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LIGHT SQUEEZING IN THE TWO-ATOM ONE-MODE MODEL WITH MULTIPHOTON TRANSITIONS

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1. INTRODUCTION

New technical developments [1,2] make it experimentally realizable to investigate the interaction of one or a few atoms with the electromagnetic field in a high-Q maser cavity [3,4]. The situation realized in the one-atom maser [1] approaches the idealized model of a two-level atom interacting with a single quantized mode of a radiation field as proposed by Jaynes and Cummings many years ago [5]. Various aspects of the dynamics and statistics of the field in this model have been studied (for reviews see [3,6]). The production of squeezed quantum states of the cavity field has been discussed in refs. [7-10]. Meystre and Zubairy [7] have found squeezing for an initially coherent field interacting with an initially excited atom. Recently, Butler and Drummond have shown the occurrence of squeezing at the onset of interaction for an alternative initial state where the initial atomic state is taken to be deexcited with the field in a coherent state [8]. Squeezing in single-mode spontaneous emission from a suitably prepared atom in an ideal cavity has been demonstrated by Knight and his co-workers [9,10]. The levels of squeezing obtained in the standard Jaynes-Cummings model are however low.

There are various ways of overcoming this difficulty. One way is to use multiphoton resonance in the framework of the so-called multiphoton Jaynes-Cummings models [11,12]. Another way is to use a large number of atoms interacting cooperatively with the cavity field [8, 13-16]. It is hence interesting to know what happens to squeezing in a cooperative system with the multiphoton-resonant interaction. The present paper is devoted to examining this problem on the basis of the simplest cooperative model consisting of two two-level atoms.

2. HAMILTONIAN AND OBSERVABLES

We consider two two-level atoms interacting with a single-mode radiation field in an ideal resonant cavity via the m-photon-transition mechanism. The effective Hamiltonian for this system in the rotating wave approximation is

$$H = \hbar\omega a^+ a + \sum_{j=1}^{2} \hbar\omega_0 R_j^z + \sum_{j=1}^{2} \hbar g (R_j^+ a^m + R_j^- a^{+m}) . \qquad (1)$$

Here R_j^z and $R_j^{\pm}$ are the pseudospin -1/2 operators of the j-th atom, a^+ and a are the creation and annihilation operators of photons in the cavity field, ω and ω_0 are the frequencies of the field mode and the atom, g is the atom-field coupling constant, and m is the photon multiple of transitions. The exact multiphoton resonance is assumed to occur: $\omega_0 = m\omega$.

We denote by $|+\rangle$, $|-\rangle$ the excited and ground states of an atom and by $|n\rangle$ the Fock states of the field. The initial state of interest for squeezing will be taken to be either a coherent (nonvacuum) state of the field together with the atomic de-excited state $|-,-\rangle$ or the vacuum state of the field together with an atomic superposition state.

The observables of interest are the variances of the slowly varying Hermitian quadratures of the field which give information on squeezing /17-20/.

The field quadratures are defined by

$$a_1 = \frac{1}{2}(a e^{i(\omega t - \theta)} + a^+ e^{-i(\omega t - \theta)}) ,$$
$$a_2 = \frac{1}{2i}(a e^{i(\omega t - \theta)} - a^+ e^{-i(\omega t - \theta)}) , \qquad (2)$$

where θ is a phase angle that may be chosen at will. The commutation of a_1 and a_2 is $[a_1, a_2] = i/2$. The variances $(\Delta a_i)^2 = \langle a_i^2 \rangle - \langle a_i \rangle^2$ (i = 1,2) satisfy the uncertainty relation $(\Delta a_1)^2 (\Delta a_2)^2 \geq 1/16$. For the vacuum and coherent states of the field one has $(\Delta a_1)^2_{vac,coh} = (\Delta a_2)^2_{vac,coh} = 1/4$. The field is in a squeezed state if there exists a phase angle θ such that $(\Delta a_i)^2 < 1/4$ for either i = 1 or 2. Squeezing states have variances smaller than the vacuum noise variance in one quadrature, and increased variances in the other quadrature. The condition for squeezing in the quadrature a_α can be written simply as

$$S_\alpha < 0 , \qquad (3)$$

where the relative variances

$$S_\alpha = \frac{(\Delta a_\alpha)^2 - (\Delta a_\alpha)^2_{coh,\,vac}}{(\Delta a_\alpha)^2_{coh,\,vac}} = 4(\Delta a_\alpha)^2 - 1 \qquad (4)$$

have been introduced and are called the factors of squeezing. In terms of the photon operators, we find readily that

$$S_1 = 2\langle a^+ a \rangle + 2\mathrm{Re}\langle a^2 e^{2i(\omega t - \theta)} \rangle - 4(\mathrm{Re}\langle a e^{i(\omega t - \theta)} \rangle)^2 ,$$
$$S_2 = 2\langle a^+ a \rangle - 2\mathrm{Re}\langle a^2 e^{2i(\omega t - \theta)} \rangle - 4(\mathrm{Im}\langle a e^{i(\omega t - \theta)} \rangle)^2 . \qquad (5)$$

Note that $S_\alpha > -1$ for an arbitrary field state. If squeezing occurs in the quadrature a_α, i.e. $S_\alpha < 0$, then the degree (per cent) of squeezing is determined by $-S_\alpha = -100 S_\alpha$ %.

