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PHOTON STATISTICS IN AN N-LEVEL (N-1)-MODE SYSTEM

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1. INTRODUCTION

In recent years, much interest has been paid to the exactly solvable in the rotating wave approximation (RWA) the Jaynes-Cummings model (J-C)[1] of a two-level atom interacting with a single mode of the quantized field radiation[2-10]. An infinite sequence of quantum collapses and revivals of Rabi oscillations in the model have been revealed by Eberly et al[2,3] and Knight and Radmore[4]. Singh[5] has studied photon statistical properties of the system. Possible exactly solvable generalizations of this model consist for example in taking into account multiphoton transitions as well as in considering an intensity dependent atom-mode coupling[5-10].

Another form of generalization of the J-C model deals with adding other levels. The first in the hierarchy of generalized models, leading to qualitative differences connected with the existence of two branches of the Rabi frequency comparing to the J-C model, is a three-level atom two-mode system. Such a system with one-photon transitions between the atomic levels has been studied by Li et al.[11,12] and Bogolubov,Jr. et al.[13-17]. The former[11,12] have presented the explicit expression of the evolution operator in the interaction picture and then they have found the mean statistical values of the level populations. Bogolubov,Jr.,et al.[13-17] have given the rigorous examination of the dynamical behaviour of the level populations and photon numbers in the Heisenberg picture. Multiphoton transitions in such a system have been considered as well[18]. Moreover, in the papers[15,17,18] statistical properties of the photons interacting with the three-level atom have been investigated.

An excellent review of the dynamical theory of γ - C - type models has recently been given by Yoo and Eberly[19]).

It has also appeared possible to obtain the rigorous operator solutions for an N -level atom the N-th level of which is coupled with the rest lower levels by N-1 modes of the radiation field[20-21]). Recently, Kotchetov[22]) has solved the problem of energy spectrum of an N -level atom in the case when its lower level is coupled with close to each other N -1 rest levels by a single near-resonant mode.

In the present paper we would like to obtain information about photon statistics in the case of the N-level atom immersed in a lossless cavity and interacting with the N -1 resonant modes (fig.1). Although, as previous calculations showed[20,21]), such a model leads rather to quantitative differences only comparing to the three-level atom in the lambda configuration, the general solution for arbitrary N is interesting in its own right. First of all, this solution embraces those for the two-level one-mode system and for the three-level two-mode lambada system with two one-photon resonances.

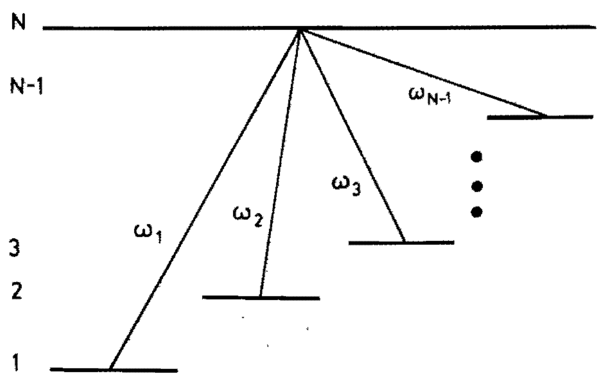

Fig. 1. Level structure

This three-level atom being initially on its lower level permits to study cross correlations between photons of the pumping and signal mode. The atom under consieration enables us to study intermodal correlations of two signal modes and to estimate the effect of the level number on the magnitude of the photon correlations.

The remainder of the paper is organized as follows. In chapters 2 and 3 the model Hamiltonian and the necessary previous results[21]) are given. In the next chapter we find general expressions for the photon-number statistical moments and correlations using the time-independent photon-number distribution function. In chapter 5 we present another possible solution to the problem of photon statistics in the system under consideration by finding the characteristic and time-dependent photon-number distribution functions. Finally, in section 6, the time behaviour of the normally ordered variances of the photon numbers and the cross correlations between signal modes are calculated.

