Z.K.Petru

ANISOTROPY OF RELAXATION
OF ACOUSTIC PHONONS SCATTERED
BY ISOTOPIC IMPURITIES

1. Introduction

The phonon Boltzmann equation (PBE) has, in general, the form of a nonlinear integro-differential equation /1/ and usually does not allow an exact treatment. The number of solvable models is very limited, and they are based on rather drastic assumptions about the collision integral. We mention the instructive Claro-Wannier model describing relaxation of acoustic phonons towards equilibrium due to anharmonic interactions /2/. In this lecture we shall consider another mechanism of relaxation which provides the solvable PBE, namely, the relaxation due to the scattering of acoustic phonons by isotopic impurities. This idea comes from recent experiments on the ballistic phonon propagation in monocrystals at low temperatures. The samples used were so large and so chemically and structurally perfect that phonons felt isotopic impurities as main scatterers /3/. Then, it seems quite resonable to consider the model in which phonons in their relaxation process are scattered only by isotopic impurities (strictly speaking, only by mass differences) /4,5/.

2. The model

The object under consideration is a phonon gas in an anisotropic homogeneous elastic continuum. Now phonons are equivalent to sound wave quanta of frequencies $\omega(\vec{k}j) = |\vec{k}|\, c(\hat{\vec{k}}j)$, with sound velocities $c(\hat{\vec{k}}j)$ depending, in general, on a wave-vector direction given by the unit vector $\hat{\vec{k}} = \vec{k}/|\vec{k}|$, and on the phonon branch index j . We shall consider two kinds of the disturbances of the phonon distribution: one which retains symmetry of the system and the other one which lowers it. In both the cases the space homogeneity is assumed. The relaxation of phonon distribution function $N(\vec{k}j;t)$ is governed by PBE of the form

$$\frac{\partial}{\partial t} N(\vec{k}j;t) = \frac{v}{(2\pi)^3} \sum_{j'} \int d\vec{k}'\, w(\vec{k}j';\vec{k}j)\, \delta(\omega(\vec{k}j') - \omega(\vec{k}j))$$
$$[N(\vec{k}j';t) - N(\vec{k}j;t)] \quad ,$$

(1)

where v is the volume of a unit cell. The probability per unit time, w, that phonon is scattered elastically from a $\vec{k}_j$ -state into a $\vec{k}'_{j'}$ -state for the sample with randomly distributed isotopic impurities of small concentration was calculated by Klemens [6] (see also [5])

$$W(\vec{k}'_{j'}; \vec{k}_j) = \frac{\pi}{2} g\, \omega(\vec{k}'_{j'})\, \omega(\vec{k}_j)\, |\vec{e}(\vec{k}'_{j'}) \cdot \vec{e}(\vec{k}_j)|^2, \tag{2}$$

where $\vec{e}$ is the normalized polarization vector $(|\vec{e}(\vec{k}_j)|^2 = 1)$.
Here $g = \sum_i f_i (1 - m_i/m)^2$ where f_i is a fraction of unit cells with the mass m_i and m is the averaged mass of the unit cell, $m = \sum_i f_i m_i$. Let us note that in the case of low-energy phonons the formula (2) is valid also for multiatomic crystals. Here we would like to stress that scattering of high-energy phonons allows only the numerical treatment [4,5].

In the system described by (1) the total energy as well as the total number of phonons are conserved. Moreover, the energy of an individual phonon scattered elastically by a mass defect is conserved, i.e. $\omega(\vec{k}_j) = \omega(\vec{k}'_{j'}) \equiv \omega$. Thus the number of phonons with given energy $\hbar\omega$ is also constant. We can then label the long-wave phonon by its frequency ω, wave-vector direction $\hat{k}$, and branch j to which it belongs, i.e. $\{\vec{k}, j\} = \{\omega, \hat{k}, j\}$. So the integration over the wave vector $\vec{k}'$ in collision integral is reduced merely to the integration over the solid angle $\Omega' \equiv \Omega(\hat{k}')$.

