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SQUEEZING IN THE MULTIPHOTON JAYNES-CUMMINGS MODEL

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A substantial interest has centered upon the recent experimental observations[1-3] of squeezed field states[4] which present a new nonclassical effect in radiation theory and may have potential application in optical communication and gravitational wave detection. Several schemes for producing squeezed states have been analysed. Among them are parametric amplifiers[5,3], four wave mixing[6,1,2], two-photon interaction with an absorber[7-9], two-photon lasers[10-12], resonance fluorescence[13,14], cooperative Dicke systems and others. On the other hand, the progress in the realization of a single Rydberg atom in a resonant cavity[15] and the first observation of quantum collapse and revival in a one-atom maser[16] make now possible testing of the simplest quantum electrodynamic models of one-atom one-mode systems[17]. It has been shown that light squeezing is possible in the Jaynes-Cummings model with a coherent cavity field[18] and in the Jaynes-Cummings-type models with special bare-type initial states[19,21]. The magnitudes of squeezing in these systems are however rather small ($\lesssim 20\%$ in[18], $\lesssim 25\%$ in[21] and $\lesssim 42\%$ in[19]). Moreover, the squeezing obtained numerically in[18] appears not at once for $t > 0$ but only after some finite interval of interaction time, and the initial conditions for squeezing in[19,21] are too specific and obviously only of academic interest. The aim of the present paper is to report the results showing that states containing a large amount of squeezing can be obtained from the exactly soluble multiphoton Jaynes-Cummings model with a coherent cavity field.

We consider a two-level atom interacting with a single-mode radiation field in a lossless resonant cavity via the m-photon-transition mechanism. The effective Hamiltonian for this system in the rotating wave approximation is

$$H = \hbar\omega a^+ a + \hbar\omega_0 R^z + \hbar g(R^+ a^m + R^- a^{+m}), \qquad (1)$$

where ω and $\omega_0 = m\omega$ are the frequencies of the field and the atom, respectively, g is the multiphoton atom-radiation coupling constant, m is the photon multiple, R^z, R^+ and R^- are the atomic pseudospin operators, and a^+ and a are the creation and annihilation operators of the field.

We denote by $|+\rangle, |-\rangle$ the excited and ground states of the atom and by $|n\rangle$ the Fock states of the field. For the atom initially in the ground state $|-\rangle$ and the field initially in a coherent $|z\rangle$,

$$|z\rangle = \exp\left(-\frac{|z|^2}{2}\right)\sum_{n=o}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle, \tag{2}$$

the wave function of the total system in the interaction picture is found from the Hamiltonian (1) to be

$$|\Psi(t)\rangle = \sum_{n=o}^{\infty}|-;n\rangle A_-^{(n)}(t) + \sum_{n=m}^{\infty}|+;n-m\rangle A_+^{(n)}(t), \tag{3}$$

where

$$A_-^{(n)}(t) = \cos\left(gt\sqrt{\frac{n!}{(n-m)!}}\right), \tag{4a}$$

$$A_+^{(n)}(t) = -i\sin\left(gt\sqrt{\frac{n!}{(n-m)!}}\right). \tag{4b}$$

Hence, the mean photon number $\langle a^+a\rangle$, the mean photon amplitude $\langle a\rangle$ and the mean square photon amplitude $\langle a^2\rangle$ are easily derived to read

$$\langle a^+a\rangle \equiv \sigma_o = \bar{n} - m\sum_{n=m}^{\infty}P_n|A_+^{(n)}|^2,$$

$$e^{i\omega t}\langle a\rangle \equiv z\sigma_1 = z\left(\sum_{n=o}^{\infty}P_n A_-^{(n)} * A_-^{(n+1)} + \sum_{n=m}^{\infty}P_n A_+^{(n)} * A_+^{(n+1)}\sqrt{1-\frac{m}{n+1}}\right),$$

$$e^{2i\omega t}\langle a^2\rangle \equiv z^2\sigma_2 = z^2\left(\sum_{n=o}^{\infty}P_n A_-^{(n)} * A_-^{(n+2)} + \sum_{n=m}^{\infty}P_n A_+^{(n)} * A_+^{(n+2)}\sqrt{\left(1-\frac{m}{n+2}\right)\left(1-\frac{m}{n+1}\right)}\right). \tag{5}$$

Here P_n is the Poissonian distribution corresponding to the coherent initial state (2) of the field

$$P_n = \exp(-\bar{n})\,\bar{n}^n/n! \tag{6}$$

and $\bar{n} = |z|^2$ is the dimensionless intensity of the field.