The existence of squeezed states is now well understood theoretically /17/ and has been experimentally observed in various systems /18-20/.

3. COHERENT FIELD TOGETHER WITH THE DE-EXCITED ATOMIC STATE

In this section we examine squeezing in the situation when the field is initially in a coherent state $|z\rangle$ and the atoms are initially in the de-excited state $|-,-\rangle$. The initial state is then given by

$$|\psi(0)\rangle = |-,-\rangle \otimes |z\rangle . \qquad (6)$$

We work here in the Schrödinger picture and expand the time-dependent wave function of the atom-field system as

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \exp[-i(n-m)\omega t] \exp(-\frac{|z|^2}{2}) \frac{z^n}{\sqrt{n!}} \times$$
$$\times \{ |-,-;n\rangle A_-^{(n)}(t) + |+,+;n-2m\rangle A_+^{(n)}(t) + \qquad (7)$$
$$+ (|+,-;n-m\rangle + |-,+;n-m\rangle) B^{(n)}(t) \} .$$

Then the Schrödinger equation gives the equations of motion for the probability amplitudes as

$$i\dot{A}_-^{(n)} = 2\sqrt{\frac{n!}{(n-m)!}}\, g B^{(n)} ,$$
$$i\dot{A}_+^{(n)} = 2\sqrt{\frac{(n-m)!}{(n-2m)!}}\, g B^{(n)} , \qquad (8)$$
$$i\dot{B}^{(n)} = \sqrt{\frac{n!}{(n-m)!}}\, g A_-^{(n)} + \sqrt{\frac{(n-m)!}{(n-2m)!}}\, g A_+^{(n)}$$

together with the initial conditions

$$A_-^{(n)}(0) = 1, \quad A_+^{(n)}(0) = B^{(n)}(0) = 0. \qquad (9)$$

The solutions of these equations are easily found to be

$$A_-^{(n)} = 1 - \frac{2q_n}{q_{n-m} + q_n} \sin^2[\sqrt{(q_{n-m} + q_n)/2}\, gt],$$

$$A_+^{(n)} = -\frac{2\sqrt{q_{n-m}q_n}}{q_{n-m} + q_n} \sin^2[\sqrt{(q_{n-m} + q_n)/2}\, gt], \qquad (10)$$

$$B^{(n)} = -i\sqrt{\frac{q_n}{2(q_{n-m} + q_n)}} \sin[\sqrt{2(q_{n-m} + q_n)}\, gt],$$

where

$$q_n = n(n-1)\dots(n-m+1) = \frac{n!}{(n-m)!}. \qquad (11)$$

Hence, the mean photon number $<a^+a>$, the mean photon amplitude $<a>$ and the mean square photon amplitude $<a^2>$ are calculated to read

$$<a^+a> \doteq \bar{n} - 2m\Big(\sum_{n=2m}^{\infty} p_n |A_+^{(n)}|^2 + \sum_{n=m}^{\infty} p_n |B^{(n)}|^2\Big) \equiv \sigma_0,$$

$$e^{i\omega t}<a> = z\Big(\sum_{n=0}^{\infty} p_n A_-^{(n)*} A_-^{(n+1)} +$$

$$+ \sum_{n=2m}^{\infty} p_n A_+^{(n)*} A_+^{(n+1)} \sqrt{1 - \frac{2m}{n+1}} + \qquad (12)$$

$$+ \sum_{n=m}^{\infty} 2p_n B^{(n)*} B^{(n+1)} \sqrt{1 - \frac{m}{n+1}}\Big) \equiv z\sigma_1,$$

$$e^{2i\omega t}<a^2> = z^2\Big(\sum_{n=0}^{\infty} p_n A_-^{(n)*} A_-^{(n+2)} +$$

$$+ \sum_{n=2m}^{\infty} p_n A_+^{(n)*} A_+^{(n+2)} \sqrt{(1 - \frac{2m}{n+1})(1 - \frac{2m}{n+2})} +$$

$$+ \sum_{n=m}^{\infty} 2p_n B^{(n)*} B^{(n+2)} \sqrt{(1 - \frac{m}{n+1})(1 - \frac{m}{n+2})}\Big) \equiv z^2\sigma_2.$$

