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A GENERALIZED N-LEVEL SINGLE-MODE JAYNES-CUMMINGS MODEL

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I. Introduction

In recent years, much interest has been paid to the Jaynes-Cummings model exactly solvable in the rotating wave approximation (RWA) (J-C) [1] of a two-level atom linearly interacting with a single mode of the quantized-field radiation [2-5]. A distinctive, from a physical point of view, feature of the J-C model is an infinite sequence of quantum collapses and revivals of Rabi oscillations revealed by Eberly et al. [2,3] and Knight and Radmore [4]. There have been considered various possible generalizations of the J-C model. In particular, Buck and Sukumar [6-9] have found an exact solution to the equations of motion for an atomic system with an interaction nonlinear in bosonic variables. Singh [5] has studied photon statistical properties of such nonlinear systems. Exact wave functions and energy levels have been found for various J-C type systems nonlinear both in bosonic and spin variables [10]. There also should be mentioned a recent series of papers by Agarwal and Puri [11-12] devoted to the generalization of the J-C model to include the effect of cavity damping. All of these rather simple models describe, however, the essential physics of radiation - matter interaction. Apart from the above-mentioned vacuum-field Rabi oscillations, these models are capable to describe such very interesting phenomena now intensively studied theoretically and experimentally as photon antibunching [13-14] and squeezing [14-15].

Another form of the generalization of the J-C model deals with adding other levels leading, in particular, to the appearance of new branches of the Rabi frequency comparing to the standard two-level J-C model. Li and Bei [16] have extended the J-C model to the case of a three-level atom interacting with a two-model radiation field. Multiphoton transitions in such a system have been considered, as well [17]. Very recently, photon antibunching [18] and squeezing [19] effects have been revealed in a three-level system. There are two possible ways to generalize the J-C model to N-level systems: considering an N-level atom interacting with a) N-1 modes or b) single mode of the radiation field. Both the cases are exactly soluble due to the existence of conserved "charges". In particular, the mean photon numbers and average values of the atomic level occupations have been studied by means of the operator equations of motion for an N-level atom, the N^{th} level of which is coupled with the rest of lower levels by N-1 modes of the radiation field [20]. There have also been considered multiphoton transitions in such a system[21].

In the present paper, I would like to consider an N-level atom immersed in a lossless cavity and interacting with a single field mode in a nonlinear way. Owing to the charge conservation, the state space of a system can be decomposed into a direct sum of finite-dimensional subspaces corresponding to feasible values of the constant of motion. An eigenstatevector of a system parametrized by possible values of this constant is determined as an expansion over the basis of an appropriate finite-dimensional subspace, with the expansion coefficients being defined from the Schrödinger equation that is reduced on every subspace to a finite system of algebraic equations.

In sec.2 a multiboson variant of the N-level single-mode J-C model is formulated; the structure of state space is thoroughly analysed, exact wave functions and energy levels of a system are found, and completeness of the obtained system of eigenfunctions is discussed. Sec.3 is devoted to the discussion of the antibunching and squeezing effects in the considered model. In sec.4 general expressions for the average atomic level occupations are found.

2. The single-model nonlinear N-level J-C problem

The Hamiltonian for an N-level single-mode system in RWA is

$$\mathcal{H} = \sum_{j=1}^{N} R_{jj}\Omega_j + \omega a^+ a + \sum_{j=1}^{N-1} \lambda_j \left(a^{\nu_j} R_{Nj} + a^{+\nu_j} R_{jN} \right). \qquad (1)$$