We introduce the two slowly varying Hermitian quadrature components a_1, a_2 of the field, defined by

$$a_1 = \frac{1}{2}\left(ae^{i(\omega t-\theta)} + a^+e^{-i(\omega t-\theta)}\right),$$
$$a_2 = \frac{1}{2i}\left(ae^{i(\omega t-\theta)} - a^+e^{-i(\omega t-\theta)}\right), \tag{7}$$

where θ is a phase angle that may be chosen at will. The condition for squeezing in the quadrature component a_α can be written simply as [4]

$$S_\alpha < 0, \tag{8}$$

where

$$S_\alpha = \frac{(\Delta a_\alpha)^2 - (\Delta a_\alpha)^2_{coh}}{(\Delta a_\alpha)^2_{coh}} = 4\langle(a_\alpha - \langle a_\alpha\rangle)^2\rangle - 1. \tag{9}$$

In terms of the photon operators, we find readily that

$$S_1 = 2\langle a^+a\rangle + 2\,\mathrm{Re}\langle a^2e^{2i(\omega t-\theta)}\rangle - 4(\mathrm{Re}\langle ae^{i(\omega t-\theta)}\rangle)^2,$$
$$S_2 = 2\langle a^+a\rangle - 2\,\mathrm{Re}\langle a^2e^{2i(\omega t-\theta)}\rangle - 4(\mathrm{Im}\langle ae^{i(\omega t-\theta)}\rangle)^2. \tag{10}$$

It is seen from Eqs.(5) and (10) that the quantities σ_o, σ_1 and σ_2 are real numbers, and therefore, the optimum choice of θ for squeezing should be $\theta = \phi$, where ϕ is the phase of z, i.e. $z = \bar{n}^{1/2}\exp(i\phi)$. Then, Eqs.(10) become

$$S_1 = 2\sigma_o + 2\bar{n}\sigma_2 - 4\bar{n}\sigma_1^2, \tag{11a}$$

$$S_2 = 2\sigma_o - 2\bar{n}\sigma_2. \tag{11b}$$

For very short times ($gt \ll 1$), we find from Eqs.(11a), (5) and (4) the asymptotic expressions

$$S_1 \simeq \begin{cases} -\frac{1}{3}\bar{n}(gt)^4 & \text{in the case } m = 1, \\ -m(m-1)\bar{n}^{m-1}(gt)^2 & \text{in the cases } m \geq 2, \end{cases} \tag{12}$$

These negative expressions indicate the immediate appearance of squeezing in a_1 for any photon multiple m and arbitrary nonzero intensity $\bar{n}$ after switching on the atom-field interaction.

Such a behaviour is absent in the case when the atom is initially in the excited state [18].

Analogously, the asymptotic expressions of S_2 at very short times ($gt \ll 1$) are found from Eqs.(11b), (5) and (4) to be

$$S_2 \simeq \begin{cases} \frac{1}{3}\bar{n}(gt)^4 & \text{in the case } m = 1 \\ m(m-1)\bar{n}^{m-1}(gt)^2 & \text{in the case } m \geq 2. \end{cases} \qquad (13)$$