2. MODEL HAMILTONIAN AND OPERATOR SOLUTIONS

The N -level atomic model considered here is shown in fig.1. The upper level N is coupled with the other N - 1 levels by one-photon dipole transitions whereas the mutual transitions between these lower levels are forbidden.

The Hamiltonian of the system in RWA is

$$\hat{H} = \sum_{\alpha=1}^{N-1} \hbar\omega_\alpha \hat{a}_\alpha^+ \hat{a}_\alpha + \sum_{i=1}^{N} \hbar\omega_{0i} \hat{R}_{ii} + \sum_{\alpha=1}^{N-1} \hbar g_\alpha \left(\hat{a}_\alpha \hat{R}_{N\alpha} + \hat{a}_\alpha^+ \hat{R}_{\alpha N} \right) \quad (1)$$

The operator $\hat{R}_{ii}$ represents the population of the level i with the energy $\hbar\omega_{0i}$ and the state vector $|i\rangle$. The operator $\hat{R}_{ij} = |i\rangle\langle j|$, $i \neq j$, describes the transition of the atom from the level j to the level i. The operators $\hat{R}_{ij}$, $i, j = 1, \ldots, N$, are generators of the unitary group SU(N) and obey the rules

$$\hat{R}_{ij}\hat{R}_{kl} = \hat{R}_{il}\delta_{jk}, \quad [\hat{R}_{ij}, \hat{R}_{kl}] = \hat{R}_{il}\delta_{jk} - \hat{R}_{kj}\delta_{il}. \tag{2}$$

Moreover, the following conservation law is satisfied:

$$\sum_{i=1}^{N} \hat{R}_{ii} = 1. \tag{3}$$

The photon creation $\hat{a}_\alpha^+$ and annihilation $\hat{a}_\alpha$ operators, $\alpha = 1, \ldots, N-1$, describe $N-1$ modes of the radiation field with resonant frequencies $\omega_\alpha = \omega_{ON} - \omega_{O\alpha}$, and g_α are the corresponding atom-mode coupling constants. The above operators satisfy the commutation relation

$$[\hat{a}_\alpha, \hat{a}_\beta^+] = \delta_{\alpha\beta}. \tag{4}$$

The photon number operator for the mode α we denote by $\hat{n}_\alpha$, $\hat{n}_\alpha = \hat{a}_\alpha^+ \hat{a}_\alpha$. Then, the photon-number statistical moments and the correlations between the modes are defined by $\hat{n}_\alpha^k$ and $\hat{n}_\alpha^k \hat{n}_\beta^l$ (k, l -arbitrary integers), respectively.

One can easily check that the following excitation number operators $\hat{\mathcal{N}}_\alpha$, $\hat{\mathcal{N}}_\alpha = \hat{n}_\alpha(t) - \hat{R}_{\alpha\alpha}(t)$, $\alpha = 1, \ldots, N-1$, commute with the Hamiltonian (1) (also with each other). They are therefore constants of motion

$$\hat{\mathcal{N}}_\alpha = \hat{n}_\alpha(t) - \hat{R}_{\alpha\alpha}(t) = \hat{n}_\alpha^0 - \hat{R}_{\alpha\alpha}^0, \tag{5}$$

where the upper symbol 0 following the operators denotes that they are taken at $t = 0$.

In the previous paper[21]) the operators $\hat{R}_{\alpha\alpha}(t)$ and $\hat{n}_\alpha(t)$ have been found explicitly and **rigorously**. They read

$$\hat{R}_{\alpha\alpha}(t) = -2\hat{\alpha}_\alpha \sin^2\tfrac{1}{2}\hat{\Omega}t + \hat{\beta}_\alpha \sin\hat{\Omega}t + \hat{\Omega}_\alpha^2 \hat{P}(t) + \hat{R}_{\alpha\alpha}^0, \quad \alpha = 1, \ldots, N-1 \tag{6}$$

$$\hat{n}_\alpha(t) = -2\hat{\alpha}_\alpha \sin^2\tfrac{1}{2}\hat{\Omega}t + \hat{\beta}_\alpha \sin\hat{\Omega}t + \hat{\Omega}_\alpha^2 \hat{P}(t) + \hat{n}_\alpha^0, \quad \alpha = 1, \ldots, N-1 \tag{7}$$

where the operator $\hat{P}(t)$ is

$$\hat{P}(t) = -2\hat{\alpha}\sin^2\hat{\Omega}t + \hat{\beta}\sin 2\hat{\Omega}t. \tag{8}$$