Here p_n is the Poissonian distribution corresponding to the coherent initial state $|z>$ of the field,

$$p_n = \exp(-\bar{n})\,\bar{n}^n/n! \qquad (13)$$

and $\bar{n} = |z|^2$ is the dimensionless initial-field intensity. Note that σ_0, σ_1 and σ_2 defined in eq. (12) are real numbers. Then, it is seen from eqs. (5) that the optimal choice of θ for squeezing should be either $\theta = \phi$ or $\theta = \phi + \pi/2$, where ϕ is the phase of z, i.e. $z = \bar{n}^{1/2}\exp(i\phi)$. Taking into account the relations $S_1(\theta + \pi/2) = S_2(\theta)$, $S_2(\theta + \pi/2) = S_1(\theta)$ we shall consider below only the choice $\theta = \phi$. In this case, equations (5) become

$$S_1 = 2\sigma_0 + 2\bar{n}\sigma_2 - 4\bar{n}\sigma_1^2, \qquad (14a)$$

$$S_2 = 2\sigma_0 - 2\bar{n}\sigma_2. \qquad (14b)$$

These equations together with eqs. (12) and (10) describe the time behaviour of the squeezing factors S_1 and S_2. For very short times ($gt \ll 1$), we find from eqs. (12) and (10) the asymptotic expressions

$$\sigma_0 \simeq \begin{cases} \bar{n} - 2\bar{n}(gt)^2 + \frac{2}{3}(\bar{n}^2 + 2\bar{n})(gt)^4, & m = 1 \\ \bar{n} - 2m\bar{n}^m(gt)^2, & m \geq 2, \end{cases}$$

$$\sigma_1 \simeq \begin{cases} 1 - (gt)^2 + \frac{1}{6}(2\bar{n} + 1)(gt)^4, & m = 1 \\ 1 - m\bar{n}^{m-1}(gt)^2, & m \geq 2, \end{cases} \qquad (15)$$

$$\sigma_2 \simeq \begin{cases} 1 - 2(gt)^2 + \frac{1}{3}(2\bar{n} + 3)(gt)^4, & m = 1 \\ 1 - [2m\bar{n}^{m-1} + m(m-1)\bar{n}^{m-2}](gt)^2, & m \geq 2. \end{cases}$$

Here we have used the property of the Poissonian distribution

$$\sum_{n=0}^{\infty} p_n n(n-1)\dots(n-k+1) = \bar{n}^k, \qquad (16)$$

$$\sum_{n=0}^{\infty} p_n (n+1)n \ldots (n-k+2) = \bar{n}^k + k\bar{n}^{k-1},$$

$$\sum_{n=0}^{\infty} p_n (n+2)(n+1) \ldots (n-k+3) = \bar{n}^k + 2k\bar{n}^{k-1} + k(k-1)\bar{n}^{k-2}.$$

By substituting expressions (15) into eqs. (14), the asymptotic expressions for S_1 and S_2 are found to be

$$S_1(gt \ll 1) = \begin{cases} -\frac{2}{3}\bar{n}(gt)^4 & \text{in the case } m = 1 \\ -2m(m-1)\bar{n}^{m-1}(gt)^2 & \text{in the case } m \geq 2, \end{cases} \qquad (17a)$$

$$S_2(gt \ll 1) = \begin{cases} \frac{2}{3}\bar{n}(gt)^4 & \text{in the case } m = 1 \\ 2m(m-1)\bar{n}^{m-1}(gt)^2 & \text{in the case } m \geq 2. \end{cases} \qquad (17b)$$

The negative expressions (17a) indicate the immediate appearance of squeezing in a_1 for any photon multiple m and arbitrary nonzero intensity $\bar{n}$ at the onset of interaction. Such a behaviour is absent in the case when the atoms are initially in the excited state [7,8,13-15]. The positive expressions (17b) indicate the corresponding increase of fluctuations in the quadrature a_2. For the particular case $m = 1$, our results are in agreement with the results obtained recently by Butler and Drummond [8] for a cooperative Dicke system. In the cases of multi-photon transitions ($m \geq 2$) the degree of squeezing in a_1 increases from the onset of interaction as $(gt)^2$ instead of $(gt)^4$ as in the one-photon case ($m = 1$). The dependence of the squeezing factor $S_1(gt \ll 1)$ upon the initial field intensity $\bar{n}$ is linear ($\sim \bar{n}$) in the cases $m = 1,2$, and nonlinear ($\sim \bar{n}^{m-1}$) in the cases $m \geq 3$. By comparison with the one-atom case [12], the cooperativity of the two atoms considered here leads to the twice larger factors $S_1(gt \ll 1)$, $S_2(gt \ll 1)$. It should be noted from eqs. (17) and (4) that to order $(gt)^4$ for $m = 1$ and $(gt)^2$ for $m \geq 2$ one has $(\Delta a_1)^2(\Delta a_2)^2 = 1/16$ indicating that a minimum uncertainty state is generated to this order.