The $R_{ij} = |i\rangle\langle j|$ are the transition $(i \neq j)$ -projection $(i=j)$ operators and also the generators of the $SU(N)$ group with the commutation relations:

$$[R_{ij}, R_{kl}] = R_{il}\delta_{jk} - R_{kj}\delta_{il}$$

$|j\rangle$ and Ω_j, $j = 1,2, \ldots, N$ are the eigenstates and eigenvalues of an N-level atom. a^+ and a, λ_j and ω are the creatinn and annihilation operators, the coupling constant and freqiency of the field mode, respectively. Note that

$$[a, a^+] = 1, \quad [a, R_{ij}] = [a^+, R_{ij}] = 0.$$

According to (1) the N^{th} level is coupled with any other j^{th} level by ν_j -photon dipole transitions ($\nu_j \geq 0$ are integer, $1 \leq j \leq N-1$),

whereas the mutual transitions between the remaining N-1 levels are forbidden. Note that (1) at N=2 and ν =1 reduces to a standard J-C problem whose exact solution is known since 1963 [1,22,23]

2.1. The struoture of state space

The state space of system (1) is generated by the basis

$$\{ |\varphi^p_m\rangle = |p\rangle|m\rangle; \quad p \geq 0, \quad 1 \leq m \leq N \},$$

where $a^+ a|p\rangle = p|p\rangle$

$R_{ii}|m\rangle = \delta_{im}|m\rangle$. Conservation of the charge of a system $\hat{M} = a^+ a - \sum \nu_j R_{jj}$, $[\mathcal{H}, \hat{M}] = 0$ limits the range of variation of numbers p and m by the condition $M = p - \nu_m(1 - \delta_{mN})$, where M is an eigenvalue of the operator $\hat{M}$. In this way, the state space can be represented by $\sum \oplus \mathcal{H}_M$. where every subspace $\mathcal{H}_M$ corresponding to a certain value of the charge M is generated by the basis

$$\{ |\varphi^M_m\rangle = |M + \nu_m(1 - \delta_{mN})\rangle |m\rangle; \quad M + \nu_m(1 - \delta_{mN}) \geq 0 \}.$$

The charge eigenvalues take the values

$$M = M_n \equiv n - \bar{\nu}, \qquad n = 0, 1, 2, \ldots$$

Here and in what follows $\max_m \nu_m \equiv \bar{\nu}$.

As a result, the basis of the subspace $\mathcal{H}_M$ may be rewritten in the following form

$$\{ |\varphi^{M=n-\bar{\nu}}_m\rangle = |n - \bar{\nu} + \nu_m(1 - \delta_{mN})\rangle |m\rangle, \quad m \in \mathcal{P}_n \}$$
$$\mathcal{P}_n = \{ m | 1, 2, \ldots, N \} \bigcap \{ m | n - \bar{\nu} + \nu_m(1 - \delta_{mN}) \geq 0, \ n \geq 0 \}. \qquad (2)$$

The state vector $|\psi^M\rangle$ obeying the Schrödinger equation

$$\mathcal{H}|\psi^M\rangle = \mathcal{E}|\psi^M\rangle. \qquad (3)$$

With Hamiltonian (1) and corresponding to the charge eigenvalue M can be represented by the following expansion over the basis (2) of the subspace $\mathcal{H}_M$:

$$|\Psi^{M=n-\bar{\nu}}\rangle \equiv |\Psi_n\rangle = \sum_{m\in\mathcal{P}_n} c_m |\Phi^M_m\rangle . \qquad (4)$$

2.2. Energy levels and eigenfunctions

Inserting (4) into the Schrödinger equation (3) and introducing the detuning parameter [21]

$$\Delta = \Omega_j - \Omega_N + \omega\nu_j , \qquad j = 1,2,\ldots,N-1 \qquad (5)$$

we obtain the following solutions:

For $n \geqslant \bar{\nu}$

$$|\Psi_n^{(\pm)}\rangle = \left(Q_{N-1} + y_\pm^2\right)^{-1/2}\left(\sum_{j=1}^{N-1} g_j |n-\bar{\nu}+\nu_j\rangle|j\rangle + y_\pm |n-\bar{\nu}\rangle|N\rangle\right), \qquad (6)$$

$$\mathcal{E}_n^{(\pm)} = \Omega_N + \omega(n-\bar{\nu}) + \Delta + y_\pm . \qquad (7)$$

where

$$y_\pm = -\frac{\Delta}{2} \pm \sqrt{(\Delta/2)^2 + Q_{N-1}} , \quad Q_{N-1} = \sum_{j=1}^{N-1} g_j^2 ,$$

