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VIOLATION OF THE CAUCHY-SCHWARZ INEQUALITY IN COLLECTIVE RESONANCE FLUORESCENCE

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In many systems involving the interaction between light and medium, the quantum statistical properties of light are predicted to violate the classical inequalities. A review of such nonclassical effects in optics is given in the paper by Loudon[1]. The most well-known example of nonclassical effects is photon antibunching which has been predicted in[5-9] and experimentally observed in the resonance fluorescence[2-4] and recently in the spontaneous parametric down-conversion[10].

Other examples of nonclassical effects are the squeezing of light[11-14] and violation of the Cauchy-Schwarz (C-S) inequality[15-18]. The violation of C-S inequality has been observed by Clausser[15] in the radiation emitted in an atomic two-photon cascade. This effect has also been predicted in the two-photon laser[16] and parametric amplifier[17]. In this paper we show that the strong violation of the C-S inequality could be observed in the collective resonance fluorescence. Like the squeezing of the light[19], the violation of the C-S inequality is presented for the case of a large number of atoms and intense fluorescent fields, and in this sense this effect is a macroscopic quantum effect.

We consider N two-level identical atoms of the Dicke model which interact with a strong monochromatic deriving field and an emitted field in the context of collective resonance fluorescence[19-20] (see fig. 1).

In treating the external field classically and using the Born and Markov approximation with respect to the coupling of the system with the vacuum field and atomic reservoir, one can obtain a master equation for the reduced density matrix for the system alone in the form[26]

$$\frac{\partial \rho}{\partial t} = -i[\frac{\delta}{2}(J_{22}-J_{11}) + G(J_{12}+J_{21}), \rho] - \gamma_{21}(J_{21}J_{12}\rho - J_{12}\rho J_{21} + \text{H.c.}), \quad (1)$$

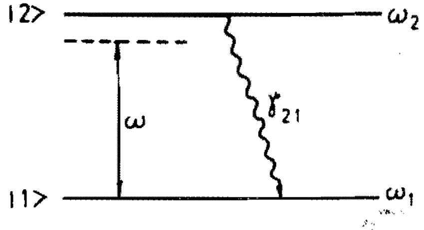

Fig. 1. Schematic representation of the two-level system interacting with the monochromatic incident field and with the emitted field.

where $2\gamma_{21}$ is radiative spontaneous transition probabilities per unit time for a single atom to change from excited level $|2\rangle$ to ground state $|1\rangle$; $G = -d_{21}E_o$ is the matrix element of the driving field and atom interaction; $\delta = \omega_{21} - \omega$ ($\omega_{21} = \omega_2 - \omega_1$; $\hbar \equiv 1$) is the frequency detuning of resonance and $J_{ij} = \sum_{k=1}^{N} |i\rangle_k {}_k\langle j|$ $(i, j = 1, 2)$ are the collective angular momenta of the atoms. They obey the commutation relation

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{ji'} - J_{i'j}\delta_{ij'}.$$

After the authors of the works[19,21], we introduce the Schwinger representation for the angular momentum

$$J_{ij} = C_i^{+} C_j \quad (i, j = 1, 2),$$

where C_i obey the boson commutation relation

$$[C_i, C_j^{+}] = \delta_{ij}.$$

Further, we investigate only the case of an intense external field or much detuning so that

$$\Omega = \left(\frac{1}{4}\delta^2 + G^2\right)^{1/2} \gg N\gamma_{21}. \tag{2}$$

After performing the canonical transformation

$$C_1 = Q_1 \cos\phi + Q_2 \sin\phi, \quad C_2 = -Q_1 \sin\phi + Q_2 \cos\phi, \tag{3}$$

where

$$\mathrm{tg}\, 2\phi = 2G/\delta,$$

one can find that the Liouville operator L appearing in equation (1) splits into two components L_o and L_1 . The component L_o is slowly varying in time whereas L_1 contains rapidly oscillating terms at frequencies 2Ω and 4Ω . For the case when relation (2) is fulfilled, one can make the secular approximation, i.e., to retain only the slowly varying part[19-20] . A correction to the results obtained in this fashion will be of an order of $(\gamma_{21}N/\Omega)^2$.

Making the secular approximation, one can find the stationary solution of the master equation (1) in the form[19]

$$\tilde{\rho} = Z^{-1} \sum_{N_1=0}^{N} X^{N_1} |N_1\rangle\langle N_1|, \tag{4}$$

where $\tilde{\rho} = U\rho U^{-1}$; here U is the unitary operators representing the canonical transformation (3)

$$X = \mathrm{ctg}^4\phi, \qquad Z = \frac{X^{N+1} - 1}{X - 1}.$$

The state $|N_1\rangle$ is an eigenstate of the operator R_{11} and $\hat{N} = R_{11} + R_{22}$ here $R_{ij} = Q_i^{+} Q_j$ $(i, j = 1, 2)$. The operators Q_i satisfy the boson commutation relation

$$[Q_i, Q_j^{+}] = \delta_{ij}, \tag{5}$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij'}\delta_{ji'} - R_{i'j}\delta_{ij'}. \tag{6}$$

In the case of resonance, i.e., when $X = 1$, solution (1) reduces to the solution by Agarwal[20].