Figure 1 presents the long time behaviour of S_1 computed numerically from eqs. (14a), (12) and (13) for, e.g., $m = 1$ and $\bar{n} = 0.2$. As soon as $t > 0$, we observe negative values of S_1 indicating the occurrence of squeezing. As time goes on, S_1 starts oscillating. Squeezing disappears and later may

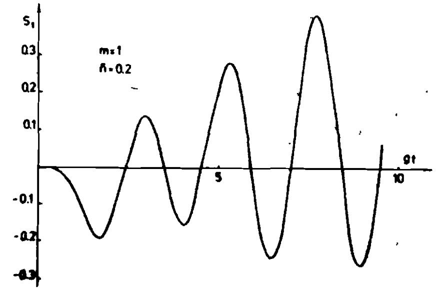


Fig. 1. Long time behaviour of the factor S_1. *The calculation has been made for* $m = 1$ *and* $\bar{n} = 0.2$.

appear again. The maximum degree of squeezing recovered again may be larger than the maximum degree of the first (short-time) squeezing. These features have been shown to exist in the situation with a single atom [7,12] and therefore are not surprising for the two-atom case.

Figures 2-6 present the behaviour of S_1 for the first squeezing, calculated for various intensities $\bar{n}$ and various photon multiples m. It can be noted that for each photon multiple m the first squeezing has a lower bound which increases as m increases from 1 to 3 and decreases as m increases from 3. For $m = 1$-5, this lower bound is reached for $\bar{n} \simeq 0.42$, 2, 4.25, 7, 10.75 and is approximately equal to 29%, 58%, 60%, 57%, 53%, respectively. It is larger than the lower bound obtained in the situation with a single atom [12].

In fig. 7, we compare the first squeezing obtained here for the two atom system with that obtained in [12] for the single-atom situation. In both the cases, the calculations have been made for the same values of m and n: $m = 1$, $\bar{n} = 0.2$

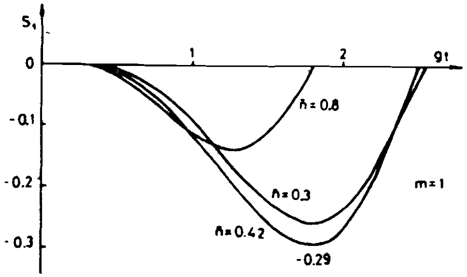


Fig. 2. Time behaviour of S_1 *for the first squeezing in the cases of* $m = 1$, $\bar{n} = 0.3$, *0.42 and 0.8. The lower bound is approximately equal to −0.29 (29% squeezing), and occurs for* $\bar{n} \simeq 0.42$.

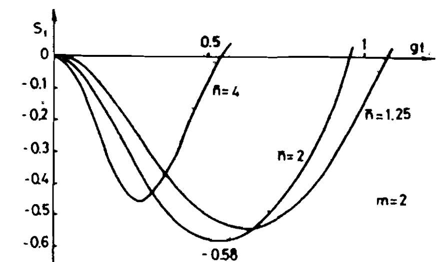


Fig. 3. Time behaviour of S_1 *for the first squeezing in the cases of* $m = 2$, $\bar{n} = 1.25$, *2 and 4. The lower bound is approximately equal to − 0.58 (58% squeezing), and occurs for* $\bar{n} \simeq 2$.

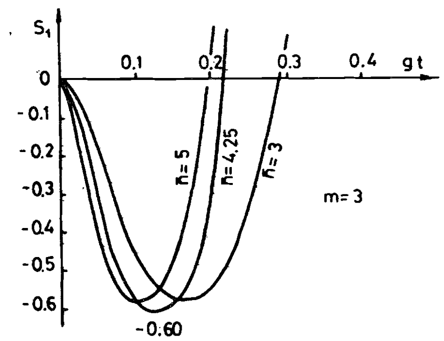


Fig.4. Time behaviour of S_1 *for the first squeezing in the cases of* $m = 3, \bar{n} = 3$, *4.25 and 5. The lower bound is approximately equal to -0.60 (60% squeezing), and occurs for* $\bar{n} \simeq 4.25$.

