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COLLECTIVE RESONANCE FLUORESCENCE IN AN INTENSE LASER FIELD WITH PHASE AND AMPLITUDE FLUCTUATIONS

1987

I. Introduction

During the last years a large number of works has been concentrated on the problems of collective interactions of atoms with a laser field and with the vacuum of radiation such as superfluorescence (see [1] and refs. therein), collective resonance fluorescence [2-8], collective double optical resonance[9], collective Raman scattering [10-14], etc.

In the one-atom case the effects due the fluctuating driving fields have been discussed for resonance fluorescence [12-19], double optical resonance [29] and Raman scattering [20-23]. Puri and Hassan [8] have studied the effects due to fluctuating driving laser field in collective spectral and statistical properties of resonance fluorescence. Contrary to the paper [8] where only the exact resonance has been considered, in this paper we discuss the off-resonance case for the collecti-ve resonance fluorescence in an intense fluctuating laser driving field and investigate the spectral and statistical properties due to the phase and amplitude fluctuations of the driving field.

II. The master equation

We consider a system of N two-level atoms concentrated in a region small compared to the wave-length of all the relevant radiation modes (Dicke model) interacting with a mode of mono-chromatic driving field of frequency ω_L and with the vacuum of other modes (fig.1). In treating the external field classically and using the Markov and rotating wave approximation when describing the coupling of the system with the vacuum field, one
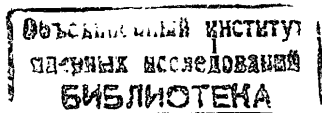
can find a master equation for the reduced density matrix ρ
for the atomic system alone in the form [6,8,26]

$$\frac{\partial \rho}{\partial t} = -i d \left[E^*(t) J_{21} + E(t) J_{12} , \rho \right]$$
$$- \frac{i}{2} \delta \left[J_{22} - J_{11} , \rho \right] - \qquad (1)$$
$$- \gamma_{21} \left(J_{21} J_{12} \rho - 2 J_{12} \rho J_{21} + \rho J_{21} J_{12} \right) ,$$

with $2\gamma_{21}$ being the transition rate from the level $|2\rangle$ to $|1\rangle$
due to the atomic interaction with the reservoir; $\delta = \omega_{21} - \omega_L$
is the detuning of laser frequency from the atomic resonance
frequency ω_{21} ; d is the dipole matrix element ; $E(t)$
is the driving laser field; $J_{k\ell}$ $(k,\ell = 1,2)$ are the collective
operators (angular momenta) describing the atomic system and
having in the Schwinger representation [27,7] the following form:

$$J_{k\ell} = c_k^+ c_\ell \qquad (k,\ell = 1,2) ,$$

where the operators c_k and c_k^+ obey the boson commutation
relations

$$\left[c_k , c_\ell^+ \right] = \delta_{k\ell}$$

and can be treated as annihilation and creation operators for .
atoms populating the level $|k\rangle$.

In the following, the laser field is assumed to have the
form

$$E(t) = \left(E_0 + E_1(t) \right) e^{-i\phi(t)} , \quad \phi_0 = \phi(0) , \qquad (2)$$

where E_0 and ϕ_0 are the nonstochastic parts of the
field amplitude and phase while $E_1(t)$, $\phi(t)$ are the
stochastic variables.

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Fig. 1. Schematic diagram of the two-level
atoms, interacting with the laser driving
field and with a vacuum of radiation.
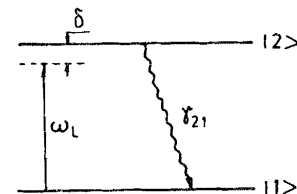
Following Puri et al. [6,8] we discuss the case in which the
phase fluctuations are described by the "phase diffusion model"
[24] and the amplitude fluctuations are described by the nonwhite
Gaussian process, i.e. the following relations are satisfied:

$$\frac{d\phi(t)}{dt} = \mu(t) , \qquad (3)$$

where $\mu(t)$ is Gaussian white noise with

$$\overline{\mu(t)} = 0 , \quad \overline{\mu(t)\mu(t')} = 2\gamma_c \delta(t-t') , \qquad (4)$$

and

$$\overline{\epsilon_1(t)} = 0 , \quad \overline{\epsilon_1(t)\epsilon_1(t')} = (\Delta\epsilon_1)^2 e^{-\gamma_a(t-t')} , \qquad (5)$$

$$\overline{\epsilon_1(t)\mu(t')} = 0 , \qquad (6)$$

where $\epsilon_1(t) = d E_1(t)$; the quantities γ_c and γ_a
describe the band-widths due to the frequency and amplitude
fluctuations of the laser field, and $(\Delta\epsilon_1)^2$ is a measure of the
fluctuations of the Rabi frequency.