Let us now define the averaged value of the distribution function according to the formula

$$\langle N(\omega; t) \rangle \equiv \frac{1}{3} \sum_j \int \frac{d\Omega}{4\pi} \left(\frac{c_D}{c(\hat{k}_j)} \right)^3 N(\omega \vec{k}_j; t), \tag{3}$$

where c_D is the Debye velocity given as usual by

$$c_D^{-3} \equiv \frac{1}{3} \sum_j \int \frac{d\Omega}{4\pi} c^{-3}(\hat{k}_j). \tag{4}$$

From the fact just mentioned that the number of phonons with a given energy $\hbar\omega$ is conserved it follows that (3) is constant in time

$$\langle N(\omega; t) \rangle = \langle N(\omega; t=0) \rangle \equiv \overline{N}(\omega) = \text{constant}. \tag{5}$$

It means that all phonons of energy $\hbar\omega$ relax towards their average value $\overline{N}(\omega)$, and so we are looking for a deviation from $\overline{N}(\omega)$ defined as

$$n(\omega \hat{k}_j; t) \equiv N(\omega \hat{k}_j; t) - \overline{N}(\omega). \tag{6}$$

Since the phonon frequency is a constant of motion and appears only as a fixed parameter, it will be omitted thereafter.

3. Symmetry of the collision integral

As we have already mentioned, the analytical solution of PBE depends on the symmetry of a system because the form of the collision integral is symmetry-dependent. To see this, let us first consider the collision-out term of PBE. The factor in front of $N(\vec{k}_j; t)$ on the right-hand side of Eq.(1), usually called the collision rate or the reciprocal of the collision time, can be written for long-wave phonons as follows

$$\nu(\vec{k}_j) = \frac{v g}{4\pi} \omega^4 \sum_{j'} \int \frac{d\Omega'}{4\pi} c^{-3}(\hat{k}'_{j'}) \left(\vec{e}(\hat{k}_j) \cdot \vec{e}(\hat{k}'_{j'}) \right)^2. \tag{7}$$

Spatial anisotropy of the collision rate is due to its dependence on polarization vectors. The right-hand side of Eq.(3) contains the following tensor

$$f_{\alpha\beta} \equiv \frac{1}{3} \sum_{j'} \int \frac{d\Omega'}{4\pi} \left(\frac{c_D}{c(\hat{k}'_{j'})} \right)^3 e_\alpha(\hat{k}'_{j'}) e_\beta(\hat{k}'_{j'}), \tag{8}$$

whose trace is normalized to unity. A similar tensor appears in the collision-out term of (1.) but now it includes the distribution function

$$F_{\alpha\beta}(t) \equiv \frac{1}{3} \sum_{j'} \int \frac{d\Omega'}{4\pi} \left(\frac{c_D}{c(\hat{k}'_{j'})} \right)^3 e_\alpha(\hat{k}'_{j'}) e_\beta(\hat{k}'_{j'}) n(\hat{k}'_{j'}; t). \tag{9}$$

The tensor (9) is traceless due to (6). The number of nonvanishing tensor components of $f_{\alpha\beta}$ and $F_{\alpha\beta}$ depends on the symmetry of the phonon system.

3.1. Cubic symmetry

For the elastic continuum having the cubic symmetry the tensor (8) is proportional to the unit matrix and the tensor (9) simply vanishes

$$f_{\alpha\beta} = \frac{1}{3}\delta_{\alpha\beta} \quad ; \qquad F'_{\alpha\beta}(t) \equiv O . \tag{10}$$

By inserting the first of Eqs.(10) into (7) we find only one collision rate common for all directions $\hat{\vec{k}}$ and branches j, given by /5/

$$\nu(\hat{\vec{k}}j) = \frac{v\,g}{4\pi}\cdot\frac{\omega^4}{c_D^3} \equiv \nu_c . \tag{11}$$

Thus we come to the conclusion that the spatial isotropy characterizes the scattering of acoustic long-wavelength phonons by mass defects in cubic crystals.