$$g_j^2 = \lambda_j^2 (n-\bar{\nu}+\nu_j)! \,/\, (n-\bar{\nu})!$$

Besides, for $n \geqslant \bar{\nu}$ N-2 eigenfunctions

$$|\Psi_n^{(k)}\rangle = \mathcal{N}_k^{-1}\left(-\sum_{i=1}^{k-1} g_i |n-\bar{\nu}+\nu_i\rangle|i\rangle + \mathcal{M}_k |n-\bar{\nu}+\nu_k\rangle|k\rangle\right) \qquad (8)$$

$$2 \leq k \leq N-1$$

correspond to eigenvalue

$$\mathcal{E}_n = \Omega_1 + \omega(n-\bar{\nu}+\nu_1) = \ldots = \Omega_{N-1} + \omega(n-\bar{\nu}+\nu_{N-1}) , \qquad (9)$$

where

$$\mathcal{N}_k^2 = \frac{Q_{k-1}Q_k}{g_k^2} , \qquad \mathcal{M}_k = Q_{k-1}/g_k .$$

It should be mentioned that the (N-2)-fold degeneration of the level $\mathcal{E}_n$ is due to (N-2)- supplementary conditions (5) imposed on the system.

For $0 \leqslant n < \bar{\nu}$ the eigenfunctions

$$|\Psi_n^{(m)}\rangle = |n-\bar{\nu}+\nu_m\rangle|m\rangle , \qquad (10)$$

where m runs over the set $\{1,2,\ldots,N-1\} \cap \{m \mid n-\bar{\nu}+\nu_m \geqslant 0\}$ correspond to the eigenvalue

$$\mathcal{E}_n = \omega(n-\bar{\nu}+\nu_m) + \Omega_m . \qquad (11)$$

Formulae (6)-(11) at N=2, ν =1 go over into well-known results of the standard two-level J-C model [1,22,23], whereas at $\nu \geqslant 0$ we arrive at the results of paper [10]. At N=3, $\nu_1 = \nu_2 = 1$ formulae (6)-(11) reduce to author's previous results [24].

2.3. Completeness of the system of eigenfunctions

Let us verify the completeness of the orthonormalized system of functions (6), (8),(10). Composing of these functions the operator $\sum |\Psi_i\rangle\langle\Psi_i|$ we get the $N \times N$ matrix

$$\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1N} \\ a_{21} & a_{22} & \ldots & a_{2N} \\ & & & \\ a_{N1} & a_{N2} & \ldots & a_{NN} \end{pmatrix} ,$$

where, for instance, $a_{NN} = \sum_0^\infty |n\rangle\langle n| \left\{ y_+^2/(Q_{N-1}+y_+^2) + y_-^2/(Q_{N-1}+y_-^2)\right\}$.

Due to the relation $y_\pm = -\frac{\Delta}{2} \pm \sqrt{(\Delta/2)^2 + Q_{N-1}}$ the expression in the braces is equal to unity and, consequently, $a_{NN} = 1$. One can easily verify also that $a_{Nj} = a_{jN} = 0$, $j = 1,2,\ldots,N-1$

Further, we have $a_{kk} = G_k^{(1)} + G_k^{(2)}$, $1 \leq k \leq N-1$, where the operator $G_k^{(2)} = \sum_1^{\nu_k} |\nu_k - i\rangle\langle\nu_k - i| = \sum_0^{\nu_k-1} |n\rangle\langle n|$ is constructed from the functions belonging to system (10); and $G_k^{(1)}$ from the appropriate functions belonging to systems (6) and (8):

$$G_K^{(1)} = \sum_{0}^{\infty} |n+\nu_K\rangle\langle n+\nu_K| \left\{ \frac{g_K^2}{Q_{N-1}} + \frac{Q_{K-1}}{Q_K} + g_K^2 \sum_{i=K+1}^{N-1} \frac{1}{\mathcal{N}_i^2} \right\} =$$

$$\sum_{n\geqslant \nu_K} |n\rangle\langle n| \left\{ \frac{g_K^2}{Q_{N-1}} + \frac{Q_{K-1}}{Q_K} + g_K^2 \sum_{i=K+1}^{N-1} \frac{g_i^2}{Q_{i-1}Q_i} \right\}$$