Moreover, with respect to the conservation law (3)

$$\hat{R}_{NN}(t) = -\hat{\Omega}^2 \hat{P}(t) + \hat{R}_{NN}^0. \tag{9}$$

The amplitude operators $\hat{\alpha}$, $\hat{\beta}$, $\hat{\alpha}_\alpha$ and $\hat{\beta}_\alpha$ are defined by the initial conditions and have the form

$$\hat{\alpha} = \frac{1}{2}\hat{\Omega}^{-4}\Big(\sum_{\beta=1}^{N-1}\hat{\Omega}_\beta^2 \hat{R}_{\beta\beta}^0 - \hat{\Omega}^2 \hat{R}_{NN}^0 + \sum_{\beta=1}^{N-1}\sum_{\gamma>\beta}^{N-1} g_\beta g_\gamma \hat{T}_{\beta\gamma}^0\Big),$$

$$\hat{\beta} = \frac{1}{2}\hat{\Omega}^{-3}\sum_{\beta=1}^{N-1}\dot{\hat{R}}_{\beta\beta}^0,$$

$$\hat{\alpha}_\alpha = \hat{\Omega}^{-4}\Big[2\hat{\Omega}_\alpha^2 \sum_{\beta=1}^{N-1}\hat{\Omega}_\beta^2(\hat{R}_{\alpha\alpha}^0 - \hat{R}_{\beta\beta}^0) + g_\alpha \hat{\Omega}^2 \sum_{\beta\neq\alpha}^{N-1} g_\beta \hat{T}_{\alpha\beta}^0 - \hat{\Omega}_\alpha^2 \sum_{\beta=1}^{N-1}\sum_{\gamma\neq\beta}^{N-1} g_\beta g_\gamma \hat{T}_{\beta\gamma}^0\Big] \tag{10}$$

$$\hat{\beta}_\alpha = \hat{\Omega}^{-3}\sum_{\beta=1}^{N-1}(\hat{\Omega}_\beta^2 \dot{\hat{R}}_{\alpha\alpha}^0 - \hat{\Omega}_\alpha^2 \dot{\hat{R}}_{\beta\beta}^0).$$

The $\hat{\Omega}_\alpha$ are operators of the one-photon Rabi frequency, and

$$\hat{\Omega}_\alpha = g_\alpha(\hat{\mathcal{N}}_\alpha + 1)^{1/2}, \tag{11}$$

whereas

$$\hat{\Omega}^2 = \sum_{\beta=1}^{N-1}\hat{\Omega}_\beta^2 \tag{12}$$

is the operator of the effective Rabi frequency.

mode at $t = 0$, i.e., at $n_i \neq 0$. In the opposite case $\Omega_i = 0$ and $R_i(i,\{n_\gamma\},t) = 1$ for any t what suggests that the atom would remain on the level i.

4. PHOTON-NUMBER STATISTICAL MOMENTS AND CORRELATIONS

The photon-number statistical moments and the correlations between the modes are by definition

$$\langle \hat{n}_\alpha^k(t) \rangle = Tr\, \hat{n}_\alpha^k(t)\hat{\rho}^0 = \sum_{\{n_\gamma\}} P(\{n_\gamma\}) \langle \hat{n}_\alpha^k(t) \rangle_{i,\{n_\gamma\}}, \quad (23)$$

$$\langle \hat{n}_\alpha^k(t)\hat{n}_\beta^l(t) \rangle = Tr\, \hat{n}_\alpha^k(t)\hat{n}_\beta^l(t)\hat{\rho}^0 = \sum_{\{n_\gamma\}} P(\{n_\gamma\}) \langle \hat{n}_\alpha^k(t)\hat{n}_\beta^l(t) \rangle_{i,\{n_\gamma\}}, \quad (24)$$

