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NONCLASSICAL CORRELATION BETWEEN LIGHT BEAMS IN A JAYNES-CUMMINGS-TYPE MODEL SYSTEM

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In many systems, involving interaction between the electromagnetic field and matter, statistical propertiers of light are predicted to have a nonclassical behaviour. Such nonclassical effects are of interest because they provide instances in which the quantum-mechanical nature of the field manifests itself. The most well-known examples of these effects are photon antibunching [1], squeezing [2] and nonclassical correlation between light beams [3,4]. The existence of nonclassical correlation indicated by violation of the Cauchy-Schwarz inequality has been observed in two-photon cascade emission [4]. It has been shown that such correlations are expected in a two-mode two-photon laser [5], parametric amplifier [6,7], resonance fluorescence [8] three-photon hyper-Raman process [9] and four-interacting-mode system [10]. In this paper, we give another example, where a nonclassical correlation between light beams may appear.

We consider a lambda - configuration three-level atom being in a lossless cavity (fig.1). The upper level 3 is connected with the lower levels 1 and 2 by dipole transitions, whereas the transition $1 \to 2$ is forbidden. The hamiltonian describing the interaction of the atom with a two-mode resonant radiation field in the dipole and rotating wave approximations is given by

$$H = \sum_{j=1}^{3} \hbar\Omega_j R_{jj} + \sum_{\alpha=1}^{2} \hbar\omega_\alpha a^+_\alpha a_\alpha + \sum_{\alpha=1}^{2} \hbar g_\alpha (a_\alpha R_{3\alpha} + a^+_\alpha R_{\alpha 3}). \qquad (1)$$

Here, the operator $R_{jj} \equiv |j><j|$ describes the population of level j, with the energy $\hbar\Omega_j$ and state vector $|j>$. $R_{ij} \equiv |i><j|$ is the operator of the atomic transition from level j to level i.

The photon operators a_α, a^+_α describe the field modes of the resonance frequencies $\omega_\alpha = \Omega_3 - \Omega_\alpha$. g_α's are the parameters of atom-field coupling.

It should be mentioned that the atomic dynamics and photon statistics of the model described above have been rigorously examined [11,12]. The time behaviour of the photon-number variances and cross correlation has been discussed [13]. Due to progress toward the realization of a single atom in a cavity [14] the model is of large interest.

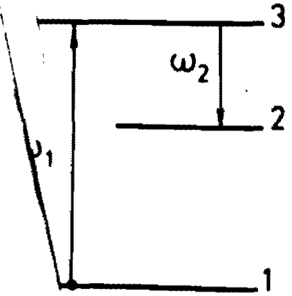

Fig.1. Energy level structure of the lambda-configuration atom considered.

We now define the degrees of second-order coherence of light in modes $g^{(2)}_{ii}$ and the degree of mode correlation $g^{(2)}_{12}$ to be [3]

$$g^{(2)}_{ij} = \frac{< a^+_i a^+_j a_j a_i >}{< a^+_i a_i > < a^+_j a_j >} . \qquad (2)$$

As is well-known [3,4], in intensity measurements on two beams the quantum translation of the Cauchy-Schwarz inequality

$$(g^{(2)}_{12})^2 \le g^{(2)}_{11} g^{(2)}_{22} \qquad (3)$$

holds for field states with positive definite Glauber representations. Another form of this inequality is

$$V \le 0, \qquad (4)$$

where the correlation function V is defined to be

$$V = < a^+_1 a^+_2 a_2 a_1 > - < a^+_1 a^+_1 a_1 a_1 > < a^+_2 a^+_2 a_2 a_2 > . \qquad (5)$$

The performances of the inequality $V > 0$ are possible for the field states with sign-indefinite Glauber representations $\mathcal{P}(\alpha_1, \alpha_2)$, $\mathcal{P}(\alpha_1, \alpha_2) \ngtr 0$ [3,10]. Such violations show the presence of nonclassical correlation between the beams and are the subject of our further consideration.

We assume that the atom is initially on the lowest level 1. In paper [11] the time-dependent photon distribution $P_t(n_1, n_2)$ has been found explicitly and rigorously. It reads

$$P_t(n_1,n_2) = P(n_1,n_2)R_{1n_1n_2}(t) + P(n_1+1, n_2-1)R_{2n_1+1\,n_2-1}(t) + $$
$$+ P(n_1+1, n_2)[1 - R_{1n_1+1n_2}(t) - R_{2n_1+1n_2}(t)], \qquad (6)$$

where

$$R_{1n_1n_2}(t) = 1 - \frac{4g_1^2 g_2^2 n_1(n_2+1)}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^2[\frac{t}{2}\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}] -$$
$$- \frac{g_1^4 n_1^2}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^2[t\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}], \qquad (7)$$

$$R_{2n_1n_2}(t) = \frac{4g_1^2 g_2^2 n_1(n_2+1)}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^4[\frac{t}{2}\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}] .$$