3.2. Tetragonal symmetry

In this case the matrix (8) is diagonal with three nonvanishing elements $f_{xx} = f_{yy}$ and f_{zz}, where we have taken the fourfold axis along the z-axis. Because, in general, $f_{xx} \neq f_{zz}$ we should, in addition to the Debye velocity, introduce another constant, say f_{zz}. Having noticed that $f_{xx} = f_{yy} = \frac{1}{2}(1 - f_{zz})$ we can rewrite the collision rate (7) in a more convenient form /7/

$$\nu(\hat{\vec{k}}j) = \nu_c\,\frac{3}{\Gamma+2}\left[1 + (\Gamma-1)\,e_z^2(\hat{\vec{k}}j)\right] , \tag{12}$$

where we have introduced the anisotropy parameter Γ defined as

$$\Gamma \equiv f_{zz}/f_{xx} . \tag{13}$$

It means that for a symmetry lower than cubic the collision rate is spatially anisotropic. The anisotropy comes from the dependence of the z-component of the polarization vector on the direction $\hat{k}$ and is different for different modes j. For crystals with the cubic symmetry as well as for isotropic media $f_{xx} = f_{zz}$ and $\Gamma = 1$, so we return to the result (11) as it should be.

4

A more important consequence of lowering of symmetry is that the tensor $F_{\alpha\beta}$ has nonzero components. For the elastic continuum of tetragonal symmetry the nonvanishing components (i.e. $F_{xx} = F_{yy}$ and F_{zz}) are connected with each other for the tensor is traceless. So we may choose one of the components, say F_{zz}, and denote it, for brevity, by F

$$F(t) \equiv F_{zz}(t) = -F'_{xx}(t) = -F_{yy}(t) \neq O . \tag{14}$$

As a result of (14) the PBE for a phonon system with tetragonal symmetry preserves its form of integro-differential equation .

3.3. Hexagonal symmetry

All results of the previous section are also valid for the anisotropic continuum of hexagonal symmetry. But here additional simplifications take place. The z-component of the polarization vector of pure transverse phonons, $j = T$, vanishes for any $\vec{k}$ by definition (see /8/). As a result, equation (12) for this mode is reduced to /7/

$$\nu_T = \frac{3}{\Gamma+2}\,\nu_c . \tag{15}$$

It means that ν_T is isotropic and differs from the cubic case (11) only by a factor which tends to unity when Γ approaches 1. For quasilongitudinal and quasitransverse modes the formula (12) is still valid but now with e_z depending only on $\hat{k}_z$ and j /7/. Simplifications just mentioned appear as a result of the symmetry. Namely, the anisotropic continuum of hexagonal symmetry is transversally isotropic /8/. Having this we are able to calculate the anisotropy parameter Γ for all hexagonal crystals for which elastic constants are known (Table). It is also easy to analyse the spatial anisotropy of the scattering rate (12). In the Figure two cases are presented: hcp helium-4 crystal for which $\Gamma < 1$ and hcp zinc with $\Gamma > 1$. For further analysis it is important to note that for $j \neq T$ the scattering rate changes monotonically with $\hat{k}_z$ and reaches extremum at $\hat{k}_z = O$.
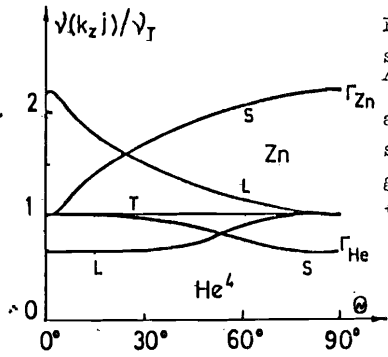
5

Fig.1. Relative value of the phonon scattering rate of two hcp crystals: ^{4}He ($\Gamma < 1$) and Zn ($\Gamma > 1$) versus the angle between wave-vector direction and sixfold axis. L, T, and S are quasilongitudinal, pure transverse, and quasitransverse modes, respectively.

4. Exact solutions of the phonon Boltzmann equation

As we have just shown, the symmetry of the system determines the final form of PBE. Now we are able to show how the solution of PBE depends both on a symmetry of a crystal itself and on a symmetry of a disturbance.