After Puri et al. [6,8], for the later use, we introduce the
transformation

$$W_m(t) = e^{-im\phi(t)} e^{-i\frac{\phi(t)}{2} J_3} \rho \, e^{i\frac{\phi(t)}{2} J_3} , \qquad (7)$$

where $J_3 = J_{22} - J_{11} .$

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It is easy to show that an equation for $W_m(t)$ has the form

$$\frac{d\,W_m(t)}{dt} = \left[L_0 - i(m + L_1)\,\dot\phi(t) - i\epsilon_1(t)L_2 \right] W_m(t),\tag{8}$$

where

$$L_0 W_m = -i\epsilon_0 \left[J_{12} + J_{21}, W_m \right] - i\frac{\delta}{2}\left[J_3, W_m \right] -$$
$$- \gamma_{21}\left(J_{21}J_{12}W_m - 2J_{12}W_m J_{21} + W_m J_{21}J_{12} \right),$$

$$L_1 W_m = \frac{1}{2}\left[J_3, W_m \right],$$

$$L_2 W_m = \left[J_{21} + J_{12}, W_m \right],$$

with $\epsilon_0 = d E_0$ being the resonance Rabi frequency. Our next step is to obtain the master equation for $\overline{W_m}(t)$, the transformed density operator averaged over the ensemble with respect to the distribution of the phases. After Van Kampen [25], using the theory of multiplicative stochastic processes, one can find

$$\frac{d\,\overline{W_m}(t)}{dt} = \left[L_0 - \gamma_c(m+L_1)^2 - i\epsilon_1(t)L_2 \right] \overline{W_m}(t)$$
$$= L\,\overline{W_m}(t).\tag{9}$$

Equation (9) has been investigated in the work by Purri and Lawande [6] where the exact steady-state operator $\overline{W_m}^{(s)}$ is given for the case without the amplitude fluctuations, i.e. for the case with $\epsilon_1(t) = 0$.

Further, we consider only the case of an intense external field or of a large detuning δ, so that the Rabi frequency Ω satisfies the following relation:

$$\Omega = \left(\frac{1}{4}\delta^2 + \epsilon_0^2 \right)^{1/2} \gg N\gamma_{21},\ \gamma_a,\ \gamma_c.\tag{10}$$

After the canonical transformation

$$C_1 = Q_1 \cos G + Q_2 \sin G,\tag{11}$$
$$C_2 = -Q_1 \sin G + Q_2 \cos G,$$

where $tg\,2G = 2\epsilon_0/\delta$ and after performing the secular approximation [3,7], i.e. ignoring the part of the operator L in eq. (9) containing rapidly oscillating terms with frequencies $n\Omega$ (n=2,4), eq.(9) reduces to the following form:

$$\frac{d}{dt}\overline{W}_m'(t) = \left[\mathcal{L}_0 + \mathcal{L}_1 - Z_3\mathcal{L}_2 - 2i\epsilon_1(t)\sin G \cdot \cos G \cdot \right.$$
$$\left. \cdot \mathcal{L}_3 \right] \overline{W}_m'(t),\tag{12}$$

where $\overline{W}_m' = U\,\overline{W}_m\,U^+$ with U being the unitary operator representing the canonical transformation (11),

$$\mathcal{L}_0 \overline{W}_m'(t) = -\gamma_c m^2\,\overline{W}_m'(t) - i\Omega\left[D_3, \overline{W}_m'(t) \right] -$$
$$- \gamma_c m(\cos^2 G - \sin^2 G)\left[D_3, \overline{W}_m'(t) \right],\tag{13}$$

$$\mathcal{L}_1 \overline{W}_m'(t) = -Z_1\left(R_{21}R_{12}\overline{W}_m'(t) - 2R_{12}\overline{W}_m'(t)R_{21} + \overline{W}_m'(t)R_{21}R_{12} \right) -$$
$$- Z_2\left(R_{12}R_{21}\overline{W}_m'(t) - 2R_{21}\overline{W}_m'(t)R_{12} + \overline{W}_m'(t)R_{12}R_{21} \right),\tag{14}$$