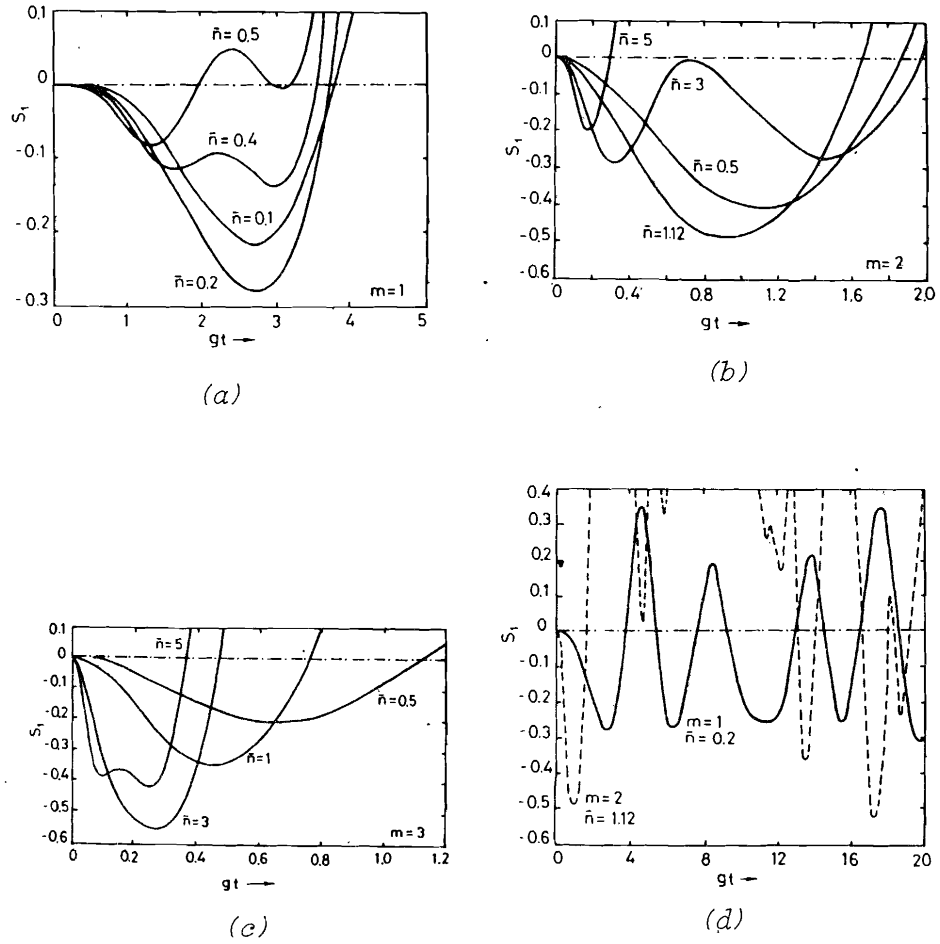
These positive expressions indicate the lack of squeezing in a_2 at the beginning of interaction for any photon multiple m and any initial field intensity $\bar{n}$.

It should be noted that for the particular case $m = 1$ the first equation in (13) and the fact that squeezing occurs in a_1 at the onset of interaction, are in agreement with the results obtained recently by Butler and Drummond [20] for a cooperative Dicke system.