By using eq.(4), the characteristic function can be defined as has been done by Louisell[22]

$$\chi_{R_{11}}(\xi) = \langle e^{i\xi R_{11}} \rangle_s = Z^{-1} \frac{Y^{N+1} - 1}{Y - 1},$$

where $Y = Xe^{i\xi}$, here $\langle A \rangle_s$ indicates the excitation value of an operator A in the steady state (4).

Once the characteristic function is known, it is easy to calculate the statistical moments

$$\langle R_{11}^n \rangle_s = \frac{\partial^n}{\partial (i\xi)^n} \chi_{R_{11}}(\xi) \Big|_{i\xi = 0} \tag{7}$$

In particular, we find

$$\langle R_{11} \rangle_s = Z^{-1}[NX^{N+2} - (N+1)X^{N+1} + X]/(X-1)^2, \tag{8}$$

$$\langle R_{11}^2 \rangle_s = Z^{-1}[N^2X^{N+3} - (2N^2+2N-1)X^{N+2} + (N+1)^2X^{N+1} - X^2 - X]/(X-1)^3 \tag{9}$$

$$\begin{aligned}\langle R_{11}^3 \rangle_s = Z^{-1}[&N^3X^{N+4} - (3N^3+3N^2-3N+1)X^{N+3} + (3N^3+6N^2-4)X^{N+2} - \\ &- (N^3+3N^2+3N+1)X^{N+1} + X^3 + 4X^2 + X]/(X-1)^4,\end{aligned} \tag{10}$$

$$\begin{aligned}\langle R_{11}^4 \rangle_s = Z^{-1}[&N^4X^{N+5} - (4N^4+4N^3-6N^2+4N-1)X^{N+4} - (6N^4+12N^3-6N^2 - \\ &- 12N+11)X^{N+3} - (4N^4+12N^3+6N^2-12N-11)X^{N+2} + (N^4+4N^3 + \\ &+ 6N^2+4N+1)X^{N+1} - X^4 - 11X^3 - 11X^2 - X]/(X-1)^5,\end{aligned} \tag{11}$$

By using the canonical transformation (3), one finds the collective angular moments J_{ij} in the form

$$J_{21} = D_3 \sin\phi \cos\phi + R_{21} \cos^2\phi - R_{12} \sin^2\phi ,$$
$$J_{12} = D_3 \sin\phi \cos\phi + R_{12} \cos^2\phi - R_{21} \sin^2\phi , \tag{12}$$

where $D_3 = R_{22} - R_{11}$.

It is easy to see[19,23] that the operators $D_3(t)\sin\phi\cos\phi$, $R_{21}(t)\cos^2\phi$ and $-R_{12}(t)\sin^2\phi$ can be considered as operator-sources of the spectrum components of the fluorescent field as frequencies ω, $\omega+2\Omega$ and $\omega-2\Omega$, and for simplicity these operators will be denoted by S_0^+, S_{+1}^+ and S_{-1}^+, respectively.

After Zubairy[16], we define the degree of second-order coherence of stationary fluorescent light to be

$$G^{(2)}_{i,j} = \frac{\langle S_i^+ S_j^+ S_j S_i \rangle_s}{\langle S_i^+ S_i \rangle_s \cdot \langle S_j^+ S_j \rangle_s} \quad (i,\, j = 0, \pm 1). \tag{13}$$

We speak that the violation of the Cauchy-Schwarz inequality is presented for the correlation between spectral components S_i and S_j if the following condition[16-18] is satisfied

$$K_{i,j} = (G^{(2)}_{i,i} \cdot G^{(2)}_{j,j})/(G^{(2)}_{i,j})^2 < 1 . \tag{14}$$

The relation (14) for the factor $K_{i,j}$ describes the violation of the Cauchy-Schwarz inequality.

For the one-atom case, after the works[23-24], we have

$$G^{(2)}_{1,1} = G^{(2)}_{-1,-1} = 0; \quad G^{(2)}_{oo} = 1, \quad G^{(2)}_{o,1} = G^{(2)}_{1,o} = G^{(2)}_{-1,o} = G^{(2)}_{o,-1} = 1,$$
$$G^{(2)}_{1,-1} > 1; \quad G^{(2)}_{-1,1} > 1.$$

and it follows that $K_{i,j} = 0$ for $i \neq j$ $(i, j = 0, \pm 1)$. It means that the inequality is violated for any two spectrum components from Mollow's triplet[25] of the one-atom resonance fluorescence. Further, we shall discuss the violation of the C-S inequality in the collective resonant fluorescence field. The following calculations show that in the collective case $(N \geq 2)$, the violation of the Cauchy-Schwarz inequality is presented only for two sidebands of the fluorescent spectrum. By using solution (4) and commutation relations (5-6), one can write