4.1. Relaxation in crystals of cubic symmetry

Let us first consider such a disturbance of the phonon distribution which does not lower its cubic symmetry. For this case the tensor (9) vanishes, $F_{\alpha\beta}(t) \equiv 0$. This essentially simplifies the collision integral, and PBE is reduced to /9/

$$\left(\frac{\partial}{\partial t} + \nu_c\right) n(\hat{\vec{k}}_j; t) = 0 .$$

(16)

Thus, in this case the exact solution of PBE is

$$n(\hat{\vec{k}}_j; t) = n(\hat{\vec{k}}_j; t=0)\, e^{-t\nu_c} .$$

(17)

In the general case equation of the form (16) is assumed in the relaxation time approximation (RTA) /6/, where the collision integrall is changed to $\nu_c \cdot n(\hat{\vec{k}}_j; t)$.

As a next example let us consider a crystal of cubic symmetry with such a disturbance of the phonon distribution function which lowers its symmetry to tetragonal. It can be realized by generation of homogeneous flows of phonons propagating in parallel with one of a fourfold axis (say, z-axis). The tensor $F_{\alpha\beta}$ containing

such disturbed distribution function should reflect the tetragonal symmetry (see Eq.(14)). It means, that PBE preserves now the form of integro-differential equation

$$\left(\frac{\partial}{\partial t} + \nu_c\right) n(\hat{\vec{k}}_j; t) = \frac{3}{2}\nu_c\left(3 e_z^2(\hat{\vec{k}}_j) - 1\right) F(t) .$$

(18)

Multiplying both sides of (18) by $e_z^2(\hat{\vec{k}}_j)$ and averaging them according to the definition (9) we obtain the equation determining the integral term

$$\left(\frac{\partial}{\partial t} + (1-\alpha)\nu_c\right) F(t) = 0 .$$

(19)

Its solution decays exponentially

$$F(t) = F(0)\, e^{-t(1-\alpha)\nu_c} .$$

(20)

The parameter α will be defined later on. Substitution of (20) into (18) gives us the final result in the form

$$n(\hat{\vec{k}}_j; t) = n(\hat{\vec{k}}_j; t=0)\, e^{-t\nu_c}$$

$$+ \frac{3}{2}\frac{F(t=0)}{\alpha}\left(3 e_z^2(\hat{\vec{k}}_j) - 1\right)\left(e^{-t(1-\alpha)\nu_c} - e^{-t\nu_c}\right) .$$

(21)

The first term on the right-hand side of (21) is nothing but the result usually obtained from RTA. The second term comes out as a consequence of the exact treatment of the collision-in part of the collision integral. Let us first note that this part of the solution vanishes for some directions in the $\vec{k}$-space, namely, for those which fulfil the condition $e_z^2(\hat{\vec{k}}_j) = 1/3$. For any other phonons the time evolution of the distribution function is described by two relaxation times. In addition to $\tau_c \equiv 1/\nu_c$ a new relaxation time appears

$$\tau_*^{-1} \equiv \nu_* \equiv (1-\alpha)\nu_c ,$$

(22)

with the parameter α given by

$$\alpha \equiv \frac{3}{2}\left(3 f_{zzzz} - f_{zz}\right) .$$

(23)

The symmetric tensor of the fourth rank, $f_{\alpha\beta\gamma\delta}$ is defined in the same way as (8) but with four polarization-vector components instead of two. It follows from the definition of α that for the crystals of cubic symmetry $1/9 < f_{zzzz} < 1/3$ or, equivalently, $0 < \alpha < 1$. The inequality $\alpha < 1$ quarantees the stability of the solutions (20) and (21). Positivity of α leads to the following inequality

$$\tau_* > \tau_c \ .$$

(24)

We can conclude from (21) and (24) that the disturbance lowering the symmetry of the phonon distribution leads to elongation of the asymptotic relaxation time.

It is worth to consider, as an example, such a disturbance which does not change the distribution of some phonons whereas the symmetry of the distribution functions of any other phonons is lowered from, say, cubic to tetragonal. In other words, the initial condition for these undisturbed phonons is given by $n(\vec{k}j;t=0) = 0$. Such phonons cannot be described in the framework of the relaxation time approximation. However, within our description their distribution function is given by the second term on the right-hand side of (21). As a function of time it starts to grow from zero, reaches its extremum at $t_{ex} = -(\tau_c/\alpha)\ln(1-\alpha)$ which is of the order of τ_c , and its asymptotical behaviour is determined by the (longer) relaxation time τ_* .