Taking into account the relationship

$$\frac{1}{Q_{N-1}} + \sum_{i=K+1}^{N-1} \frac{g_i^2}{Q_{i-1}Q_i} = \frac{1}{Q_K}, \qquad 1 \leqslant K \leqslant N-1$$

that can be easily proved by induction, we obtain

$$G_K^{(1)} = \sum_{n\geqslant \nu_K} |n\rangle\langle n| \left\{ \frac{Q_{K-1}}{Q_K} + \frac{g_K^2}{Q_K} \right\} = \sum_{n\geqslant \nu_K} |n\rangle\langle n|$$

and finally

$$a_{KK} = \sum_{n<\nu_K} |n\rangle\langle n| + \sum_{n\geqslant \nu_K} |n\rangle\langle n| = 1.$$

Analogously, it is easy to see that

$$a_{Ki} = 0, \quad k \neq i, \quad k, i = 1, 2, \ldots, N-1.$$

3. Antibunching and squeezing effects

Here we apply the above results to discuss the photon antibunching and squeezing effects in the considered model. Both the photon antibunching and squeezing are aspects of the quantum nature of light; the former being a particle effect and the latter, a wave one. These unique properties are related to the decrease of quantum fluctuations in the photon number and phase below those of coherent light. The photon antibunching is characterised by a nonolassical state of the field in which the variance of the number of photons is less than the mean number of photons, i.e. the photons exhibit sub-Poissonian statistics.

Let us introduce the normally ordered variance $V(t)$ of the

photon number in an arbitrary state:

$$V(t) = \langle n^2(t)\rangle - \langle n(t)\rangle - \langle n(t)\rangle^2. \tag{12}$$

where

$$n(t) = a^{\dagger}a(t) = e^{i\mathcal{H}t}\, a^{\dagger}(0)\, a(0)\, e^{-i\mathcal{H}t}.$$

The quantity (12) is proportional to the excess of coincidences in counting rates measured in the Hanburry Brown and Twiss-type experiment /25,26/ . The sign (+) or (-) of V(t) shows that the photon statistics of the field is super- or sub-Poissonian and indicates whether the photon bunching or antibunching occurs /27-28/, respectively. Usually, to examine the number and phase-dependent field fluctuations in the J-C - type models, the radiation field is initially assumed to be in a coherent state interacting with an initially excited atom. Very recently, Knight /15/ has considered the two-level J-C model for an atom interacting at t=0 not with a coherent state but with a vacuum field, provided the atom is excited to a coherent superposition of upper and lower states. To generalize Knight's prodedure to an N-level J-C model, we assume

$$|\Psi(t=0)\rangle \equiv |\Phi\rangle = \sum_{j=1}^{N} C_j |j\rangle|0\rangle, \tag{13}$$

where $\sum |C_j|^2 = 1$, $|0\rangle$ is a field-vacuum state and $|j\rangle$ is one of the pure atomic states.

Let us put $\Delta = 0$ for simplicity and assume also that all $\nu_j > 0$ j= 1,2, ..., N-1. Then using the complete set of functions (6),(8), (10) one can easily obtain the state vector at $t \geqslant 0$

$$|\Psi(t)\rangle = e^{-i\mathcal{H}t}|\Phi\rangle = \sum_{n\geqslant \bar{\nu}} |\Psi_n^{(+)}\rangle\langle\Psi_n^{(+)}|\Phi\rangle e^{it\mathcal{E}_n^{(+)}}$$

$$+ \sum_{n\geqslant \bar{\nu}} |\Psi_n^{(-)}\rangle\langle\Psi_n^{(-)}|\Phi\rangle e^{-it\mathcal{E}_n^{(-)}} + \sum_{m=1}^{N-1}\sum_{n=0}^{\bar{\nu}-1} |\Psi_n^{(m)}\rangle\langle\Psi_n^{(m)}|\Phi\rangle e^{-it\mathcal{E}_n^{(m)}}$$