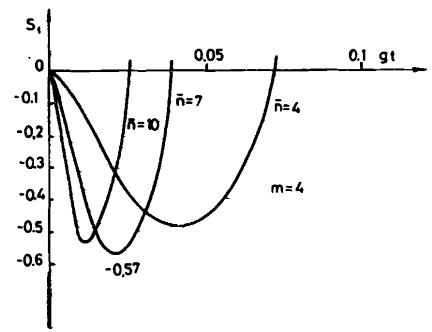


Fig.5. Time behaviour of S_1 *for the first squeezing in the cases of* $m = 4$, $\bar{n} = 4$, *7 and 10. The lower bound is approximately equal to -0.57 (57% squeezing), and occurs for* $\bar{n} \simeq 7$.

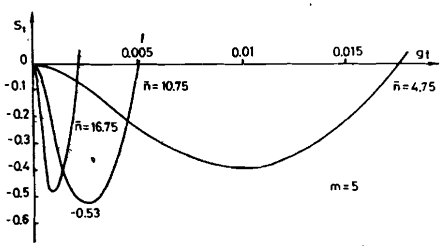
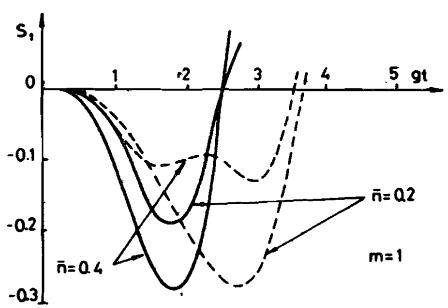


Fig.6. Time behaviour of S_1 *for the first squeezing in the cases of* $m = 5, \bar{n} = 4.75$, *10.75 and 16.75. The lower bound is approximately equal to -0.53 (53% squeezing), and occurs for* $\bar{n} \simeq 10.75$.

Fig.7. Comparison of the squeezing factors S_1 *obtained in the single-atom- and two-atom-models. The full line corresponds to the two-atom case. The dashed line corresponds to the single-atom case. In both the cases, the calculations have been made for* $m = 1$, $n = 0.2$ *and 0.4.*



and 0.4. It is noted from the figure that for $\bar{n} = 0.4$, the maximum degree of the first squeezing in the two-atom case is larger than in the single-atom case, whereas for $\bar{n} = 0.2$ the contrary relation occurs. For the same and very small times, the squeezing in the two-atom case is twice as large as that in the single-atom case.

4. VACUUM FIELD AND ATOMIC SUPERPOSITION STATE

In this section, on the basis of the model (1) we investigate the possibility of generating squeezing by the cooperative interaction of two suitably prepared atoms with a cavity mode initially in the vacuum state. Of course, no spontaneous emission squeezing will be produced if both the atoms are initially fully excited. However, the atoms can be injected into the cavity in coherent superpositions of excited and ground states and subsequent squeezing is possible [9,10]. To get physical insight we concentrate on three cases where simple analytical solutions can be derived.

Case 1. Let one atom be prepared in a coherent superposition state; while the other atom, in the ground state. The initial state of the total system is then

$$|\psi(0)\rangle = \cos\frac{\alpha}{2}|+,-;0\rangle + e^{-i\beta}\sin\frac{\alpha}{2}|-,-;0\rangle , \quad 0 \le \alpha \le \pi, \quad 0 \le \beta \le 2\pi, \tag{18}$$

The interaction Hamiltonian couples the state $|+,-;0\rangle$ to the states $|-,-;m\rangle$ and $|-,+;0\rangle$ but the zero quantum state $|-,-;0\rangle$ is decoupled and does not evolve. This means that the atoms can emit m photons into the field initially in the vacuum state and then reabsorb them. At the time t, the wave function is

$$|\psi(t)\rangle = \cos\frac{\alpha}{2}(A(t)|+,-;0\rangle + B(t)|-,+;0\rangle + C|-,-;m\rangle) + e^{i(m\omega t-\beta)}\sin\frac{\alpha}{2}|-,-;0\rangle . \tag{19}$$

The Schrödinger equation gives the equations of motion for the probability amplitudes

$$i\dot{A} = g\sqrt{m!}C, \quad i\dot{B} = g\sqrt{m!}C, \quad i\dot{C} = g\sqrt{m!}(A+B), \tag{20}$$

where $A(0) = 1$, $B(0) = 0$, $C(0) = 0$. These equations yield the solutions

$$A(t) = \cos^2[g\sqrt{m!/2}\,t],$$
$$B(t) = -\sin^2[g\sqrt{m!/2}\,t], \qquad (21)$$
$$C(t) = -\frac{1}{\sqrt{2}}\sin[g\sqrt{2(m!)}\,t].$$

It is seen from eq. (19) that if $m \geq 3$ then $\langle a\rangle = \langle a^2\rangle = 0$, hence $S_1 = S_2 = 2\langle a^+a\rangle > 0$, and therefore, no squeezing occurs.