$$\mathcal{L}_2 \overline{W}_m'(t) = D_3^2\,\overline{W}_m'(t) - 2D_3\overline{W}_m'(t)D_3 + \overline{W}_m'(t)D_3^2,\tag{15}$$

$$\mathcal{L}_3 \overline{W}_m'(t) = \left[D_3, \overline{W}_m'(t) \right],\tag{16}$$

where $D_3 = R_{22} - R_{11}$ with $R_{ij} = Q_i^+ Q_j$ ($i,j = 1,2$) are the collective angular momenta of "dressed" atoms ;

$$Z_1 = \gamma_{21} \cos^4 G + \gamma_c \sin^2 G \cdot \cos^2 G \quad ,$$

$$Z_2 = \gamma_{21} \sin^4 G + \gamma_c \sin^2 G \cdot \cos^2 G \quad ,$$

$$Z_3 = \gamma_{21} \sin^2 G \cos^2 G + \frac{\gamma_c}{4} (\cos^2 G - \sin^2 G)^2 .$$

Now we derive an equation for $\overline{\overline{W}}{}'_m(t)$ (i.e. $\overline{\overline{W}}{}'_m(t)$ averaged over the distribution of the amplitude fluctuations). Again, we use the theory of multiplicative stochastic processes [25] and arrive at the equation for $\overline{\overline{W}}{}'_m(t)$ in the form

$$\frac{d}{dt} \overline{\overline{W}}{}'_m(t) = \left[\mathcal{L}_0 + \mathcal{L}_1 - (X_0 + X_1(t)) \mathcal{L}_2 \right] \overline{\overline{W}}{}'_m(t) \quad , \qquad (17)$$

where

$$X_0 = \gamma_{21} \sin^2 G \cdot \cos^2 G + \frac{\gamma_c}{4} (\cos^2 G - \sin^2 G)^2$$
$$+ 4 \frac{(\Delta \epsilon_1)^2}{\gamma_a} \cdot \sin^2 G \cdot \cos^2 G \quad ,$$

$$X_1(t) = -4 \frac{(\Delta \epsilon_1)^2}{\gamma_a} \sin^2 G \cdot \cos^2 G \cdot e^{-\gamma_a t} \quad ,$$

and the Liouville operators $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2$ have the same forms as given in (13-15). For the case of exact resonance, i.e. $ctg^2 G$ equation (17) reduces to that derived by Puri and Hassan [8].

The stationary solution of equation (17) takes the form

$$\overline{\overline{W}}{}''_m{}^{(s)} = \begin{cases} 0 & \text{for } m \neq 0 \ , \\ A^{-1} \sum_{M=0}^{N} Z^M \, |M\rangle\langle M| & \text{for } m = 0 \ , \end{cases}$$

$$(18)$$

where

$$Z = \frac{Z_1}{Z_2} = \frac{\gamma_{21} \, ctg^4 G + \gamma_c \, ctg^2 G}{\gamma_{21} + \gamma_c \, ctg^2 G} \quad ,$$

$$A = \frac{Z^{N+1} - 1}{Z - 1} \quad ,$$

$|M\rangle$ is an eigenstate of the operators R_{11} and $\hat{N} = R_{11} + R_{22}$ where $R_{ij} = Q_i^+ Q_j$ ($i, j = 1, 2$).. The operators Q_i and Q_i^+ satisfy the boson commutation relations

$$[Q_i , Q_j^+] = \delta_{ij} \quad , \qquad (19)$$

so that

$$[R_{ij} , R_{i'j'}] = R_{ij'} \, \delta_{i'j} - R_{i'j} \, \delta_{ij'} . \qquad (20)$$

For the later use, we introduce the characteristic function

$$\chi_{R_{11}}(\zeta) = Tr \left\{ e^{i\zeta R_{11}} \cdot \overline{\overline{W}}_0^{(s)} \right\} = \langle e^{i\zeta R_{11}} \rangle_s$$

$$= A^{-1} \frac{Y^{N+1} - 1}{Y - 1} \quad ,$$

where $Y = Z \cdot e^{i\zeta}$.