where the brackets $\langle\ \rangle_{i,\{n_\gamma\}}$ stand for the **expectation values** calculated as previously in the state $|i,\{n_\gamma\}\rangle$. The time dependence of the photon number operator $\hat{n}_\alpha(t)$ is rather complicated. Hence, it is cumbersome to calculate immediately the mean quantum value of its k-th power and so much the worse to calculate the mean quantum values of the products of the k-th and l-th powers of the operators $\hat{n}_\alpha(t)$ and $\hat{n}_\beta(t)$. One can overcome these difficulties using the relation (5) and the fact that $\hat{R}_{\alpha\alpha}^k(t) = \hat{R}_{\alpha\alpha}(t)$ and $\hat{R}_{\alpha\alpha}(t)\hat{R}_{\beta\beta}(t) = 0$ for $\alpha \neq \beta$. Then[15]

$$\hat{n}_\alpha^k(t) = \hat{N}_\alpha^k + [(\hat{N}_\alpha+1)^k - \hat{N}_\alpha^k]\hat{R}_{\alpha\alpha}(t), \quad (25)$$

$$\begin{aligned}\hat{n}_\alpha(t)\hat{n}_\beta(t)_{\alpha\neq\beta} = {} & \hat{N}_\alpha^k\hat{N}_\beta^l + \hat{N}_\alpha^k[(\hat{N}_\beta+1)^l - \hat{N}_\beta^l]\hat{R}_{\beta\beta}(t) \\ & + \hat{N}_\beta^l[(\hat{N}_\alpha+1)^k - \hat{N}_\alpha^k]\hat{R}_{\alpha\alpha}(t). \quad (26)\end{aligned}$$

On substitution of (25) and (26) into (23) and (24), respectively one simply finds

$$\begin{aligned}\langle \hat{n}_\alpha^k(t) \rangle = \sum_{\{n_\gamma\}} P(\{n_\gamma\}) \{ & (n_\alpha - \delta_{\alpha i})^k + [(n_\alpha - \delta_{\alpha i}+1)^k \\ & - (n_\alpha - \delta_{\alpha i})^k]\, R_\alpha(i,\{n_\gamma\},t) \}, \end{aligned} \quad (27)$$

$$\begin{aligned}\langle \hat{n}_\alpha^k(t)\hat{n}_\beta^l(t) \rangle = {} & \sum_{\{n_\gamma\}} P(\{n_\gamma\}) \{ (n_\alpha - \delta_{\alpha i})^k (n_\beta - \delta_{\beta i})^l \\ & + (n_\alpha - \delta_{\alpha i})^k [(n_\beta - \delta_{\beta i}+1)^l - (n_\beta - \delta_{\beta i})^l]\, R_\beta(i,\{n_\gamma\},t) + \\ & + (n_\beta - \delta_{\beta i})^l [(n_\alpha - \delta_{\alpha i}+1)^k - (n_\alpha - \delta_{\alpha i})^k]\, R_\alpha(i,\{n_\gamma\},t) \}. \end{aligned} \quad (28)$$

Thus the time dependence of eqs.(27) and (28) is given by the first powers of $R_\alpha(i,\{n_\gamma\},t)$ and the latter is represented by eq.(18). Note that the relation (27) for $k = 1$ could directly be obtained from (19).

5. TIME DEPENDENT PHOTON-DISTRIBUTION FUNCTIONS

In the statistical averages (27) and (28) the time dependence is included in the **expectation values of the level** population operators. One can also calculate the photon-number statistical moments and the correlations between the modes from the following definitions:

$$\langle \hat{n}_\alpha^k(t) \rangle = \sum_{\{n_\gamma\}} P(\{n_\gamma\},t)\, n_\alpha^k, \quad (29)$$

$$\langle \hat{n}_\alpha^k(t)\hat{n}_\beta^l(t) \rangle = \sum_{\{n_\gamma\}} P(\{n_\gamma\},t)\, n_\alpha^k n_\beta^l, \quad (30)$$