For $m = 1$, the expectation values of the operators a, a^2 and a^+a are easily found from eqs. (19) and (21) to be

$$\langle a\rangle = -\frac{1}{2\sqrt{2}}\sin\alpha\,\sin(\sqrt{2}\,gt)\,e^{i(\beta-\omega t)},$$
$$\langle a^2\rangle = 0, \qquad (22)$$
$$\langle a^+a\rangle = \frac{1}{2}\cos^2\frac{\alpha}{2}\sin^2(\sqrt{2}\,gt).$$

According to eqs. (5), the factors S_1 and S_2 are then

$$S_1 = [\cos^2\frac{\alpha}{2} - \frac{1}{2}\sin^2(\theta-\beta)\sin^2\alpha]\sin^2(\sqrt{2}\,gt),$$
$$S_2 = [\cos^2\frac{\alpha}{2} - \frac{1}{2}\cos^2(\theta-\beta)\sin^2\alpha]\sin^2(\sqrt{2}\,gt). \qquad (23)$$

We take the parameters $\theta = \beta + \pi/2$ and $\alpha = 2\pi/3$ which are optimum for squeezing in the quadrature a_1. Equations (23) then become

$$S_1 = -\frac{1}{8}\sin^2(\sqrt{2}\,gt), \qquad S_2 = \frac{1}{4}\sin^2(\sqrt{2}\,gt). \qquad (24)$$

It is seen that the fluctuations in a_1 are reduced to be below the vacuum noise level whereas the fluctuations in a_2 are enhanced. The minimum value of S_1 is $\min S_1 = -1/8$ and is reached periodically at the times $t = (2k+1)\pi/(2\sqrt{2}\,g)$, $k = 0,1,2,\ldots$ The corresponding maximum degree of squeezing is 12.5%. It is twice as small as the degree 25% of the maximum squeezing obtained in the one-atom one-photon-transition ($m = 1$) situation /10/.

For $m = 2$, we find for $\langle a\rangle$, $\langle a^2\rangle$ and $\langle a^+a\rangle$ the expressions

$$\langle a\rangle = 0,$$
$$\langle a^2\rangle = -\frac{1}{2}\sin\alpha\,\sin(2gt)\,e^{i(\beta-2\omega t)}, \qquad (25)$$
$$\langle a^+a\rangle = \cos^2\frac{\alpha}{2}\sin^2(2gt).$$

The factors S_1 and S_2 are therefore

$$S_1 = 2\cos^2\frac{\alpha}{2}\sin^2(2gt) - \sin(2\theta-\beta)\sin\alpha\,\sin(2gt),$$
$$S_2 = 2\cos^2\frac{\alpha}{2}\sin^2(2gt) + \sin(2\theta-\beta)\sin\alpha\,\sin(2gt). \qquad (26)$$

After making the optimum choice: $\theta = \beta/2 + \pi/4$ and $\alpha = 3\pi/4$ we get

$$S_1 = \sin(2gt)\,[(1 - \frac{1}{\sqrt{2}})\sin(2gt) - \frac{1}{\sqrt{2}}],$$
$$S_2 = \sin(2gt)\,[(1 - \frac{1}{\sqrt{2}})\sin(2gt) + \frac{1}{\sqrt{2}}]. \qquad (27)$$

These equations show periodical changes of squeezing in a_1 by that in a_2 and vice versa. It is found that $\min S_1 = \min S_2 = -(\sqrt{2}-1) \simeq -0.416$. The corresponding maximum degree of squeezing is about 41.6%. It is less than the degree $(\sqrt{6}-2) \simeq 45\%$ that may be obtained in a one-atom two-photon-transition system /10/. The times for reaching the maximum squeezing are $t = (2k+1)\pi/4g$, $k = 0,1,\ldots$.

Thus, the presence of the unexcited atom leads to decrease of the maximum degree of squeezing in spontaneous emission from the superposition-state-excited atom.

Case 2. Let the atoms be prepared in a superposition state such that

$$|\psi(0)\rangle = \cos\frac{\alpha}{2}|+,+;0\rangle + e^{-i\beta}\sin\frac{\alpha}{2}|-,-;0\rangle, \qquad (28)$$
$$0 \leq \alpha \leq \pi, \quad 0 \leq \beta \leq 2\pi.$$

The interaction Hamiltonian couples the state $|+,+;0\rangle$ to the states $|+,-;m\rangle$, $|-,+;m\rangle$ and $|-,-;2m\rangle$ but the zero quantum state $|-,-;0\rangle$ is decoupled and does not evolve. The time-evolved wave function has the form

$$|\psi(t)\rangle = e^{-i\omega_0 t}\cos\frac{\alpha}{2}[\,|+,+;0\rangle A_+(t) + |-,-;2m\rangle A_-(t) +$$
$$+ (|+,-;m\rangle + |-,+;m\rangle)B(t)] + e^{-i(\beta-\omega_0 t)}\sin\frac{\alpha}{2}|-,-;0\rangle. \qquad (29)$$