All statistical moments of "dressed" atoms of the form $\langle R_{11}^q \rangle_s$ can be found from the characteristic function $\chi_{R_{11}}(\zeta)$ and have the following form:

$$\langle R_{11}^q \rangle_s = \frac{\partial^q}{\partial (i\zeta)^q} \chi_{R_{11}}(\zeta) \Big|_{i\zeta = 0} \quad . \qquad (21)$$

The expectation values of the atomic observables averaged over the distributions of the phase and amplitude are given by

$$\langle \overline{\overline{J_{21}^{\,n+m} J_3^{\,p} J_{12}^{\,n}}} \rangle = Tr \left\{ \overline{\overline{J_{21}^{\,n+m} J_3^{\,p} J_{12}^{\,n}}} \, \mathcal{S}(t) \right\}$$

$$= Tr \left\{ \left(J_{21}^{\,n+m} J_3^{\,p} J_{12}^{\,n} \right)' \overline{\overline{W}}_m'(t) \right\} \quad , \qquad (22)$$

where, as denoted above, $A' = U A U^{+}$ with U being the unitary operator representing the canonical transformation (11).

It is easy to show from the stationary solution (18) that

$$\langle \overline{\overline{J_{21}^{\,n+m} J_3^{\,p} J_{12}^{\,n}}} \rangle_{ss} =$$

$$= \begin{cases} Tr \left\{ \left(J_{21}^{\,n} J_3^{\,p} J_{12}^{\,n} \right)' \overline{\overline{W}}_0'(s) \right\} & \text{if} \qquad m = o \;, \\ o & \text{if} \qquad m \neq o \;. \end{cases} \qquad (23)$$

By using the canonical transformation (18) one can write the expectation value (23) via the statistical moments $\langle R_{11}^{\,q} \rangle_s$ given in the relation (21).

As it is seen from (18) and (23), in the secular approximation, only the phase fluctuations (but not the amplitude fluctuations) of the laser driving field affect the stationary atomic density matrix and consequently atomic observables and their correlation functions (23). In the case of exact resonance ($ctg^2 G = 1$) we have $z = 1$ for all values γ_c and the phase fluctuations also do not influence the atomic observables and their correlation functions of the form (23). The influence of the phase fluctuations on the steady-state intensities and photon statistics of the spectral components will be considered in the following sections.

III. Steady-state fluorescence spectrum

In this section we consider the effects that may arise in the collective steady-state fluorescence spectrum due to the phase and amplitude fluctuations.

Since the operator $\mathcal{L}_2$ in master equation (17) commutes with the operators $\mathcal{L}_o$ and $\mathcal{L}_1$ this equation describes a Markov process despite the time-dependent coefficient $\chi_1(t)$. Thus, the two-time averages may derived from the one-time averages by taking the advantage of the quantum refression theorem [28]. The steady-state spectrum of the fluorescent light is proportional to the Fourier transforms of the following atomic correlation functions

$$\langle \overline{\overline{J_{21}(\tau) J_{12}}} \rangle_{ss} = \lim_{t \to \infty} \langle \overline{\overline{J_{21}(t+\tau) J_{12}(t)}} \rangle \;. \qquad (24)$$

By using the transformation (7) and canonical transformation (11) one finds

$$\langle \overline{\overline{J_{21}}} \rangle = Tr \left\{ \overline{\overline{J_{21}}} \, \mathcal{S}(t) \right\} = \qquad (25)$$

$$= \sin G \cos G \, \langle D_3 \rangle_1 + \cos^2 G \langle R_{21} \rangle_1 - \sin^2 G \langle R_{12} \rangle_1,$$

where

$$\langle R_{ij} \rangle_m = Tr \left\{ R_{ij} \, \overline{\overline{W}}_m'(t) \right\} \;. \qquad (26)$$