Let us finally notice that the results just obtained remain also valid for an isotropic elastic medium (instead of the cubic one) with the initial distribution function of the axial symmetry. In this simplest case using only symmetry arguments we get $f_{zzzz} = 1/5$ so that $\alpha = 2/5$ and $\tau_* = \frac{5}{3}\tau_c$.

4.2. Relaxation in crystals of tetragonal symmetry

As we have shown in Section 3.2, in a crystal of a symmetry lower than cubic the collision rate is anisotropic. If we disturb the phonon distribution in such a crystal we shall obtain that $F(t) \neq 0$. So, in the case of elastic continuum of tetragonal symmetry the relaxation of the disturbance which does not lower the initial symmetry is governed by PBE having the form

$$\left(\frac{\partial}{\partial t} + \nu(\vec{k}_j)\right) n(\vec{k}_j;t) = \frac{3}{2}\nu_c(3e_z^2(\vec{k}_j)-1) F(t) \ .$$

(25)

The integro-differential equation (25) can also be treated exactly. The discussion can be made more simple by transforming (25) into a double-integral equation of the form

$$n(\vec{k}_j;t) = n(\vec{k}_j;t=0) e^{-t\nu(\vec{k}_j)}$$
$$+ \frac{3}{2}\nu_c(3e_z^2(\vec{k}_j)-1)\int_0^t e^{-(t-t')\nu(\vec{k}_j)} F(t') dt' \ .$$

(26)

Now we should find an equation determining the unknown integral $F(t)$. As in the previous section, we can get it easily by substitution of (26) into the definition (9). We have

$$F(t) = g(t) + \int_0^t K(t-t') F(t') dt' \ ,$$

(27)

where, for brevity, we have denoted the new integral kernel by

$$K(t) \equiv \frac{\nu_c}{2}\sum_j\int\frac{d\Omega}{4\pi}\left(\frac{c_D}{c(\vec{k}_j)}\right)^3 e_z^2(\vec{k}_j)(3e_z^2(\vec{k}_j)-1) e^{-t\nu(\vec{k}_j)} \ ,$$

(28)

and the inhomogeneous term by

$$g(t) \equiv \frac{1}{3}\sum_j\int\frac{d\Omega}{4\pi}\left(\frac{c_D}{c(\vec{k}_j)}\right)^3 e_z^2(\vec{k}_j) n(\vec{k}_j;t=0) e^{-t\nu(\vec{k}_j)} \ .$$

(29)

In this way we have reduced the PBE to a set of two inhomogeneous Volterra integral equations of the second kind with kernels depending only on the difference of arguments. Thus, equation (27) is simply an integral equation of the convolution type. The solution of (27) can be put in the standard form /10/

$$F(t) = g(t) + \int_0^t R(t-t') g(t') dt' \ ,$$

(30)

with the resolvent $R(t)$ given as the inverse Laplace transform

$$R(t) = \int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{dz}{2\pi i}\frac{\widetilde{K}(z)}{1-\widetilde{K}(z)} e^{zt} \ .$$

(31)

Here $\widetilde{K}(z)$ is the Laplace transform of the kernel $K(t)$. Moreover, let us note that $z/(1-z)$ as a function of the complex variable is analytic in the neighbourhood of zero including the point $z=0$ at which it vanishes. The last two statements in accordance with the theorem presented in /11/ guarantee the existence of the inverse Laplace transform (31). Equations (26)-(31) give the general solution of PBE (25). We note that the resolvent (31) depends only on analytical properties of the Laplace transform of the kernel $\widetilde{K}(z)$ and not on initial conditions. The slow relaxation time is given by this value of z at which $\widetilde{K}(z) = 1$. The last condition is fully determined by internal phonon characteristics of a given crystal sample: sound velocities, polarization vectors, and collision rates.

Let us note that the general solution (26)-(31) has exactly the same form also for a hexagonal symmetry and therefore we shall present a detailed discussion of (26)-(31) in the next section.

