

**ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА**

**E17-87-198**

**N.M.Plakida, V.L.Aksenov, S.L.Drechsler\***

**ANHARMONIC MODEL  
FOR HIGH- $T_c$  SUPERCONDUCTORS**

Submitted to "Europhysics Letters"

---

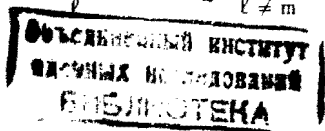
\* Central Institute for Solid State Physics and Material  
Science, Dresden, GDR

In recent experiments new records for superconducting transition temperatures  $T_c$  in oxide superconductors have been obtained:  $T_c = 30 \div 40$  K in the La-Ba-Cu-O<sup>/1-3/</sup> and La-Sr-Cu-O<sup>/4/</sup> systems and  $T_c = 93$  K in the Y-Ba-Cu-O<sup>/5,6/</sup> system. (For additional references see these papers). As has been shown in the subsequent papers<sup>/7-9/</sup>, a two-dimensional character of electron spectrum, that brings about a Peierls instability in a pure La<sub>2</sub>CuO<sub>4</sub>, and strong-deformation potentials can offer the possibility of enhancement of  $T_c$  in these materials. The anharmonic soft modes, structural instability and interfacial effects arising from mixed phases<sup>/5/</sup> also play an essential role, as generally accepted, in reaching a record value of  $T_c$ .

To explain considerable enhancement of  $T_c$  in structurally unstable systems an anharmonic model for high  $T_c$  superconductors has been proposed in<sup>/10/</sup>. In this model highly anharmonic quasi-local vibrations in a double-well potential with large amplitudes  $d \gg \sqrt{\langle u^2 \rangle}$  were admitted (here  $\langle u^2 \rangle$  is a mean-square displacement of ions in the harmonic approximation). Since the effective electron-electron coupling  $\lambda$  is proportional to square of ion displacements, the anharmonic coupling constant  $\lambda_s$  is much larger than  $\lambda_{ph}$  in the harmonic approximation:  $\lambda_s / \lambda_{ph} = d^2 / \langle u^2 \rangle \gg 1$  (see (8) below). This model was employed in<sup>/11/</sup> to estimate the enhancement of  $T_c$  in metal glasses and to explain the inverse isotope effect in the Zr, Hf-(H,D) systems<sup>/12/</sup>. The paper<sup>/13/</sup> should also be pointed out where  $T_c \approx 100$  K was obtained in this model for a sufficiently high concentration of two-level anharmonic excitations.

In the present paper the high-temperature superconductivity in oxide materials is considered on the basis of the anharmonic model<sup>/10/</sup>. According to the results of electronic-structure calculations<sup>/8,9/</sup> the strong coupling of the oxygen bond-stretching modes with electrons is maintained. In the tetragonal phase a planar breathing type displacement of oxygen atoms coupled with tilting of CuO<sub>6</sub> octahedra (the latter being frozen in the orthorhombic phase<sup>/7/</sup>) as well as a planar B<sub>1g</sub> mode of oxygen atoms due to the Jahn-Teller effect for Cu<sup>2+</sup>, are highly anharmonic. The ferroelectric mode of Cu<sup>2+</sup> or Cu<sup>3+</sup> can also be very anharmonic as in other perovskites of BaTiO<sub>3</sub> type. Considering for simplicity only one local anharmonic normal mode (e.g., breathing type) we write down the Hamiltonian of the model in the pseudo-spin notation<sup>/10/</sup>:

$$H_s = -\Omega \sum_{\ell} S_{\ell}^x - \frac{1}{2} \sum_{\ell \neq m} C_{\ell m} S_{\ell}^x S_m^x, \quad (1)$$



where  $\Omega = E_a - E_s > 0$  is the energy difference of the lowest two levels in the anharmonic double-well potential: the anti-symmetric ( $\Psi_a$ ) and symmetric ( $\Psi_s$ ) ones;  $C_{\ell m}$  is an effective coupling for the excitations in different lattice cells  $\ell, m$ ,  $S_{\ell}^{\alpha}$  is the pseudospin operator for  $S = 1/2$  acting in the ( $\Psi_s, \Psi_a$ ) space. The interaction of electron with anharmonic ion vibrations is also written in the pseudospin representation:

$$H_{e-s} = \sum_{\vec{p}, \vec{p}', \vec{q}, \sigma} V_{\vec{q}}^{\alpha}(\vec{p}, \vec{p}') 2x_{sa}^{\alpha} S_{\vec{q}}^{\alpha} a_{\vec{p}\sigma}^{\dagger} a_{\vec{p}'\sigma}, \quad (2)$$