It is straighforward to solve the Schrödinger equation for the amplitudes $A_+(t)$, $A_-(t)$ and $B(t)$. As a result we get

$$A_+(t) = 1 - \frac{g^2 m!}{\Omega_R^2} \sin^2(\Omega_R t) ,$$

$$A_-(t) = - \frac{g^2 \sqrt{(2m)!}}{\Omega_R^2} \sin^2(\Omega_R t) , \qquad (30)$$

$$B(t) = -i \frac{g\sqrt{m!}}{2\Omega_R} \sin(2\Omega_R t) ,$$

where Ω_R is the effective Rabi frequency given by

$$\Omega_R = g \sqrt{\frac{1}{2}[\, m! + \frac{(2m)!}{m!}\,]} . \qquad (31)$$

For $m \geq 2$, we find from eq.(29) that $\langle a\rangle = \langle a^2\rangle = 0$, and therefore, no squeezing exists. For $m = 1$, $\langle a\rangle$, $\langle a^2\rangle$ and $\langle a^+a\rangle$ are calculated to read

$$\langle a\rangle = 0$$

$$\langle a^2\rangle = -\frac{2}{3} e^{i(\beta - 2\omega t)} \sin\alpha \sin^2(\frac{\sqrt{6}}{2} gt) , \qquad (32)$$

$$\langle a^+a\rangle = \frac{4}{3} \cos^2\frac{\alpha}{2} \sin^2(\frac{\sqrt{6}}{2} gt)\, [\, 1 + \frac{1}{3} \sin^2(\frac{\sqrt{6}}{2} gt)] .$$

The factors S_1 and S_2 are then

$$S_1 = \frac{4}{3} \sin^2(\frac{\sqrt{6}}{2} gt)\, \{[\, 1 + \frac{1}{3}\sin^2(\frac{\sqrt{6}}{2} gt)]\,(1 + \cos\alpha) - \cos(2\theta - \beta) \sin\alpha\} ,$$

$$S_2 = \frac{4}{3} \sin^2(\frac{\sqrt{6}}{2} fgt)\, \{[\, 1 + \frac{1}{3}\sin^2(\frac{\sqrt{6}}{2} gt)]\,(1 + \cos\alpha) + \cos(2\theta - \beta) \sin\alpha\} . \qquad (33)$$

It can be shown that the optimum choice of θ and α should be $\theta = \beta/2$, $\alpha = \mathrm{arctg}(-3/4)$ $(0 \leq \alpha \leq \pi)$. For these parameters equations (33) take the form

$$S_1 = -\frac{4}{45} \sin^2(\frac{\sqrt{6}}{2} gt)\, [\, 6 - \sin^2(\frac{\sqrt{6}}{2} gt)] ,$$

$$S_2 = \frac{4}{45} \sin^2(\frac{\sqrt{6}}{2} gt)\, [\, 12 + \sin^2(\frac{\sqrt{6}}{2} gt)] . \qquad (34)$$

It is seen that squeezing occurs in $a_1 (S_1 \leq 0)$ but not in $a_2 (S_2 \geq 0)$. At the times $t = (2k+1)\,\pi/g\sqrt{6}$, $k = 0, 1, 2, \ldots$, the factor S_1 reaches the minimum value $\min S_1 = -4/9$. It represents more than 44.4% squeezing which is marginally larger than the 25% squeezing associated with the spontaneous emission in the standard Jaynes-Cummings model [9,10].

Case 3. Let the initial state of the total system be

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \cos\frac{\alpha}{2} (\,|+,-;0\rangle + |-,+;0\rangle\,) + e^{-i\beta} \sin\frac{\alpha}{2} |-,-;0\rangle , \qquad (35)$$

$$0 \leq \alpha \leq \pi, \quad 0 \leq \beta \leq 2\pi .$$

The interaction couples the states $|+,-;0\rangle$, $|-,+;0\rangle$ to the state $|-,-;m\rangle$. The wave function at the time t is

$$|\psi(t)\rangle = \cos\frac{\alpha}{2} \{ \frac{1}{\sqrt{2}} \cos[\sqrt{2(m!)}\, gt\,]\,(\,|+,-;0\rangle + |-,+;0\rangle) - i \sin[\sqrt{2(m!)}\, gt]\, |-,-;m\rangle \} + e^{i(m\omega t - \beta)} \sin\frac{\alpha}{2} |-,-;0\rangle . \qquad (36)$$

For $m \geq 3$, we find from eq.(36) that $\langle a\rangle = \langle a^2\rangle = 0$, and hence no squeezing occurs.