4.3. Relaxation in crystals of hexagonal symmetry

The elastic continuum of hexagonal symmetry is the only anisotropic medium for which the Christoffel equation can be solved analytically, and formulae for sound velocities and polarization vectors can be written down in an explicit form /8,7/. Hence, we can determine the value of the anisotropy parameter Γ (13) and the behaviour of relaxation rates (12) and (15) as functions of the wave vector and branch index. Having these we can study analytical properties of the Laplace transform of the resolvent (31). The singularities of it in the complex z-plane are the following:

(a) A cut along the spectrum of collision rate, $z = -\nu(\hat{k}_z, j)$, i.e., from $z = -\nu_T$ to $z = -\Gamma\nu_T$, due to the integral of the Cauchy-type in $\widetilde{K}(z)$;

(b) An isolated pole located at a point $z = -\nu_*$ at which $\widetilde{K}(z) = 1$. It is important to underline that $0 < \nu_* < \nu_T$ for $\Gamma > 1$ or $0 < \nu_* < \Gamma\nu_T$ for $\Gamma < 1$. To see this, let us recall that $\nu(\hat{k}_z, j)$ is a monotonic function of $\hat{k}_z$ for $j \neq T$ (see the Figure) and so $\widetilde{K}(z)$ is also monotonic for $z = x > \max\{-\nu_T, -\Gamma\nu_T\}$. Next, we should check that for $\Gamma > 1$ the following inequalities take place: $\widetilde{K}(0) < 1$ and $\widetilde{K}(-\nu_T) > 1$ (for $\Gamma < 1$ instead of the latter inequality we have $\widetilde{K}(-\Gamma\nu_T) = +\infty$).

As a consequence of the above set of singularities, $R(t)$

consists of two contributions: first contribution comes from the integral along the cut and the second one originates from the pole at $z = -\nu_*$

$$R(t) = a_* e^{-t\nu_*} + \frac{\nu_c}{2} \sum_j \int \frac{d\Omega}{4\pi} \left(\frac{c_D}{c(\hat{k}_z j)}\right)^3 e_z^2(\hat{k}_z j)(3e_z^2(\hat{k}_z j) - 1) \cdot$$
$$\cdot |\widetilde{K}(-\nu(\hat{k}_z j) + i0)|^{-2} e^{-t\nu(\hat{k}_z j)}, \quad (32)$$

where $a_* = \lim_{z \to -\nu_*} (z + \nu_*)/(1 - \widetilde{K}(z))$.

We get the solution of PBE (25) by substituting the resolvent (32) into (30) and then $F(t)$ into (26). The final solution for the phonon distribution function consists, in general, of three terms: (i) a RTA-like term proportional to $\exp\{-t\nu(\hat{k}_z j)\}$, (ii) the integral along the cut leading to exponential decay with the effective decay time being between $1/\nu_T$ and $1/\Gamma\nu_T$, i.e. within the spectrum of $\tau(\hat{k}_z j) \equiv 1/\nu(\hat{k}_z j)$; and (iii) a qualitatively new term decaying exponentially with a new relaxation time $\tau_* \equiv 1/\nu_* > 1/\nu_T$ for $\Gamma > 1$ or $\tau_* > 1/\Gamma\nu_T$ for $\Gamma < 1$. Numerical values presented in the Table show that τ_* is a few times as large as τ_T.

Table. Anisotropy parameter Γ and the ratio of the relaxation time τ_* to τ_T for crystals of hexagonal symmetry

	Be	C	Cd	Hf	He[4]	H$_2$	Mg
Γ	0.847	51.46	2.117	0.920	0.647	0.821	0.945
τ_*/τ_T	2.8	11.1	2.6	3.3	5.3	6.7	3.57

	Re	Ti	Tl	Y	Zn	Zr	
Γ	0.924	0.849	0.385	0.965	2.215	0.997	
τ_*/τ_T	4.2	3.7	6.2	3.6	2.1	3.8	