$$= \frac{C_N e^{-it\Omega_N}}{\sqrt{S_{N-1}}} \left\{ (-i)\sin t\sqrt{S_{N-1}} \sum_{j=1}^{N-1} \lambda_j [(\nu_j)!]^{1/2} |\nu_j\rangle\langle\nu_j| + \right. \tag{14}$$

$$\left. \sqrt{S_{N-1}}\cos t\sqrt{S_{N-1}}\, |0\rangle|N \right\} + \sum_{m=1}^{N-1} C_m e^{-it\Omega_m} |0\rangle|m\rangle.$$

where S_{N-1} stands for $Q_{N-1}\big|_{n=\nu} = \sum_{j=1}^{N-1} \lambda_j^2 (\nu_j)!$

The average value of a time-dependent function $f_t(a^+a) = e^{i\mathcal{H}t} f\, e^{i\mathcal{H}t}$ is then

$$\langle \varphi | f_t(a^+a) | \varphi \rangle = \langle \Psi(t) | f(a^+a) | \Psi(t) \rangle$$

$$= \frac{|C_N|^2}{S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_{i=1}^{N-1} \lambda_i^2\, \nu_i!\, f(\nu_i), \tag{15}$$

where it is assumed that $f(x=0)=0$. With the help of (15) we have

$$V(t) = \frac{|C_N|^2}{S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_{i=1}^{N-1} \lambda_i^2\, \nu_i!\, \nu_i^2 - \frac{|C_N|^2}{S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \tag{16}$$

$$\times \sum_{i=1}^{N-1} \lambda_i^2\, \nu_i!\, \nu_i - \left[\frac{|C_N|^2}{S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_{i=1}^{N-1} \lambda_i\, \nu_i!\, \nu_i \right]^2$$

For $\nu_i = \nu > 0$, $i = 1,2,\dots, N-1$, $S_{N-1} = \nu! \sum \lambda_i^2$ and formula (16) goes over into

$$V(t) = \nu |C_N|^2 \sin^2 t\sqrt{S_{N-1}} \left\{ \nu - 1 - \nu |C_N|^2 \sin^2 t\sqrt{S_{N-1}} \right\}. \tag{17}$$

This implies that the light field generated from the initial vacuum by an atomic-field interaction has sub-Poissonian photon statistics, and, hence exhibits the photon antibunching for all times t satisfying the condition

$$|C_N|^2 \sin^2 t\sqrt{S_{N-1}} > \frac{\nu - 1}{\nu}. \tag{18}$$

It should be stressed that formula (17) reveals an interesting ν-dependence of antibunching phenomena. Namely, for any $\nu > 0$ there exists a sequence of such time intervals, that on each of them antibunching occurs. As ν increases, i.e. the total number of emitted (absorbed) photons per an atom grows, the length of each of these intervals decreases, and the effect asymptotically disappears . . There is also a simple N-dependence of antibunching through the Rabi frequency $\sqrt{S_{N-1}}$:

Another nonclassical effect , the so-called squeezing effect, has recently become a subject of intense studies. As for antibunching, the light field in a squeezed state has no classical counterpart. Strictly speaking, squeezing is characterized by a field state in which the variance of one of two noncommuting observables is less than half the absolute value of their commutator [15,29-33]). We define the Hermitian field operators a_1 and a_2 through [15]

$$a_1 = \tfrac{1}{2}(a + a^+) \qquad \text{and} \qquad a_2 = \tfrac{1}{2i}(a - a^+)$$

so that $[a_1, a_2^+] = i/2$ and the variances $(\Delta a_i)^2 = \langle a_i^2 \rangle - \langle a_i \rangle^2$ $i = 1,2$ satisfy the incertainty relation

$$(\Delta a_1)^2 (\Delta a_2)^2 \geqslant 1/16 .$$

The field is squeezed if $(\Delta a_i)^2 < 1/4$ for either $i = 1$ or 2. With the aid of the state-vector (14) it is easy to obtain