where now the time dependence is included in the time-dependent photon-number distribution function $P(\{n_\gamma\},t)$. It is our aim to find in this chapter the form of $P(\{n_\gamma\},t)$. Moreover, we are going to show that contrary to the relation (16) the time-dependent photon-number distribution function does not factorize, i.e. that

$$\mathcal{P}(\{n_\gamma\},t) \neq \prod_{\alpha=1}^{N-1} \mathcal{P}_\alpha(n_\alpha,t) \tag{31}$$

for $t>0$, what in another manner implies the appearance of correlations between the modes because of the interaction with the atom.

Let us introduce the following operators

$$\hat{\chi}(\{\xi_\gamma\}) = \exp\left[i\sum_{\alpha=1}^{N-1}\xi_\alpha \hat{n}_\alpha(t)\right], \tag{32}$$

$$\hat{\chi}_\alpha(\xi) = \exp\left[i\xi\hat{n}_\alpha(t)\right], \tag{33}$$

where, for instance, $\hat{\chi}_1(\xi) = \hat{\chi}(\xi, 0,\dots,0)$ and $\hat{\chi}_{N-1}(\xi) = \hat{\chi}(0,0,\dots,\xi)$. The characteristic functions corresponding to the above operators are, respectively[24])

$$\chi(\{\xi_\gamma\}) = \langle\hat{\chi}(\{\xi_\gamma\})\rangle = Tr\,\hat{\chi}(\{\xi_\gamma\})\hat{\rho}^0, \tag{34}$$

$$\chi_\alpha(\xi) = \langle\hat{\chi}_\alpha(\xi)\rangle = Tr\,\hat{\chi}_\alpha(\xi)\hat{\rho}^0. \tag{35}$$

In turn, these functions are related with the photon-number distribution functions $\mathcal{P}(\{n_\gamma\},t)$ and $\mathcal{P}_\alpha(n_\alpha,t)$ by the relations

$$\chi(\{\xi_\gamma\}) = \sum_{\{n_\gamma\}} \exp\left[i\sum_{\alpha=1}^{N-1}\xi_\alpha n_\alpha\right] P(\{n_\gamma\},t), \tag{36}$$

$$\chi_\alpha(\xi) = \sum_{n_\alpha} \exp(i\xi n_\alpha)\, P_\alpha(n_\alpha,t). \tag{37}$$

With respect to the relation (5) and already mentioned equalities: $\hat{R}^k_{\alpha\alpha}(t) = \hat{R}_{\alpha\alpha}(t)$, $\hat{R}_{\alpha\alpha}(t)\hat{R}_{\beta\beta}(t) = 0$ for $\alpha\neq\beta$, from the definitions (32) and (33) we find

$$\hat{\chi}(\{\xi_\gamma\}) = \exp\left(i\sum_{\alpha=1}^{N-1}\xi_\alpha\hat{N}_\alpha\right)\left\{1+\sum_{\alpha=1}^{N-1}\left[\exp(i\xi_\alpha)-1\right]\hat{R}_{\alpha\alpha}(t)\right\}, \tag{38}$$

$$\hat{\chi}_\alpha(\xi) = \exp(i\xi\hat{N}_\alpha)\left\{1+\left[\exp(i\xi)-1\right]\hat{R}_{\alpha\alpha}(t)\right\}. \tag{39}$$

Substituting the expressions (38) and (39) into the definitions (34) and (35) and using the density operator (17) together with (14) and (15), we arrive at

$$\chi(\{\xi_\gamma\}) = \sum_{\{n_\gamma\}} P(\{n_\gamma\})\exp\left[i\sum_{\alpha=1}^{N-1}\xi_\alpha(n_\alpha-\delta_{\alpha i})\right]\Big\{1 + \sum_{\alpha=1}^{N-1}\left[\exp(i\xi_\alpha)-1\right]R_\alpha(i,\{n_\gamma\},t)\Big\}, \tag{40}$$

$$\chi_\alpha(\xi) = \sum_{\{n_\gamma\}} P(\{n_\gamma\})\exp\left[i\xi(n_\alpha-\delta_{\alpha i})\right]\Big\{1 + \left[\exp(i\xi)-1\right]R_\alpha(i,\{n_\gamma\},t)\Big\}, \tag{41}$$

where the quantity $R_\alpha(i,\{n_\gamma\},t)$ is given by (18).