The function $P(n_1, n_2)$ is the initial photon distribution. It is of interest to consider the case when both the pump beam (the field in mode 1) and the signal beam (the field in mode 2) are initially in Glauber coherent states. The initial photon distribution $P(n_1, n_2)$ then takes the form

$$P(n_1, n_2) = \exp(-\bar n_1 - \bar n_2)\, \bar n_1^{\,n_1}\, \bar n_2^{\,n_2} / (n_1!\, n_2!). \tag{8}$$

Let us introduce the notation $\overline{(\ldots\ldots)} \equiv \sum_{n_1 n_2} (\ldots\ldots) P(n_1, n_2)$. It can be easily shown that

$$\overline{n_1 f_{n_1 n_2}} = \bar n_1 \overline{f_{n_1+1\, n_2}}, \quad \overline{n_2 f_{n_1 n_2}} = \bar n_2 \overline{f_{n_1 n_2+1}}, \tag{9}$$

where $f_{n_1 n_2}$ is an arbitrary function of n_1 and n_2. Using the relations (9) and eq.(6) we can obtain

$$\langle a_1^+ a_2^+ a_2 a_1 \rangle = \bar n_1 \bar n_2 + \bar n_2 (\overline{R_{1n_1 n_2+1}} - 1) + \bar n_1 \overline{R_{2n_1+1\, n_2}} - \overline{R_{2n_1 n_2}},$$

$$\langle a_1^+ a_1^+ a_1 a_1 \rangle = \bar n_1^2 + 2\bar n_1 (\overline{R_{1n_1+1\, n_2}} - 1) - 2(\overline{R_{1n_1 n_2}} - 1), \tag{10}$$

$$\langle a_2^+ a_2^+ a_2 a_2 \rangle = \bar n_2^2 + 2\bar n_2 \overline{R_{2n_1 n_2+1}}.$$

We assume that the initial mean photon numbers are large, i.e., $\bar n_1, \bar n_2 \gg 1$. Then the functions $\overline{R_{1n_1 n_2}}$ and $\overline{R_{2n_1 n_2}}$ can be approximated by [13]

$$\overline{R_{1n_1 n_2}} \simeq 1 - \frac{4 g_1^2 g_2^2 \bar n_1 (\bar n_2 + 1) F(t;\, g_2^2) + g_1^4 \bar n_1^2 F(2t;\, g_2^2)}{2[g_1^2 \bar n_1 + g_2^2 (\bar n_2 + 1)]^2},$$

$$\overline{R_{2n_1 n_2}} \simeq \frac{g_1^2 g_2^2 \bar n_1 (\bar n_2 + 1)}{2[g_1^2 \bar n_1 + g_2^2 (\bar n_2 + 1)]^2} [4F(t;\, g_2^2) - F(2t;\, g_2^2)], \tag{11}$$

where the function F is given by

$$F(t;\, g^2) = 1 - \int_0^\infty dx_1 \int_0^\infty dx_2 \cos[t\sqrt{g_1^2 x_1 + g_2^2 x_2 + g^2}]\,(4\pi^2 x_1 x_2)^{-1/2} \times$$
$$\times \exp[-\bar n_1 - \bar n_2 + x_1 + x_2 - x_1 \ln(x_1/\bar n_1) - x_2 \ln(x_2/\bar n_2)]. \tag{12}$$

The approximate expressions for the other functions $\overline{R_{\alpha\, n_1+k_1 n_2+k_2}}$ ($\alpha = 1,2$; $k_1, k_2 = 0,1$) can be obtained from (11) by changes $\bar n_1 \to \bar n_1 + k_1$, $\bar n_2 \to \bar n_2 + k_2$ and $F(\ldots;\, g_2^2) \to F(\ldots;\, k_1 g_1^2 + k_2 g_2^2 + g_2^2)$. Hence, taking into account the assumption $\bar n_1, \bar n_2 \gg 1$ we can easily see that $\overline{R_{\alpha\, n_1+k_1 n_2+k_2}} \approx \overline{R_{\alpha\, n_1 n_2}}$. By using the latter and after substituting eqs.(10) into eq.(4) we get the approximate expression

$$V \simeq [\bar n_1 \overline{R_{2n_1 n_2}} - \bar n_2 (1 + \overline{R_{1n_1 n_2}})][\bar n_1 \overline{R_{2n_1 n_2}} + \bar n_2 (1 - \overline{R_{1n_1 n_2}})]. \tag{13}$$