For $m = 1$, we get for $\langle a\rangle$, $\langle a^2\rangle$ and $\langle a^+a\rangle$ the expressions

$$\langle a\rangle = -\frac{i}{2} e^{i(\beta - \omega t)} \sin\alpha \sin(\sqrt{2}\, gt) ,$$

$$\langle a^2\rangle = 0 , \qquad (37)$$

$$\langle a^+a\rangle = \cos^2\frac{\alpha}{2} \sin^2(\sqrt{2}\, gt) .$$

The factors S_1 and S_2 are then

$$S_1 = \sin^2(\sqrt{2}\, gt)\, [\, 1 + \cos\alpha - \sin^2(\theta - \beta) \sin^2\alpha] ,$$

$$S_2 = \sin^2(\sqrt{2}\, gt)\, [\, 1 + \cos\alpha - \cos^2(\theta - \beta) \sin^2\alpha] . \qquad (38)$$

The optimum choice of parameters for squeezing in, e.g., a_1 should be $\theta = \beta + \pi/2$, $a = 2\pi/3$. For these parameters, equations (38) take the form

$$S_1 = -\frac{1}{4}\sin^2(\sqrt{2}\,gt)\,, \qquad S_2 = \frac{1}{2}\sin^2(\sqrt{2}\,gt)\,. \tag{39}$$

At the times $t = (2k+1)\pi/2\sqrt{2}\,g$ the factor S_1 takes the minimum value $\min S_1 = -1/4$ which represents 25% squeezing and is equal to the maximum degree obtained in the Jaynes-Cummings model /9,10/.

For m = 2, we find from eq.(36) that

$$\langle a\rangle = 0\,,$$

$$\langle a^2\rangle = -\frac{i}{\sqrt{2}}e^{i(\beta-2\omega t)}\sin a\,\sin(2gt)\,, \tag{40}$$

$$\langle a^+a\rangle = 2\cos^2\frac{a}{2}\sin^2(2gt)\,.$$

Hence, the factors S_1 and S_2 are found to be

$$S_1 = 4\cos^2\frac{a}{2}\sin^2(2gt) - \sqrt{2}\sin(2\theta-\beta)\sin a\,\sin(2gt)\,,$$

$$S_2 = 4\cos^2\frac{a}{2}\sin^2(2gt) + \sqrt{2}\sin(2\theta-\beta)\sin a\,\sin(2gt)\,, \tag{41}$$

The optimum choice of the parameters θ and a should be $\theta = \beta/2+\pi/4$, $a = \mathrm{arctg}(-1/\sqrt{2})$. For these parameters, equations (41) take the form

$$S_1 = \sqrt{\frac{2}{3}}\sin(2gt)\,[2(\sqrt{\frac{3}{2}}-1)\sin(2gt) - 1]\,,$$

$$S_2 = \sqrt{\frac{2}{3}}\sin(2gt)\,[2(\sqrt{\frac{3}{2}}-1)\sin(2gt) + 1]\,. \tag{42}$$

We find that $\min S_1 = \min S_2 = -(\sqrt{6}-2)$ representing the maximum squeezing about 45% in a_1 and a_2 at the times $t = (4k+1)\pi/4g$ and $t = (4k+3)\pi/4g$ ($k = 0,1,2,\ldots$), respectively. This maximum degree of squeezing is equal to the maximum degree obtained in the situation with a single atom (the two-photon Jaynes-Cummings model).

5. SUMMARY

We have investigated the generation of squeezing states of the cavity radiation field in the two-atom one-mode model with multiphoton transitions. The cases when the field is initially in a coherent state together with the de-excited atoms, and when the field is initially in the vacuum state together with an atomic superposition state have been examined. The time-dependent squeezing factors have been calculated. The conditions for the optimum squeezing have been shown. The comparision with the single-atom situation has been made. These results are potentially of interest to experimentalists studying Rydberg atoms in high-Q single-mode cavities in which the squeezed states can be generated.

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Фам Ле Киен, Каданцева Е.П., Шумовский А.С. E17-87-410
Сжатие света в двухатомной одномодовой модели с мультифотонными переходами

Исследована генерация сжатых состояний в двухатомной одномодовой модели с мультифотонными переходами. Вычислены времязависимые факторы сжатия. Приведены условия для оптимального сжатия.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Fam Le Kien, Kadantseva E.P., Shumovsky A.S. E17-87-410
Light Squeezing in the Two-Atom One-Mode Model with Multiphoton Transitions

The generation of squeezing states of the cavity radiation field in the two-atom one-mode model with multiphoton transitions is investigated. The time-dependent squeezing factors are calculated. The conditions for the optimum squeezing are shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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