where the deformation potential is given by

$$V_{\vec{q}}^{\alpha} = \frac{1}{\sqrt{N}} \sum_{\ell} e^{i\vec{q}\cdot\vec{\ell}} \langle \vec{p} | \nabla_{\vec{r}}^{\alpha} U(\vec{\ell} + \vec{r}) | \vec{p}' \rangle,$$

and  $x_{sa}^{\alpha} = \langle \Psi_s | x_{\ell}^{\alpha} | \Psi_a \rangle$  is the matrix element for the local normal mode displacement  $x_{\ell}^{\alpha}$  between the states  $\Psi_s$  and  $\Psi_a$ . The interaction of electrons with acoustic phonons and other harmonic phonon modes as well as electron-electron Coulomb interaction can be allowed for in the framework of the well-known electron-phonon theory of superconductivity<sup>/14/</sup>.

By applying the double-time Green function method<sup>/15/</sup> the Eliashberg-type equation can easily be obtained (see<sup>/10, 11/</sup>) for the anharmonic model (1), (2) and the coupling constant can be calculated:

$$\lambda_s = \frac{1}{N(0) S_F} \int \frac{d^2 \vec{p}}{v_{\vec{p}}} \int \frac{d^2 \vec{p}'}{S_F v_{\vec{p}'}} |V_{\vec{q}}^{\alpha}(\vec{p}, \vec{p}')|^2 \chi_s(\vec{q}) \approx N(0) \overline{J_s^2} \chi_s \equiv \eta_s \chi_s, \quad (3)$$

where  $\overline{J_s^2}$  is the deformation potential averaged over the Fermi surface  $S_F$ . The averaged static susceptibility for the anharmonic mode is given by

$$\chi_s = \overline{\chi_s(\vec{q})} \approx (2d)^2 \Omega S^z / E_{\vec{q}}^2. \quad (4)$$

Here  $2d = 2|x_{sa}|$  is the characteristic distance between two minima of the anharmonic double-well potential,  $S^z = \langle S_{\ell}^z \rangle = (\Omega/2h) \tanh(h/2kT)$ ,  $h^2 = \Omega^2 + (C(0) \langle S_{\ell}^x \rangle)^2$  and the excitation spectrum in the lattice

$$E_{\vec{q}}^2 = \Omega [\Omega + C(\vec{q}) S^z] + (C(0) \langle S_{\ell}^x \rangle)^2, \quad (5)$$

where  $C(\vec{q})$  is the Fourier transform of  $C_{\ell m}$  in (1). For  $T > T_0$  ( $T_0$  is the temperature of structural phase transition to the distorted state with  $\langle S_{\ell}^x \rangle \neq 0$ )

one has  $\overline{E_{\vec{q}}^2} \approx \overline{E_{\vec{q}}^2} = \Omega^2$  and for the static susceptibility one gets:

$$\chi_s = \frac{2d^2}{\Omega} \tanh \frac{\Omega}{2kT}. \quad (4a)$$

2

To compare  $\lambda_s$  (3) with  $\lambda_{ph}$  in the harmonic approximation, we write the latter in the same form as (3) and (4)

$$\lambda_{ph} = N(0) \overline{J_{ph}^2} \chi_{ph} \equiv \eta_{ph} \chi_{ph}, \quad (6)$$

where the static susceptibility for the harmonic phonons

$$\chi_{ph} = \frac{1}{M \overline{\omega^2}} \equiv \frac{2 \langle u^2 \rangle}{h \overline{\omega}} \tanh \frac{h \overline{\omega}}{2kT}. \quad (7)$$

Here we introduce the mean-square displacement of ions  $\langle u^2 \rangle = (\hbar/2M\overline{\omega}) \times \cotanh(\hbar\overline{\omega}/2kT)$  in the harmonic approximation with averaged frequency  $\overline{\omega} = \sqrt{\overline{\omega^2}}$ . By assuming  $\overline{J_{ph}^2} \approx \overline{J_s^2}$  and  $\Omega \approx \hbar\overline{\omega}$  one gets the estimation

$$\lambda_s / \lambda_{ph} \approx \chi_s / \chi_{ph} \approx d^2 / \langle u^2 \rangle \gtrsim 10, \quad (8)$$

where values  $\langle u^2 \rangle \approx 10^{-3} \text{ \AA}^2$  and  $d \gtrsim 0.1 \text{ \AA}$  can be taken from Debye-Waller factors in harmonic crystal and from atomic displacements at structural phase transition, respectively.