Equations of motion for $\langle R_{ij} \rangle_m$ can be derived by using master equation (17) and have the following forms

$$\frac{d}{dt} \langle D_3 \rangle_m = -(\gamma_c m^2 + 2Z_1 + 2Z_2) \langle D_3 \rangle_m -$$

$$- \gamma_{21} (\sin^2 G - \cos^2 G) \langle D_3^2 \rangle_m$$

$$+ (\sin^2 G - \cos^2 G) \langle \hat{N}^2 + 2 \hat{N} \rangle_m , \qquad (27)$$

$$\frac{d}{dt} \langle R_{21} \rangle_m = 2i\Omega \langle R_{21} \rangle_m - [\gamma_c m^2 + 4 m \gamma_c (\sin^2 G -$$

$$- \cos^2 G) + 4X_0 + 4X_1(t) + 2Z_1] \langle R_{21} \rangle_m$$

$$- \gamma_{21} (\sin^2 G - \cos^2 G) \langle D_3 R_{21} \rangle_m , \qquad (28)$$

In the case of exact resonance, $ctg^2 G = 1$, equations (27–28) reduce to the linear differential equations derived by Puri and Hassan [8].

For the one-atom case, one can use the well-known operator relation

$$R_{ij} R_{i'j'} = R_{ij'} \delta_{ji'} \qquad (i, j; i', j' = 1, 2)$$

and equations (27–28) also reduce to the linear differential equations of the paper [12,19].

For a general case, we use the factorization

$$\langle D_3 R_{ij} \rangle_m = \langle D_3 \rangle_s \cdot \langle R_{ij} \rangle_m . \qquad (29)$$

By using the steady-state density matrix (18) one can show that the factorization (29) yields an error of an order of $N^{-1/2}$ in the calculation of the steady-state fluorescent spectrum which can be neglected when N is large [2,11].

It is easy to see that with factorization (29) the equations (27–28) have simple exponential solutions. Using the relation (25), the solutions of equations (27–28), and applying the quantum regression theorem one obtains the following expressions for the correlation function (24)

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$$\overline{\langle J_{21}(\tau) J_{12} \rangle_{ss}} = \sin^2 G \cdot \cos^2 G \left(\langle D_3^2 \rangle_s - \frac{\Gamma_a}{\Gamma_0} \right) e^{-\Gamma_0 \tau}$$

$$+ \sin^2 G \cdot \cos^2 G \cdot \frac{\Gamma_a}{\Gamma_0}$$

$$+ \cos^4 G \langle R_{21} R_{12} \rangle_s e^{2i\Omega \tau - \Gamma_1 \tau - \zeta (e^{-\gamma_a \tau} - 1)}$$

$$+ \sin^4 G \langle R_{12} R_{21} \rangle_s e^{-2i\Omega \tau - \Gamma_1 \tau - \zeta (e^{-\gamma_a \tau} - 1)} ,$$

where

$$\zeta = 16 \sin^2 G \cdot \cos^2 G \cdot \frac{(\Delta \epsilon_1)^2}{\gamma_a^2} ,$$

$$\Gamma_0 = 2 \gamma_{21} (\cos^4 G + \sin^4 G) + \gamma_c (1 + 4 \sin^2 G \cdot \cos^2 G)$$

$$+ \gamma_{21} (\sin^2 G - \cos^2 G) (N - 2 \langle R_{11} \rangle_s) , \qquad (30)$$

$$\Gamma_1 = 2 \gamma_{21} \cos^2 G (1 + \sin^2 G) + \gamma_c (1 + 5 \sin^4 G - 3 \cos^4 G)$$

$$+ 16 \frac{(\Delta \epsilon_1)^2}{\gamma_a} \sin^2 G \cdot \cos^2 G + \gamma_{21} (\sin^2 G - \cos^2 G)(N - 2 \langle R_{11} \rangle_s) \qquad (31)$$

$$\Gamma_a = \gamma_{21} (\sin^2 G - \cos^2 G) (N^2 + 2N) \langle D_3 \rangle_s . \qquad (32)$$

By using the commutation relations (19–20) one can write

$$\langle D_3^2 \rangle_s = 4 \langle R_{11}^2 \rangle_s - 4 N \langle R_{11} \rangle_s + N^2 , \qquad (33)$$

$$\langle R_{12} R_{21} \rangle_s = - \langle R_{11}^2 \rangle_s + (N+1) \langle R_{11} \rangle_s , \qquad (34)$$

$$\langle R_{21} R_{12} \rangle_s = - \langle R_{11}^2 \rangle_s + (N-1) \langle R_{11} \rangle_s + N , \qquad (35)$$

where the statistical moments $\langle R_{11} \rangle_s$ and $\langle R_{11}^2 \rangle_s$ are found from equation (21).