$$\langle a^p(t) \rangle = \langle \Psi(t) | a^p(0) | \Psi(t) \rangle$$

$$= -\frac{\sin t\sqrt{S_{N-1}}}{\sqrt{S_{N-1}}} \sum_{i=1}^{N-1} C_N C_i^*\, p!\, e^{-itp\omega} \lambda_i \delta_{\nu_i, p}, \quad p > 0 \tag{19}$$

provided that as before $\Delta = 0$ and all $\nu_j > 0$. Relation (19) leads to

$$\langle a_1(t) \rangle = \frac{i \sin t\sqrt{S_{N-1}}}{2\sqrt{S_{N-1}}} \sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i, 1} \left(C_N^* C_i e^{it\omega} - C_N C_i^* e^{-it\omega} \right)$$

$$\langle a_2(t) \rangle = -\frac{\sin t\sqrt{S_{N-1}}}{2\sqrt{S_{N-1}}} \sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i, 1} \left(C_N^* C_i e^{it\omega} + C_N C_i^* e^{-it\omega} \right).$$

Taking into account the relations

$$\langle a_1^2 \rangle = \tfrac{1}{4} + \tfrac{1}{2}\langle a^+a \rangle + \tfrac{1}{4}\langle a^2 \rangle + \tfrac{1}{4}\langle a^{+2} \rangle$$

$$\langle a_2^2 \rangle = \tfrac{1}{4} + \tfrac{1}{2}\langle a^+a \rangle - \tfrac{1}{4}\langle a^2 \rangle - \tfrac{1}{4}\langle a^{+2} \rangle$$

$$\langle a^+a \rangle = \frac{|C_N|^2}{S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_{1}^{N-1} \lambda_i^2 (\nu_i)!\, \nu_i$$

we have for the variances

$$
(\Delta a_1)^2 = \frac{1}{4} + \frac{|C_N|^2}{2S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_1^{N-1} \lambda_i^2 \nu_i!\, \nu_i + \frac{i \sin t\sqrt{S_{N-1}}}{2\sqrt{S_{N-1}}} \sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i,2} \times
$$

$$
\left(c_N^* c_i e^{2it\omega} - c_N c_i^* e^{-2it\omega} \right) + \frac{\sin^2 t\sqrt{S_{N-1}}}{4S_{N-1}} \times \tag{20}
$$

$$
\left[\sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i,1} \left(c_N^* c_i e^{it\omega} - c_N c_i^* e^{-it\omega} \right) \right]^2 ,
$$

$$
(\Delta a_2)^2 = \frac{1}{4} + \frac{|C_N|^2}{2S_{N-1}} \sin^2 t\sqrt{S_{N-1}} \sum_1^{N-1} \lambda_i^2 \nu_i!\, \nu_i
$$

$$
- \frac{i \sin t\sqrt{S_{N-1}}}{2\sqrt{S_{N-1}}} \sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i,2} \left(c_N^* c_i e^{2it\omega} - c_N c_i^* e^{-2it\omega} \right)
$$

$$
- \frac{\sin^2 t\sqrt{S_{N-1}}}{4S_{N-1}} \left[\sum_{i=1}^{N-1} \lambda_i \delta_{\nu_i,1} \left(c_N^* c_i e^{it\omega} + c_N c_i^* e^{-it\omega} \right) \right]^2 . \tag{21}
$$

As a consequence of relations (20)-(21) there is no squeezing if all $\nu_j \geq 3$. The squeezing might occur provided at least one of ν_j belongs to the set $\{1,2\}$.