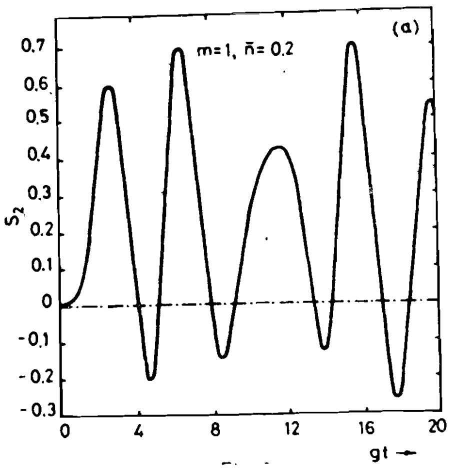
Figs. 1 present the time evolution of S_1 computed numerically from Eqs.(11), (5) and (4) for various intensities $\bar{n}$ of the coherent initial field and various photon multiplies m. As soon as $t > 0$, we observe nonclassical negative values of S_1. For the cases ($m = 1$, $\bar{n} = 0{,}2$), ($m = 2$, $\bar{n} = 1.12$) and ($m = 3$, $\bar{n} = 3$) the maximum magnitudes of squeezing in the region of short times are $S_1 \simeq -0.28$, -0.49 and -0.57 (i.e. 28%, 49% and 57%), respectively. As time goes on, S_1 starts oscillating and then reaches positive values. The long-time behaviour of S_1 is characterized by recoveries of squeezing. The squeezing in a_1 appears, disappears, and later may appear again, see Fig. 1(d). The maximum magnitude of squeezing recovered again (e.g., $\simeq$ .52% for $gt \simeq 17.28$, $\bar{n} = 1.12$, $m = 2$ see Fig. 1(d)) may be larger than the maximum magnitude of squeezing in the short-time region ($\simeq$ 49% for $gt \simeq 0.92$). It is seen from the figures that for the larger intensity $\bar{n}$ the duration of the first squeezing is generally shorter (except for the cases when overlapping of the first and second squeezing regions occurs).

In Figs. 2(a) and 2(b), we plot S_2 versus time gt for the cases ($m = 1$, $\bar{n} = 0.2$) and ($m = 2$, $\bar{n} = 5$). It is clear from the figures that squeezing may also occur in the field component a_2. The delay of squeezing in a_2 for the cases examined here is seen.

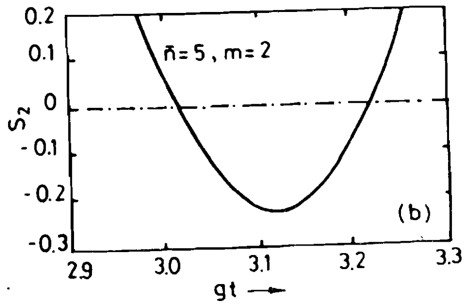
To conclude briefly, we have obtained squeezing states in the exactly soluble multiphoton Jaynes-Cummings model, where the atom is initially in the ground state and the fields are



Figs.1. Time evolution of S_1 *. (a)* $m = 1$*, short times:* $gt < 5$*. Here, the maximum magnitude* $S_1 \simeq -0.28$ *of the first squeezing occurs for* $\bar{n} = 0.2$ *at* $gt \simeq 2.75$*. (b)* $m = 2$*, short times* $gt < 2$*. The maximum magnitude* $S_1 \simeq -0.49$ *of the first squeezing occurs for* $\bar{n} \simeq 1.12$ *at* $gt \simeq 0.92$*. (c)* $m = 3$*, short times* $gt < 1.2$*. The maximum magnitude* $S_1 \simeq -0.57$ *of the first squeezing occurs for* $\bar{n} = 3$ *at* $gt \simeq 0.27$*. (d) Long times:* $gt < 20$*. The full line corresponds to the case* $m = 1$*,* $\bar{n} = 0.2$*. The dashed line corresponds to the case* $m = 2$*,* $\bar{n} = 1.12$*. The large magnitudes of squeezing:* $S_1 \simeq -0.31$ *for* $m = 1$*,* $\bar{n} = 0.2$*,* $gt \simeq 19.65$ *and* $S_1 \simeq -0.52$ *for* $m = 2$*,* $\bar{n} = 1.12$*,* $gt \simeq 17.28$ *are obtained.*



in a coherent state. The immediate appearance of squeezing in a_1 for short times of interaction has been established. Due to the photon multiplicity and the suitable choice of the field intensity, the large magnitudes of squeezing have been obtained. A more detailed investigation of the squeezing in the model will be a subject of a subsequent paper.



Figs. 2. Time evolution of S_2 *. (a)* $m = 1$, $\bar{n} = 0.2$, *long times:* $gt < 20$. *(b)* $m = 2$, $\bar{n} = 5$, *the time interval* $2.9 < gt < 3.3$ *for the first squeezing in* a_2 .

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Squeezing in the exactly soluble multiphoton Jaynes-Cummings model is examined.

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