The steady-state spectrum of the fluorescent light is proportional to the Fourier transform of the correlation

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function $\overline{\langle J_{21}(\tau) J_{12} \rangle}_{ss}$ and has the following form

$$S(\nu) \sim \frac{1}{2} Re \left[\int_0^\infty e^{-i(\nu - \omega_L)\tau} \overline{\langle J_{21}(\tau) J_{12} \rangle}_{ss} \, d\tau \right]$$

$$= sin^2 G \cdot cos^2 G \cdot \left(\langle D_3^2 \rangle_s - \frac{\Gamma_a}{\Gamma_0} \right) \cdot \frac{\Gamma_0}{(\nu - \omega_L)^2 + \Gamma_0^2}$$

$$+ cos^4 G \langle R_{21} R_{12} \rangle_s \cdot e^{\frac{2}{3}} \sum_{n=0}^\infty \frac{(-1)^n z^n}{n!} \cdot \frac{\Gamma_1 + n \gamma_a}{(\nu - \omega_L - 2\Omega)^2 + (\Gamma_1 + n\gamma_a)^2}$$

$$+ sin^4 G \langle R_{12} R_{21} \rangle_s \cdot e^{\frac{2}{3}} \sum_{n=0}^\infty \frac{(-1)^n z^n}{n!} \cdot \frac{\Gamma_1 + n\gamma_a}{(\nu - \omega_L + 2\Omega)^2 + (\Gamma_1 + n\gamma_a)^2}$$

$$+ \frac{\pi \Gamma_a}{2 \Gamma_0} \cdot sin^2 G \cdot cos^2 G \cdot \delta(\nu - \omega_L) \, . \qquad (36)$$

The steady-state fluorescence spectrum (36) contains three spectral lines centered at $\nu = \omega_L$, $\omega_L \pm 2\Omega$. In the off-resonance case, i.e. when $ctg^2 G \neq 1$ the central line at $\nu = \omega_L$ contains the elastic component with the intensity proportional to N^2 and the Lorentzian shaped component with the width Γ_0 and intensity $sin^2 G \cdot cos^2 G \left(\langle D_3^2 \rangle_s - \frac{\Gamma_a}{\Gamma_0} \right)$.
The two sidebands are sums of the Lorentzians of widths $\Gamma_1 + n\gamma_a$, $n = 0,1,...$ centered at frequencies $\nu = \omega_L - 2\Omega$ and $\nu = \omega_L + 2\Omega$, and having the intensities which are proportional to $sin^4 G \langle R_{12} R_{21} \rangle_s$ and $cos^4 G \langle R_{21} R_{12} \rangle_s$, respectively. For off-resonance case and a large number of atoms, as for the nonfluctuating laser driving field [2], the widths and intensities of the three inelastic lines are proportional to N (i.e. their peak intensities are independent of N).

In the case of exact resonance ($ctg^2 G = 1$) one can see that $\Gamma_a = 0$ and the elastic component vanishes. In this case

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the spectrum (36) is in agreement with the work [8] and is the same as that for N=1, except for the fact that the intensities of all inelastic components are proportional to N^2.

In contrast with the exact resonance case [8] the spectrum (36) is not symmetric whenever $ctg^2 G \neq 1$. In fig.2 the relative intensities of the two sidebands, i.e. the values $I_{-1}/N = sin^4 G \langle R_{12} R_{21} \rangle_s / N$ (solid curves) and $I_{+1}/N = cos^4 G \langle R_{21} R_{12} \rangle_s / N$ (dashed curves) are plotted as functions of the parameter $ctg^2 G$. As is clear from fig.2, the intensities of the two sidebands are equal only in the case of exact resonance ($ctg^2 G = 1$) or in the case of $\gamma_c = 0$ which means that the asymmetry of the spectrum is caused by the phase fluctuations of the laser driving field. We note that the values $I_{\pm 1}$ are the integrated intensities of the two sidebands and the value $I_0 =$ $= sin^2 G \cdot cos^2 G \langle D_3^2 \rangle_s$ is the integrated intensity (the sum of
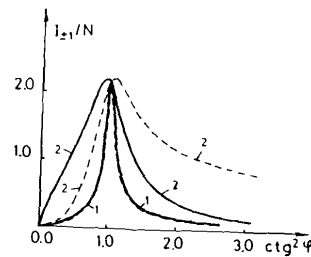