It is interesting to point out that structural transformation suppresses this enhancement of  $\lambda_s$  and hence  $T_c$ . Since for  $T < T_0$   $\langle S_{\ell}^x \rangle \neq 0$  in (5) the spectrum of excitation becomes more hard, the susceptibility  $\chi_s$  (4) decreases thus suppressing  $\lambda_s$  (3) (for details see<sup>/10/</sup>).

Now we can estimate  $\lambda_s$  for the La(Y)-Ba-Cu-O systems on the basis of the obtained formulae. To calculate  $T_c$  we use the formula in the intermediate coupling limit proposed in<sup>/16/</sup>

$$kT_c \approx 0.05 \Omega (\lambda_s - 0.25). \quad (9)$$

For further estimations the Coulomb contribution  $\mu^*$  as well as contributions from acoustic phonon modes  $\lambda_{ph}$  will be neglected since usually  $\lambda_{ph} - \mu^* \ll 1$ .

To calculate  $\lambda_s$  one should know three parameters  $\eta_s$ ,  $d$  and  $\Omega$  in (3), (4a). According to the results of electronic-structure calculations for La-Ba-Cu-O system<sup>/8/</sup> one can adopt for density of states  $N(0)$  at  $E_F$ :  $N(0) = 1.3$  states/eV. cell and for the deformation potential for breathing mode  $J_s = 1.6-3.9$  eV/\AA near the BZ boundary. But it is very difficult at present

to propose some reliable value for the averaged value of  $\eta_s = N(0) \overline{J_s^2}$  for the considered anharmonic mode. Therefore we take for crude estimations  $\eta_s \approx 2$  eV/\AA<sup>2</sup> that seems reasonable (compare, e.g., with  $\eta_s \approx 2.4$  eV/\AA<sup>2</sup> for

superconducting Pb). For the energy of anharmonic vibrations  $\Omega$  one can take some average values of optic mode frequencies in perovskites,  $\Omega \approx 25$  meV. Then, if one chooses  $d \approx 0.14$  Å in (4a), one gets  $\lambda_s \approx 3$  and  $T_c \approx 40$  K for the anharmonic model of La-Ba-Cu-O system. The adopted value of  $d$  seems to be reasonable in comparison with oxygen atoms displacements in the orthorhombic phase of La-Ba-Cu-O:  $d \approx 0.2$  Å, due to  $5^\circ$  tilting of the  $\text{CuO}_6$  octahedra<sup>17</sup>. For the same values of  $\eta_s = 2$  eV/Å<sup>2</sup>,  $\Omega = 25$  meV but with  $d \approx 0.3$  Å one gets  $\lambda_s = 7$  and  $T_c \approx 100$  K that can be compared with  $T_c \approx 93$  K in the Y-Ba-Cu-O system<sup>5, 6</sup>. In the latter system some additional highly anharmonic local vibrations due to structural instability and interfacial effects arising from mixed phases<sup>5</sup> may also be of great importance as in the glass state<sup>11</sup>. The upper limit for  $T_c$  in the present model one can get by choosing the higher value for  $\eta_s = 7$  eV/Å<sup>2</sup> (as in the transition metals). Then for  $\Omega \approx 25$  meV and  $d \approx 0.3$  Å one gets  $\lambda_s \approx 27$  and  $T_c \approx 270$  K by using the formulae<sup>17</sup>:  $kT_c \approx 0.18 \Omega \sqrt{\lambda_s}$  in the strong coupling limit ( $\lambda > 10$ ).

In conclusion, experiments<sup>1-6</sup> show that high temperature superconductivity of oxide perovskites can be understood in essential in the framework of the electron-phonon mechanism though at larger electron-phonon coupling constant. The proposed model gives principal explanation of this enhancement of  $\lambda$  due to strongly anharmonic vibration of ions.

The estimations given above for  $\lambda_s$  on the basis of the presently known parameters allow us to obtain the experimentally observed results for  $T_c$  for the La-Ba-Cu-O system as well as for the Y-Ba-Cu-O system. To get more reliable numerical results for these new materials in the framework of the proposed model some additional experimental information is needed concerning phonon spectrum, anharmonic parameters of lattice vibration, etc., that can be obtained from X-ray or neutron scattering investigation on a single crystal.