On the other hand, we have $\overline{R_{\alpha\, n_1 n_2}} \leq 1$. Hence, we can see from eq.(13) that the Cauchy-Schwarz inequality (4) is violated if

$$v \equiv \bar n_1 \overline{R_{2n_1 n_2}} - \bar n_2 (1 + \overline{R_{1n_1 n_2}}) > 0. \tag{14}$$

Note that for small times the correlation function V is nonpositive, i.e., $V \leq 0$. Using eqs.(11) and (12) we can describe the time behaviour of v and V. It has a feature of collapses and revivals very similar to those examined in [12,13 and refs. therein]. We restrict ourselves to the so-called quasisteady value of v which is reached either in the time regions between collapse and revival or at very large times $t \to \infty$. In these regions of times the function $F(\ldots;\, g_2^2)$ approaches unity [12,13]. Therefore, the steady value of v can easily be found. Taking into account the assumption $\bar n_2 \gg 1$ this value is

$$v_{st} = \frac{\bar n_2}{2(g_1^2 \bar n_1 + g_2^2 \bar n_2)^2} \{3 g_1^2 (g_2^2 - g_1^2) \bar n_1^2 - 4 g_1^2 g_2^2 \bar n_1 \bar n_2 - 4 g_2^4 \bar n_2^2\}. \tag{15}$$

It is seen that $v_{st} > 0$ if: 1) the coupling of the atom with the signal mode is stronger than that of the atom with the pump mode, i.e.,

$$\left|\frac{g_2}{g_1}\right| > 1 \tag{16}$$

and, 2) the mean number $\bar n_1$ of the initial photons in the pump mode is so large that

$$\frac{\bar n_1}{\bar n_2} > \left(\frac{\bar n_1}{\bar n_2}\right)_{thresh}. \tag{17}$$

Here the threshold value $(\bar n_1/\bar n_2)_{thresh}$ is

$$\left(\frac{\bar n_1}{\bar n_2}\right)_{thresh} = \frac{2(g_2/g_1)^2}{\sqrt{3(g_2/g_1)^2 - 2} - 1}. \tag{18}$$

Under the conditions (16) and (17) nonclassical correlation between the beams will appear in the quasi-steady time-regions. In fig.2 we plot the dependence of the threshold value $(\bar n_1/\bar n_2)_{thresh}$ of the photon-number ratio for the appearance of nonclassical correlation upon the ratio g_2/g_1 of the coupling constants. The minimum value of the quantity $(\bar n_1/\bar n_2)_{thresh}$ is $4(\sqrt{3} + 1)/3$ and occurs at $g_2/g_1 = [2(1 + 1/\sqrt{3})]^{1/4}$.

Finally, we notice another simple case for the violation of the Cauchy-Schwarz inequality in the system. This is the case when no photon is initially in the signal beam, i.e., when

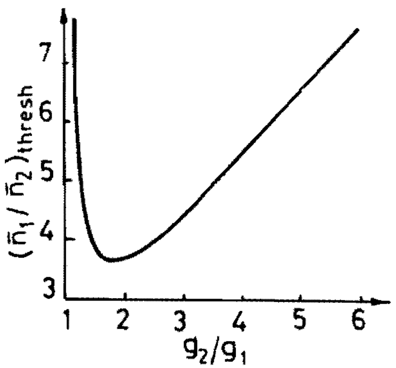

Fig.2. The dependence of $(\bar{n}_1/\bar{n}_2)_{thresh}$ *upon* (g_2/g_1).

$P(n_1,n_2) = \delta_{n_2 0} P_1(n_1)$. From eqs.(2) and (6) we can easily see that $g_{22}^{(2)}$ is then equal to zero. Therefore, if the pump beam is initially in a nonvacuum state, i.e., if $P_1(n_1 = 0) < 1$ the inequality (3) is violated for $t > 0$. This means that when the signal beam is generated from its initial vacuum by pumping the atom from the ground level 1, the nonclassical correlation between the beams occurs for all times $t > 0$, arbitrary relations of coupling constants and for arbitrary (nonvacuum) initial states of the pump beam.

In conclusion, we have shown that even the simplest model of light beams interacting with an atom follows a dynamics leading to nonclassical correlation between the beams. The conditions for the appearance of such a nonclassical correlation have been presented.

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Неклассическая корреляция между пучками света в модельной системе типа Джейнса - Каммингса

Исследована генерация неклассической корреляции между пучками света в трехуровневой двухмодовой модели типа Джейнса - Каммингса.

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Nonclassical Correlation between Light Beams in a Jaynes-Cummings-Type Model System

The appearance of nonclassical correlation between light beams due to the interaction with an atom in a three-level two-mode Jaynes-Cummings-type model is examined.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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