Fig. 2. The relative intensities I_{+1}/N (dashed curves) and I_{-1}/N (solid curves) as functions of the parameter $ctg^2 G$ for fixed N=50. Curves (1)-(2) correspond to γ_c/γ_{21} =0 and 4, respectively.
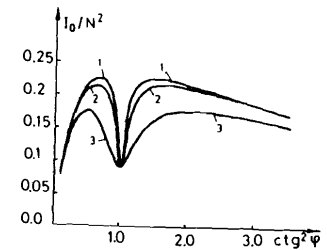
Fig. 3. The relative intensity I_0/N^2 as a function of the parameter $ctg^2 G$ for fixed N=50. Curves (1)-(3) correspond to γ_c/γ_{21}= =0; 0.5; 4, respectively.

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elastic and inelastic components) of the central peak which is plotted in fig.3. As has been noted above, in the off-resonance case (see figs. 2-3) the phase fluctiations strongly affect the integrated intensities of the spectral lines while the amplitude fluctuations have no influence whatsoever on these quantities. In the case of exact resonance, the integrated intensities of the spectral lines are the same as these for the case when the phase and amplitude fluctuations of the laser driving field are absent.

IV. Photon statistics of the spectral components

In this section we discuss the influence of the phase and amplitude fluctuations on the photon statistics of the spectral components. By using the canonical transformation (11) one can write the collective angular momentum of atoms J_{21} in the following form:

$$ J_{21} = \sin G \cos G\, D_3 + \cos^2 G\, R_{21} - \sin^2 G\, R_{12} . \qquad (37) $$

As is clear from the previous section, the operators $\cos^2 G\, R_{21}$, $\sin G \cos G\cdot D_3$ and $-\sin^2 G\, R_{12}$ can be considered as operator sources of the spectral lines centered at $\nu = \omega_L + 2\Omega,\ \omega_L$ and $\omega_L - 2\Omega$ and for later use these operators will be denoted by S_{+1}^+, S_0^+ and S_{-1}^+, respectively.

We introduce the degree of second-order coherence for the spectral line S_ℓ ($\ell = 0, \pm 1$) in the following form:

$$ G_{\ell,\ell}^{(2)} = \frac{\langle S_\ell^+ S_\ell^+ S_\ell S_\ell \rangle_s}{(\langle S_\ell^+ S_\ell \rangle_s)^2} . \qquad (38) $$

By using the stationary atomic density matrix (18) and commutation relations (19-20) one can find

$$ G_{0,0}^{(2)} = \langle D_3^4 \rangle_s / (\langle D_3^2 \rangle_s)^2 , \qquad (39) $$

$$ G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = \langle R_{12} R_{12} R_{21} R_{21} \rangle_s / (\langle R_{12} R_{21} \rangle_s)^2 , \qquad (40) $$

where

$$ \langle D_3^4 \rangle_s = 16 \langle R_{11}^4 \rangle_s - 32 N \langle R_{11}^3 \rangle_s + 24 N^2 \langle R_{11}^2 \rangle_s - 8 N^3 \langle R_{11} \rangle_s + N^4 , \qquad (41) $$

$$ \langle R_{12} R_{12} R_{21} R_{21} \rangle_s = \langle R_{11}^4 \rangle_s - 2(N+2) \langle R_{11}^3 \rangle_s + (N^2 + 5N + 5) \langle R_{11}^2 \rangle_s - (N^2 + 3N + 2) \langle R_{11} \rangle_s , \qquad (42) $$

the statistical moments $\langle D_3^2 \rangle_s$ and $\langle R_{12} R_{21} \rangle_s$ are found from (33-34) and $\langle R_{11}^n \rangle_s$ can be found according to equation (21). As has been mentioned in Sec.2, the statistical moments $\langle R_{11}^n \rangle_s$ are independent of the amplitude fluctuations and as a consequence the amplitude fluctuations of the laser driving field do not influence the photon statistics of the spectral components.