For $t \gg \tau_T$ relaxation is dominated by the term (iii) which has the form

$$\frac{1}{2}\nu_c a_*(3e_z^2(\hat{k}_z j) - 1)\sum_{j'} \int \frac{d\Omega'}{4\pi}\left(\frac{c_D}{c(\hat{k}_z' j')}\right)^3 \frac{e_z^2(\hat{k}_z' j') n(\hat{k}_z' j'; t=0)}{\nu_* - \nu(\hat{k}_z' j')}.$$

$$\cdot\left(\frac{e^{-t\nu_*}-e^{-t\nu(\hat{k}_{zj})}}{\nu_*-\nu(\hat{k}_{zj})}-\frac{e^{-t\nu(\hat{k}'_{zj'})}-e^{-t\nu(\hat{k}_{zj})}}{\nu(\hat{k}'_{zj'})-\nu(\hat{k}_{zj})}\right). \quad (33)$$

Let us note that the new relaxation time τ_* is the same for all $\hat{k}$ and j. The only exceptions are those wave vectors which fulfil the condition $3e_z^2(\hat{k}_{zj})=1$. With the last condition the terms (ii) and (iii) vanish (see (26)). The corresponding modes decay exponentially with a decaying time exactly equal to τ_c (Eq.(11) but with c_D taken for crystals of hexagonal symmetry). For the case of hexagonal symmetry the wave vectors of these modes form two cones, one for a longitudinal mode, the other for a slow transverse mode. Let us add that for some crystals the cone for the latter mode intersects the surface of constant energy almost at the same points as the cone of collinear points does. For example for hcp ^{4}He the corresponding angles are equal 44° and 48°, respectively, and for hcp Zn 37° and 36°, respectively.

References:

1. R.E.Peierls, Quantum Theory of Solids(Oxford,Clarendon Press,1955).
2. F.H.Claro and C.H.Wannier, J.Math.Phys. 12, 92 (1971).
3. R.G.Ulbrich, V.Narayanamurti, and M.A.Chin, Phys.Rev.Lett., 45, 1432 (1980).
4. M.Lax, V.Narayanamurti, P.Hu, and W.Weber, J.Phys. (Paris), Colloq. 42, C6-161 (1981).
5. S.Tamura, Phys.Rev. B27, 858 (1983); B30, 849 (1984).
6. P.G.Klemens, in: Solid State Physics, F.Seitz and D.Turnbull, eds. (Academic Press, New York, 1958), vol.7.
7. Z.K.Petru, in: Proc. 2nd Int.Conf. on Phonon Physics, J.Kollàr, N.Croò, N.Menyhàrd, and T.Siklòs, eds. (World Scientific, Singapore, 1985).
8. F.I.Fedorov, Theory of Elastic Waves in Crystals (Plenum, New York 1968).
9. Z.K.Petru, Communication of JINR, No. E17-86-632 (1986).
10. E.C.Titchmarsh, Introduction to the Theory of Fourier Integrals (Oxford, Clarendon Press, 1948);G.Doetsch,Anleitung zum Praktischen Gebrauch der Laplace-Transformation and Z-Transformation (Oldenbourg, Munchen, 1967).
11. G.Doetsch, Handbuch der Laplace-Transformation (Verlag Birkhauser, Basel, 1950), Vol. 1, Chap. 8.

Петру З.К.   E17-87-236
Анизотропия релаксации акустических фононов, рассеянных изотопическими примесями

Для случая пространственно-однородных систем получены точные решения фононного уравнения Больцманна, описывающего релаксацию акустических фононов, рассеянных изотопическими примесями. Решения оказываются зависящими от симметрии системы. Если симметрия кристалла или симметрия неравновесного распределения фононов ниже кубической, тогда эффективное время релаксации удлиняется по сравнению с величиной, полученной в,так называемом, приближении времени релаксации.

Petru Z.K.   E17-87-236
Anisotropy of Relaxation of Acoustic
Phonons Scattered by Isotopic Impurities

Exact solutions of the phonon Boltzmann equation describing relaxation of acoustic phonons scattered by isotopic impurities are found for the case of spatially homogeneous systems. The solutions depend on the symmetry of the system. If the symmetry of a crystal or the symmetry of the disturbed phonon distribution function is lower than cubic, the effective relaxation time is elongated as compared to the value obtained within the relaxation-time approximation. This new relaxation time is strongly direction-dependent.