Let us put for instance

$$
\nu_j = 1, \quad j = 1, 2, \ldots, N-1, \quad \lambda_1 = \lambda_2 = \ldots = \lambda_{N-1} \equiv \lambda .
$$

then (20), (21) go over into

$$
(\Delta a_1)^2 = \frac{1}{4} + \frac{|C_N|^2}{2} \sin^2 t\lambda\sqrt{N-1} + \frac{\sin^2 t\lambda\sqrt{N-1}}{4(N-1)} \left(\sum_{i=1}^{N-1} \left(c_i c_N^* e^{it\omega} - c_i^* c_N e^{-it\omega} \right) \right)^2 , \tag{22}
$$

$$
(\Delta a_2)^2 = \frac{1}{4} + \frac{|C_N|^2}{2} \sin^2 t\lambda\sqrt{N-1} - \frac{\sin^2 t\lambda\sqrt{N-1}}{4(N-1)} \left[\sum_i \left(c_N c_i^* e^{-it\omega} + c_N^* c_i e^{it\omega} \right) \right]^2 . \tag{23}
$$

Not that for N=2 and $C_2 = \cos\theta/2$, $c_1 = e^{i\varphi} \sin\theta/2$ formulae (22),(23) reduce to the appropriate results of paper /15/.

For

$$
c_N = r, \quad 0 \leq r \leq 1, \quad c_1 = c_2 = \ldots = c_{N-1} = \left(\frac{1-r^2}{N-1} \right)^{1/2} \tag{24}
$$

$$
(\Delta a_1)^2 = \frac{1}{4} + r^2 \sin^2 \lambda t\sqrt{N-1} \left\{ \frac{1}{2} - (1-r^2) \sin^2 \omega t \right\} \tag{25}
$$

and $(\Delta a_1)^2$ is squeezed for all times satisfying the relation

$$
\sin^2 \omega t > \frac{1}{2(1-r^2)} , \quad r \sin \lambda t\sqrt{N-1} \neq 0 .
$$

In the case

$$
\nu_j = 2, \quad j = 1, 2, \ldots, N-1, \quad \lambda_1 = \lambda_2 = \ldots = \lambda_{N-1} \equiv \lambda .
$$

taking into account (24) it is easy to obtain

$$
(\Delta a_1)^2 = \frac{1}{4} + r^2 \sin^2 \lambda t\sqrt{2(N-1)} - \frac{\sin \lambda t\sqrt{2(N-1)}}{\sqrt{2}} r (1-r^2)^{1/2} \sin 2\omega t \tag{26}
$$

and $(\Delta a_1)^2$ is squeezed for all times satisfying the relation

$$
\sin 2\omega t \gtrless \left(\frac{2r^2}{1-r^2} \right)^{1/2} \cdot \sin \lambda t\sqrt{2(N-1)} , \quad r \sin \lambda t\sqrt{2(N-1)} \gtrless 0 .
$$

Squeezing considered above is due to the initially excited atomic coherent state /15/. There is a number of papers dealing with that type of squeezing /34-35/, including the models where many atoms coherently interact with the field /36-38/. It is interesting to note that like in the antibunching formula (17), the only N-dependence of squeezing in (25) and (26) is through the Rabi frequencies. That seems to be a general feature of single-mode J-C type models.

4. The average atomic level occupations

Here we apply the results of sec.2 to find the N-dependence of the mean atomic level occupations for system (1). To simplify the calculations, we put

$$\lambda_1 = \lambda_2 = \dots = \lambda_{N-1} = \lambda\ , \quad \nu_1 = \nu_2 = \dots = \nu_{N-1} = 1\ , \quad \Delta = 0. \tag{27}$$

The wave functions and energy levels (6)-(11) go over into

for $n \geqslant 1$

$$|\psi_n^{(\pm)}\rangle = \frac{1}{\sqrt{2(N-1)}}\left(\sum_{j=1}^{N-1} |n\rangle|j\rangle \pm \sqrt{N-1}\,|n-1\rangle|N\rangle\right) \tag{28}$$

$$\mathcal{E}_n^{(\pm)} = \Omega + \omega n \pm \lambda\sqrt{n(N-1)} \tag{29}$$

$$|\psi_n^{(k)}\rangle = \left[k(k-1)\right]^{-1/2}\left(-\sum_{j=1}^{k-1} |n\rangle|j\rangle + (k-1)|n\rangle|k\rangle\right) \tag{30}$$

$$\mathcal{E}_n^{(k)} = \Omega + \omega n\ , \qquad 2 \leqslant k \leqslant N-1 \tag{31}$$

for $n = 0$

$$|\psi_{n=0}^{(j)}\rangle = |0\rangle|j\rangle\ , \qquad 1 \leqslant j \leqslant N-1 \tag{32}$$

$$\mathcal{E}_{n=0}^{(i)} = \Omega\ , \qquad \Omega = \Omega_N - \omega. \tag{33}$$

