{X}^+ \circ \tilde{X}\Big)^{\!} + \frac{1}{2}\tilde{X}^+ \hat{\Phi}\tilde{X}\Big\} - N\frac{A^2}{4B} \qquad , \tag{8}$$

where $\hat{\Phi}$ is diagonal NxN matrix with non-negative, $\varphi_0 - \varphi_q$, elements. Assumption (7), which is not very strong and in fact has been used in the above mentioned papers /2,3/, allows one to find the absolute minima of the potential energy in (8)

$$(V + U)\Big|_{\tilde{x} = \tilde{x}^*} = - N \frac{A^2}{4B} , \tag{9}$$

as determined by the condition (4) where only the first component of the $\tilde{x}^*$ -vector differs from zero:

$$\tilde{x}^* = \frac{1}{\sqrt{N}} \begin{pmatrix} Q_0^* \\ 0 \\ 0 \\ \vdots \end{pmatrix} , \tag{10a}$$

$$(Q_0^*)^2 = N \frac{A}{B} . \tag{10b}$$

Therefore, the phase transition in the model (1) is associated with a spontaneously broken discrete symmetry (10b), and gapless mode found in /2,3/ is not the Goldstone type mode.

Now, let us study the spectrum of small oscillations around one of the minima (10)

$$\tilde{x} = \tilde{x}^* + \delta \tilde{x} .$$

A quadratic, in variables $\delta \tilde{x}$, part of the Hamiltonian (8) takes the simple form

$$\mathcal{H} = N \sum_q \left\{ \frac{1}{2M} P_{-q} P_q + A(\delta x_0)^2 + \frac{1}{2}(\varphi_0 - \varphi_q)\, \delta x_{-q} \delta x_q \right\} - N \frac{A^2}{4B} . \tag{11}$$

Therefore, the spectrum of small oscillations around minimum of potential energy

$$\omega_q^2 = \frac{1}{M}(\varphi_0 - \varphi_q) , \quad (q \neq 0) \tag{12a}$$

$$\omega_{q=0}^2 = \frac{2A}{M} , \tag{12b}$$

resembles, throughout almost all the Brillouine zone an acoustic-type spectrum, but frequency for q =0 is finite (as it should be in principle, what follows from (5), (8) and (10)). Although, it is obviously not the Goldstone mode, we shall call it the pseudo-Goldstone mode because of two reasons:

Firstly, we want to emphasize a special symmetry of V which, in consequence, leads to formula (12).

Secondly, we expect that the susceptibility function, χ_q exhibits the following behaviour in the ordered phase of the system (1) (after taking the thermodynamical limit)

$$\chi_q^{-1} \sim \omega_q^2 \sim \varphi_0 - \varphi_q , \quad (q \neq 0) . \tag{13a}$$

$$\chi_{q=0}^{-1} \sim \omega_{q=0}^2 \sim 2B\langle Q\rangle^2 , \quad (\chi_{q=0}^{-1}(T_c) = 0) . \tag{13b}$$

However, such a singular behaviour of χ_q at the centre of the Brillouin zone may not be visible in the thermodynamical consideration and instead of formula (13) a formula of the acoustic-type for the susceptibility function

$$\chi_q^{-1} \sim \varphi_0 - \varphi_q \tag{14}$$

may be found. (There is an explicit difference in formula for χ_q , (13) and (14), but there is no difference between the free energies calculated within both treatments as a contribution of each mode is of the order $\frac{1}{N}$, vanishing in the thermodynamical limit - see also the discussion dealing with the region $T < T_c$ in /3/ and /4/).

This last remark needs some explanation. Our above discussion deals with some properties of a mechanical (T=0), finite system. On this basis, we may use a variational method with the trial Hamiltonian

$$H_0 = \frac{1}{2}\sum_q \{ P_{-q} P_q + \Delta_q Q_{-q} Q_q \}, \tag{15}$$

where Δ_q (and $\langle Q \rangle$)are the variational parameters,to justify the second conclusion. However, as the first conclusion is true for a class of systems with the Hamiltonian of type (1a), where V is the function of r

$$r^2 = X^T \circ X \tag{16}$$

only, and the second one is expected to be true at least for some class of exactly soluble models, e.g. /5,6/ and /4/

$$V(r) = N\left\{\frac{A}{2}r^2 + B\exp(-Cr^2)\right\}, \tag{17a}$$

$$V(\{r_\alpha\}) = N\left\{-\frac{A}{2}\sum_{\alpha=1,2} r_\alpha^2 + \frac{B_1}{4}\Big(\sum_\alpha r_\alpha^2\Big)^2 + \frac{B_2}{4}\sum_\alpha (r_\alpha^2)^2\right\}, \tag{17b}$$

we shall discuss in detail the properties of χ_q in the next, more extended paper within another treatment.

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Псевдоголдстоновская мода в точно-решаемой модели фазового перехода

Показано, что фазовый переход в точно-решаемой модели, предложенной Шнейдером, Штоллом и Беком, связан с нарушением дискретной симметрии; таким образом, бесщелевая мода, которая появляется в упорядоченной фазе, не должна рассматриваться как возбуждение голдстоновского типа. В связи с результатом, полученным для T = 0, предполагается, что это возбуждение /названное псевдоголдстоновской модой/ имеет особое поведение при $T < T_c$ в точке q = 0. Такое явление должно быть характерным для целого класса точно решаемых моделей.

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A Pseudo-Goldstone Mode in the Exactly Soluble Model of Phase Transition

It is shown that a phase transition within a model proposed by Schneider, Stoll and Beck is associated with a spontaneously broken discrete symmetry; so, the gapless mode in the ordered phase is not the Goldstone-type excitation. On the basis of the results for T = 0, it is suggested that this excitation (called the pseudo-Goldstone mode) exhibits an unusual behaviour at q = 0 for all $T < T_c$. Such a phenomenon should be common for some class of the exactly soluble models of phase transitions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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