Comparing (40) with (36) and (41) with (37), respectively we finally find the time-dependent photon-number distribution functions $\mathcal{P}(\{n_\gamma\},t)$ and $P_\alpha(n_\alpha,t)$. With respect to the conservation law (3) the first of them is

$$\begin{aligned}\mathcal{P}(\{n_\gamma\},t) &= \sum_{\alpha=1}^{N-1} P(\{n_\gamma+\delta_{\gamma i}-\delta_{\gamma\alpha}\})R_\alpha(i,\{n_\gamma+\delta_{\gamma i}-\delta_{\gamma\alpha}\},t) \\ &\quad + P(\{n_\gamma+\delta_{\gamma i}\})R_N(i,\{n_\gamma+\delta_{\gamma i}\},t) \equiv \\ &\equiv \sum_{\alpha=1}^{N} P(\{n_\gamma+\delta_{\gamma i}-\delta_{\gamma\alpha}\})R_\alpha(i,\{n_\gamma+\delta_{\gamma i}-\delta_{\gamma\alpha}\},t),\end{aligned} \tag{42}$$

where we have used the fact that $\delta_{\gamma\alpha}=0$ for $\alpha=N$ since γ does not reach N.

shows the photon statistics of the mode is super- or sub-Poissonian and indicates whether photon **bunching** or antibunching occurs[27-31]), respectively. The quantity $V_{\alpha\beta}(t)$ is measured in a HBT-type experiment with two different light beams[28,31]). If the $V_{\alpha\beta}(t)$ is negative we speak about photon anticorrelation.

By the pumping mode we obviously understand that one which transits the atom from an initial level $i \neq N$ to the level N (for this mode $\alpha = i$). The rest of modes ($\alpha \neq i$) we call the signal modes. Here, we are interested in the photon statistics and the correlations of the signal modes. Hence we must put in eq.(46) $\delta_{\alpha i} = 0$ and to do the same in eqs.(18) and (20). Since, moreover, we assume $i \neq N$, therefore $\delta_{iN} = 0$ too. Then, from eqs. (18)-(21) we get

$$R_\alpha(i,\{n_f\},t) = \frac{4g_\alpha^2 g_i^2 (n_\alpha+1)n_i}{\Omega^4} \sin^4 \frac{1}{2}\Omega t \,, \tag{49}$$

where

$$\Omega = \Big[\sum_{\beta=1}^{N-1} g_\beta^2 (n_\beta+1) - g_i^2\Big]^{1/2} . \tag{50}$$

The form of $R_\beta(i,\{n_f\},t)$ is readily given from (49) by replacing $\alpha \to \beta$.

The magnitude of the Rabi frequency Ω increases while that of the quantity (49) decreases as the number of levels increases.

Let us further assume for simplicity that only the pumping mode i and the two signal modes under consideration α and β are initially excited and we put $g_1 = \dots g_{N-1} = g$. Then

$$R_\alpha(i,n_i,n_\alpha,n_\beta,t) = \frac{4(n_\alpha+1)n_i}{(N-2+n_i+n_\alpha+n_\beta)^2} \sin^4 \frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta} \tag{51}$$

Note that now N must be $\geqslant$ 4.