Let us assume that at an initial moment $t = 0$ the atom is on a level j ($j=1,2, \dots N$) and the field is in a coherent state $|z\rangle$

$$|\Psi(t=0)\rangle \equiv |\varphi_j\rangle = |j\rangle|z\rangle. \tag{34}$$

The expectation value of the j^{th} level occupation operator R_{jj} in the state (34) at a time $t \geqslant 0$ is then

$$\langle\varphi_j|R_{jj}(t)|\varphi_j\rangle = \sum_{\alpha,\beta}\langle\varphi_j|\psi_\alpha\rangle\langle\psi_\alpha|R_{jj}(0)|\psi_\beta\rangle\langle\psi_\beta|\varphi_j\rangle e^{it(\mathcal{E}_\alpha - \mathcal{E}_\beta)},$$

where $\{\psi_\alpha\}$, $\mathcal{E}_\alpha$ is a complete set of orthonormalized wave functions (28), (30), (32) and energy levels (29), (30), (33). As a result, the average atomic level occupations are found to be

$$\langle\varphi_j|R_{jj}(t)|\varphi_j\rangle = |\langle z|0\rangle|^2 + \left(\frac{N-2}{N-1}\right)^2\sum_1^\infty |\langle z|n\rangle|^2 + \frac{2(N-2)}{(N-1)^2}\sum_1^\infty |\langle z|n\rangle|^2 \cos\lambda t\sqrt{n(N-1)} + \frac{1}{2(N-1)^2}\sum_1^\infty |\langle z|n\rangle|^2\left(\cos 2\lambda t\sqrt{n(N-1)} + 1\right),\ 1 \leqslant j \leqslant N-1 \tag{35}$$

$$\langle\varphi_N|R_{NN}(t)|\varphi_N\rangle = \frac{1}{2}\sum_1^\infty |\langle z|n-1\rangle|^2\left(1 + \cos 2\lambda t\sqrt{n(N-1)}\right). \tag{36}$$

It should be pointed out that $\langle\varphi_j|R_{jj}(t)|\varphi_j\rangle$ does not depend on j due to the relations (27). From (35), (36) we conclude that the Rabi frequencies of the vacuum-field oscillations grow with N as $\sqrt{N}$, whereas the oorresponding amplitudies, in general, diminish. For $N=2$ (35) and (36) reduce to the results of paper /6/ .

5. Summary

Thus, we have constructed the complete set of wave functions such energy levels of the nonlinear (ν -dependent) N-level single--mode J-C model and then used the obtained spectrum to investigate the antibunching and squeezing effects. There have been revealed the ν - and N-dependences of parameters characterizing the strength of these effects. It would be interesting to compare the obtained results with those of the multimode N-level J-C model to clear up in detail the ν, N -behaviour of the physical parameters now experimentally observed.

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Кочетов Е.А. E17-87-574

Обобщенная N-уровневая одномодовая модель Джейнса - Каммингса

Исследуется квантовостатистическая модель N-уровневого атома, взаимодействующего с единственной модой поля излучения. Получена полная система собственных функций и уровней энергии. Обсуждаются применения этой модели к исследованию эффектов сжатия света и разгруппировки фотонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kochetov E.A. E17-87-574

A Generalized N-Level Single-Mode Jaynes-Cummings Model

A quantum statistical model for the interaction of an N-level atom with a single mode radiation field is presented. A complete system of eigenfunctions and eigenvalues is found. Application of this model to the investigation of photon antibuncing and squeezing effects is also discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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