$$K_{1,-1} = \frac{\langle R_{21}R_{21}R_{12}R_{12}\rangle_s \cdot \langle R_{12}R_{12}R_{21}R_{21}\rangle_s}{(\langle R_{21}R_{12}R_{21}R_{12}\rangle_s)^2} , \tag{15}$$

$$K_{-1,1} = \frac{\langle R_{21}R_{21}R_{12}R_{12}\rangle_s \cdot \langle R_{12}R_{12}R_{21}R_{21}\rangle_s}{(\langle R_{12}R_{21}R_{12}R_{21}\rangle_s)^2} , \tag{16}$$

where

$$\langle R_{12}R_{12}R_{21}R_{21}\rangle_s = \langle R_{11}^4\rangle_s - 2(N+2)\langle R_{11}^3\rangle_s + (N^2+5N+5) \times \langle R_{11}^2\rangle_s - (N^2+3N+2)\langle R_{11}\rangle_s , \tag{17}$$

$$\langle R_{21}R_{21}R_{12}R_{12}\rangle = \langle R_{11}^4\rangle_s - (2N-4)\langle R_{11}^3\rangle_s + (N^2-7N+5) \times \langle R_{11}^2\rangle_s + (3N^2-7N+2)\langle R_{11}\rangle_s + 2N^2 - 2N , \tag{18}$$

$$\langle R_{21}R_{12}R_{21}R_{12}\rangle_s = \langle R_{11}^4\rangle_s - (2N-2)\langle R_{11}^3\rangle_s + (N^2-4N+1) \times \langle R_{11}^2\rangle_s + (2N^2-2N)\langle R_{11}\rangle_s + N^2 , \tag{19}$$

$$\langle R_{12}R_{21}R_{12}R_{21}\rangle_s = \langle R_{11}^4\rangle_s - 2(N+1)\langle R_{11}^3\rangle_s + (N+1)^2 \langle R_{11}^2\rangle_s , \tag{20}$$

here the statistical moments $\langle R_{11}^n\rangle_s$ $(n = 1,2,3,4)$ can be found in eqs.(8-11). The behaviour of the factors $K_{-1,1}$ and $K_{+1,-1}$ as functions against the parameter $X = \mathrm{ctg}^4\phi$ is shown in Figs.2 and 3, respectively. In contrast with the case of two-photon laser[16] where the effect of violation of C-S inequality is very small (of an order of $(\langle n_1\rangle + \langle n_2\rangle)^{-1}$ where $\langle n_1\rangle$ and $\langle n_2\rangle$ are mean photon numbers), the violation of the C-S inequality for

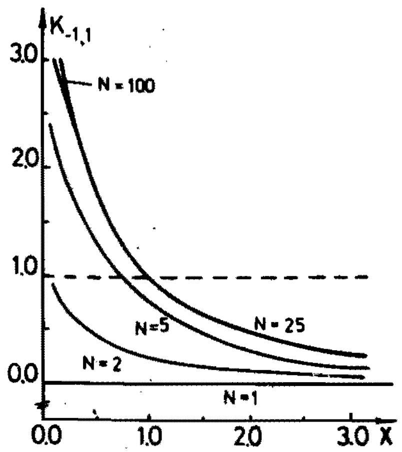

Fig. 2. Factor $K_{-1,+1}$ *as a function of the parameter* $X = \mathrm{ctg}^4\phi$.

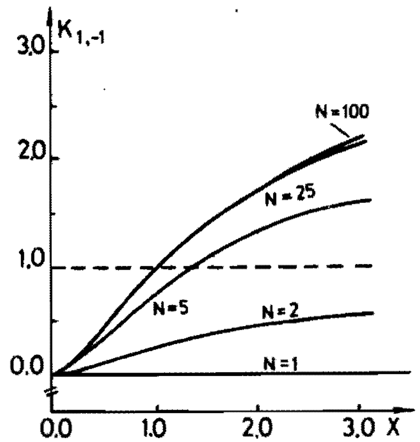

Fig. 3. Factor $K_{1,-1}$ *as a function of the parameter* $X = \mathrm{ctg}^4\phi$.

two sidebands of the collective resonance fluorescence, as is shown in figs. 2-3, is strong and is present for the case of many atoms. In this sense the violation of the C-S inequality is a macroscopic quantum effect. The examining of the violation of the C-S inequality in the collective resonance fluorescence expands the applicability of the test of quantum electrodynamics.

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Боголюбов Н.Н./мл./, Шумовский А.С., Чан Куанг E17-87-25
Нарушение неравенства Коши-Шварца
в коллективной резонансной флуоресценции

Обсуждено нарушение неравенства Коши-Шварца в коллективной резонансной флуоресценции. Показано сильное нарушение этого неравенства для двух крайних спектров коллективного флуоресцентного поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bogolubov N.N.Jr., Shumovsky A.S., Tran Quang E17-87-25
Violation of the Cauchy-Schwarz Inequality
in Collective Resonance Fluorescence

The violation of the Cauchy-Schwarz inequality in the collective resonance fluorescence is discussed. The strong violation of this inequality for the two sidebands of the collective fluorescent spectrum is shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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