For the one-atom case, by using the well-known operator relation

$$ R_{ij} R_{i'j'} = R_{ij'} \delta_{i'j} \qquad (i, j, i', j' = 1, 2) , $$

one can obtain

$$ G_{0,0}^{(2)} = \langle R_{11} + R_{22} \rangle_s / (\langle R_{11} + R_{22} \rangle_s)^2 = 1 , $$

$$ G_{1,1}^{(2)} = G_{-1,-1}^{(2)} = 0 , $$

Thus, the photon statistics of the central component remains Poissonian and the sidebands have subpoissonian statistics as for the case when the phase fluctuations are absent. In other words, for the one-atom case the phase fluctuations do not affect the photon statistics of the spectral components.

Contrary to the one-atom case, the phase fluctuations strongly affect the photon statistics of the collective spectral components. The degrees of second-order coherence $G_{0,0}^{(2)}$ and $G_{\pm 1,\pm 1}^{(2)}$ as functions of the parameter $ctg^2\varphi$ are plotted in figs. 4 and 5, respectively. As is seen from figs. 4-5, except for the point of exact resonance, i.e. $ctg^2\varphi = 1$, the phase fluctuations of the laser field play an important role in determining photon statistics of particular spectral components in the collective resonance fluorescence. The character of the photon statistics (Poissonian, superpoissonian) can even be changed due to phase fluctuations as is evident from fig.4.
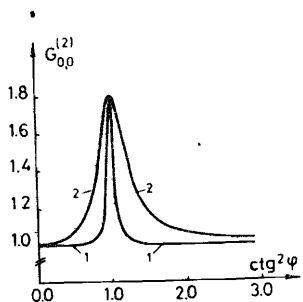
Fig. 4. The degree of second-order coherence $G_{0,0}^{(2)}$ as a function of the parameter $ctg^2\varphi$ for fixed N=50. Curves (1)-(2) correspond to γ_c/γ_{21} =0; 4, respectively.
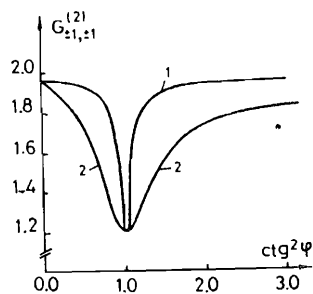
Fig. 5. The degree of second-order coherence $G_{\pm 1,\pm 1}^{(2)}$ as a function of the parameter $ctg^2\varphi$ for fixed N=50. The curves (1)-(2) correspond to γ_c/γ_{21} =0; 4, respectively.

V. Conclusions

We have considered the problem of collective resonance fluorescence in the strong laser field with the phase and amplitude fluctuations. The steady-state solution for the atomic density matrix averaged over the phase and amplitude fluctuations has been obtained for the general off-resonance case within the secular approximation. Analytical formulas for the spectrum of resonance fluorescence have been derived. It has been shown that in the off-resonance case the spectrum is asymmetric with the asymmetry which is determined by the phase fluctuations solely. The explicit analytical formulas for the degrees of the second order coherence for particular spectral components have also been obtained. It has been shown that, unlike the one-atom case, in the collective resonance fluorescence the photon statistics of individual spectral components depends strongly on the phase fluctuations of the laser field. It should be noted here that the approach presented in this paper can be used to investigate the influence of the phase and amplitude fluctuations on the two-time intensity-intensity correlation functions, the cross-correlations between the spectral components and the macroscopic nonclassical effects such as squeezing and violation of the Cauchy-Schwarz inequality. The research in this field is in progress.

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Шумовский А.С., Танась Р., Чан Куанг   E17-87-436
Коллективная резонансная флуоресценция в интенсивном флуктационном по фазе и амплитуде лазерном поле

Рассмотрена коллективная резонансная флуоресценция в системе двухуровневых атомов, взаимодействующих с интенсивным флуктуационным по фазе и амплитуде лазерным полем. Для общего случая с частотной расстройкой обсуждены эффекты, возникающие в результате влияния флуктуаций лазерного поля на спектр и на степень когерентности второго порядка спектральных компонент флуоресцентного поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Shumovsky A.S., Tanas R., Tran Quang   E17-87-436
Collective Resonance Fluorescence in an Intense Laser Field with Phase and Amplitude Fluctuations

The collective resonance fluorescence from a system of two-level atoms resonantly driven by a strong laser field with the phase and amplitude fluctuations is considered. The effect of the field fluctuations on the spectrum as well as on the degrees of the second-order coherence of the fluorescent field is discussed in detail for the off-resonance case.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.