The authors are grateful to Academician N.N.Bogolubov for fruitful discussions.

## REFERENCES

1. Bednorz J.G., Müller K.A. – *Z.Phys.B*, 1986, 64, 189.
2. Bednorz J.G., Takashige M., Müller K.A. – *Europhys.Lett.*, 1987, 3, 379.
3. Chu C.W., Hor P.H., Meng R.L., Gao L., Huang Z.J., Wang Y.O. – *Phys.Rev.Lett.*, 1987, 58, 405.
4. Cava R.J., van Dover R.B., Batlogg B., Rietman E.A. – *Phys.Rev.Lett.*, 1987, 58, 408.
5. Wu M.K., Ashburn J.R., Torng C.J., Hor P.H., Meng R.L., Gao L., Huang Z.J., Wang Y.Q., Chu C.W. – *Phys.Rev.Lett.*, 1987, 58, 908.
6. Hor P.H., Gao L., Meng R.L., Huang Z.J., Wang Y.Q., Forster K., Vassiliou J., Chu C.W., Wu M.K., Ashburn J.R. and Torng C.J. – *Phys.Rev.Lett.*, 1987, 58, 911.

7. Jorgensen J.D., Schuttler H.-B., Hinks D.G., Capone D.W., Zhang H.K., Brodsky M.B. – *Phys.Rev.Lett.*, 1987, 58, 1024.
8. Mattheiss L.F. – *Phys.Rev.Lett.*, 1987, 58, 1028.
9. Jaejun Yu., Freeman A.F., Xu J.-H. – *Phys.Rev.Lett.*, 1987, 58, 1035.
10. Vujičić G.M., Aksenov V.L., Plakida N.M., Stamenković S. – *Phys.Lett.*, 1979, 73A, 439; *J.Phys.C: Solid St.Phys.*, 1981, 14, 2377.
11. Vujičić G.M., Plakida N.M. – *Fiz.Nizk.Temp. (in Russian)* 1983, 9, 269; Drechsler S.L., Vujičić G.M. *phys.stat.sol.(b)*, 1983, 117, 569.
12. Drechsler S.L., Vujičić G.M., Plakida N.M. – *J.Phys.F: Met.Phys.*, 1984, 14, L243.
13. Vujičić G.M. – *phys.stat.sol. (b)*, 1984, 123, K93.
14. Bogolubov N.N. – *Nuovo Cim.*, 1958, 7, 794.;  
Bogolubov N.N., Tolmachev V.V., Shirkov D.V. – *A New Method in the Theory of Superconductivity*. New York, Consultants Bureau, London, Chapman and Hall, 1959, VII, 121;
15. Bogolubov N.N., Tyablikov S.V. – *Dokl.Akad.Nauk SSSR*, 1959, 126, 53.
16. Rowell J.M. – *Solid St.Comm.*, 1976, 19, 1131.
17. Allen P.B., Dynes R.C. – *Phys.Rev.*, 1975, B12, 905.

Received by Publishing Department  
on March 30, 1987.

Плакида Н.М., Аксенов В.Л., Дрекслер Ш.Л.  
Ангармоническая модель сверхпроводников  
с высокой  $T_c$

E17-87-198

Заметное повышение температуры сверхпроводящего перехода  $T_c$  в перовскитовых оксидных соединениях объясняется на основе ангармонической модели сверхпроводника с структурнонеустойчивой решеткой. Показано, что наличие ангармонических локальных возбуждений, амплитуда флуктуаций которых значительно превышает амплитуду гармонических колебаний, приводит к увеличению константы связи  $\lambda$ . Полученные оценки для  $T_c$  согласуются с данными для соединений  $\text{La}(\text{Y})\text{BaCuO}$ .

Работа выполнена в Лаборатории теоретической физики ОИЯИ.  
Препринт Объединенного института ядерных исследований. Дубна 1987

Plakida N.M., Aksenov V.L., Drechsler S.L.  
Anharmonic Model for High- $T_c$  Superconductors.

E17-87-198

A considerable enhancement of the superconducting transition temperature  $T_c$  in perovskite oxide compounds is explained in the framework of the anharmonic model for superconductors with structurally unstable lattices. It is shown that anharmonic local excitations with fluctuation amplitudes much greater than harmonic vibrations amplitudes lead to a considerable enhancement of the coupling constant  $\lambda$ . The obtained estimations for  $T_c$  are in agreement with experimental data for  $\text{La}(\text{Y})\text{BaCuO}$  systems.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987