On substitution of (51) into (46) and (48)

$$V_\alpha(t) = V_\alpha^0 + 8\sum_{n_i,n_\alpha,n_\beta} P_i(n_i)P_\alpha(n_\alpha)P_\beta(n_\beta)\frac{(n_\alpha-\bar{n}_\alpha)(n_\alpha+1)n_i}{(N-2+n_i+n_\alpha+n_\beta)^2} \times$$

$$\cdot \sin^4\frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta} - 16\Big[\sum_{n_i,n_\alpha,n_\beta} P_i(n_i)P_\alpha(n_\alpha)P_\beta(n_\beta) \times \tag{52}$$

$$\frac{(n_\alpha+1)n_i}{(N-2+n_i+n_\alpha+n_\beta)^2}\sin^4\frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta}\Big]^2 .$$

Replacing $\alpha \to \beta$ we obtain the variance of the photon number in the mode β .

$$V_{\alpha\beta}(t) = V_{\alpha\beta}^0 + 4\sum_{n_i,n_\alpha,n_\beta} P_i(n_i)P_\alpha(n_\alpha)P_\beta(n_\beta) \times$$

$$\times \frac{[(n_\alpha-\bar{n}_\alpha)(n_\beta+1)+(n_\beta-\bar{n}_\beta)(n_\alpha+1)]n_i}{(N-2+n_i+n_\alpha+n_\beta)^2}\sin^4\frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta} -$$

$$-16\Big[\sum_{n_i,n_\alpha,n_\beta} P_i(n_i)P_\alpha(n_\alpha)P_\beta(n_\beta)\frac{(n_\alpha+1)n_i}{(N-2+n_i+n_\alpha+n_\beta)^2} \times$$

$$\times \sin^4\frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta}\Big]\Big[\sum_{n_i,n_\alpha,n_\beta} P_i(n_i)P_\alpha(n_\alpha)P_\beta(n_\beta) \times$$

$$\times \frac{(n_\beta+1)n_i}{(N-2+n_i+n_\alpha+n_\beta)^2}\sin^4\frac{gt}{2}\sqrt{N-2+n_i+n_\alpha+n_\beta}\Big] . \tag{53}$$

The above formulas can be studied numerically. But we can still simplify them, assumming both signal model initially in the vacuum. Then

$$P_\alpha(n_\alpha)P_\beta(n_\beta) = \delta_{n_\alpha 0}\delta_{n_\beta 0} . \tag{54}$$

In this case from (52) and (27) we arrive at the particularly simple result:

$$V_\alpha(t) = -\langle \hat{n}_\alpha(t)\rangle^2 =$$

$$= -16\Big[\sum_{n_i} P_i(n_i)\frac{n_i}{(N-2+n_i)^2}\sin^4\frac{gt}{2}\sqrt{N-2+n_i}\Big]^2 \leqslant 0. \tag{55}$$

The non-delayed coincidence counting rate

$$g^{(2)}(t) = 1 + \frac{V_\alpha(t)}{\langle \hat{n}_\alpha(t)\rangle^2} = 0 \qquad (56)$$

because of one-photon transition at every atomic jump.

From eq. (53) we find

$$V_{\alpha\beta}(t) = V_\alpha(t) \leq 0. \qquad (57)$$

The equality of these quantities takes place since we assummed $g_\alpha = g_\beta$.

The result (55) or (56) implies that the signal modes generated from the initial vacuum by arbitrary light pumping has sub-Poissonian photon statistics and, hence, exhibit photon antibunching for all times $t>0$. The photons of the signal modes manifest moreover anticorrelation for all times (57). The quantities (55) and (57) will obviously show a sequence of collapse and revivals at coherent pumping. Exactly speaking,

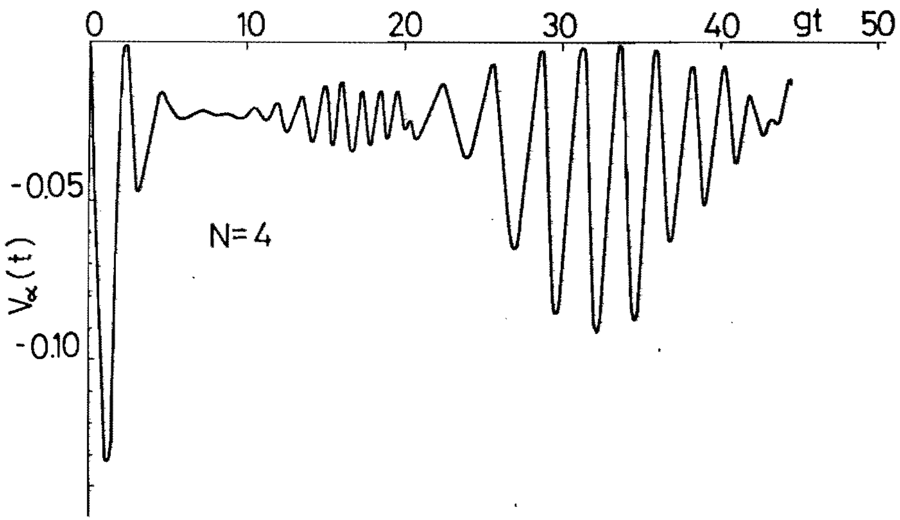

Fig. 2. Temporal behaviour of the $V_\alpha(t)$ in the signal mode produced from the vacuum at coherent pumping: N=4, $\bar{n}_1=5$, $P_i(n_i) = \exp(-\bar{n}_i)\,\bar{n}_i^{\,n_i}/n_i!$.

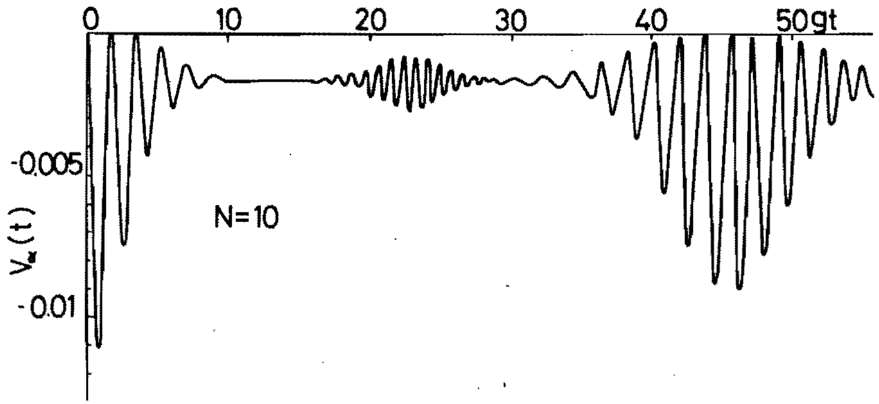

Fig. 3. Time dependence of the $V_\alpha(t)$ in the signal mode produced from the vacuum at coherent pumping: N=10, $\bar{n}_1=5$, $P_i(n_i) = \exp(-\bar{n}_i)\,\bar{n}_i^{\,n_i}/n_i!$.

we deal with two kinds of collapses and revivals with different periods and maxima what is related to the existence of two branches of the Rabi frequency: Ω and 2Ω [16]. In fact, $4\sin^4(\Omega t/2) = 4\sin^2(\Omega t/2) - \sin^2\Omega t$, and the maxima of the revivals connected with the low-frequency branch will be noticeably greater than those of the high-frequency branch.

The greater the number of levels N, the smaller the mean number of photons $\langle \hat{n}_\alpha(t)\rangle$ and their dispersion $\langle[\Delta\hat{n}_\alpha(t)]^2\rangle = \langle \hat{n}_\alpha(t)\rangle[1-\langle \hat{n}_\alpha(t)\rangle]$ (the greater the $V_\alpha(t)$), will be (figs. 2 and 3).

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Козеровски М., Шумовский А.С. E17-87-23

Статистика фотонов в N-уровневой (N-1)-модовой системе

Изучены характеристические функции, функции распределения и статистические моменты числа фотонов, а также междумодовые корреляции. Вычислена нормально упорядоченная вариация числа фотонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Kozierowski M., Shumovsky A.S. E17-87-23

Photon Statistics in an N-Level (N-1)-Mode System

The characteristic and photon number distribution functions, the statistical moments of photon numbers and the correlations of modes are studied. The normally ordered variances of the photon numbers and the cross-correlation functions are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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