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SPECTRUM OF SQUEEZING IN COLLECTIVE DOUBLE OPTICAL RESONANCE

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I. Introduction

In recent years, a large amount of theoretical (Walls 1983, and refs. cited there) and experimental (Slusher et al. 1985, Ling-An Wu et al. 1986) works are concentrated on the problem of generation of squeezed light characterizing by that its noise in a one electric-field quadrature is less than that of a coherent state. A possible application of one mode (Caves 1983) and two modes (Bondurant and Shapiro 1984) squeezed light in the detection of gravity waves it discussed.

The squeezing in the resonance fluorescence field for a one-atom case (Mandel 1982, Walls and Zoller 1981, Loudon 1984, Collet et al. 1984) and a multiatom case (Lakshmi and Agarwal 1984, Ficek et al. 1984, Bogolubov et al. 1986) has been an object of investigation. In our previous work (Bogolubov et al. 1987) we have shown that a substantial squeezing is present in some mixtures of the fluorescent spectral band from a collective double resonant process for the case of intense external fields when squeezing is absent for separate spectral bands. In this work we wish to give a spectral analysis for our previous work (Bogolubov et al. 1987). We also discuss a possible experimental scheme with the use of the Fabry-Perot filters for an observation of the spectral squeezing and show a condition to receive nearly perfect squeezing.

II. Basic equations

We consider a system of N three-level atoms of the Dicke model (fig.1) interacting with two external fields $\vec{\mathcal{E}}_1$, $\vec{\mathcal{E}}_2$ with frequencies Ω_1 , Ω_2 and with an emitted field. The external fields $\vec{\mathcal{E}}_1$ and $\vec{\mathcal{E}}_2$ are assumed to be intense and can

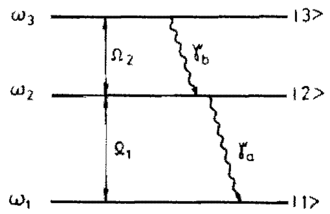

Fig. 1. Three-level atoms interacting with two monochromatic external fields and an emitted field.

be treated classically. For simplicity the external field $\vec{\mathcal{E}}_2$ is assumed to be in resonance with the level separation $\omega_3 - \omega_2 = \omega_{32}$ and the field $\vec{\mathcal{E}}_1$ is assumed to be in-resonance with $\omega_2 - \omega_1 = \omega_{21}$ $(\hbar \equiv 1)$. After Agarwal (1974), using the Markovian and rotating-wave approximation, one can find the master equation for the atomic system

$$\begin{aligned} \frac{\partial \rho}{\partial t} = {} & iG\,[\cos\alpha\, J_{12} + \sin\alpha\, J_{23} + H.C., \rho] \\ & - \gamma_a \left(J_{21}J_{12}\rho - J_{12}\rho J_{21} + H.C.\right) \\ & - \gamma_b \left(J_{32}J_{23}\rho - J_{23}\rho J_{32} + H.C.\right) = L\rho , \end{aligned} \tag{1}$$

where ρ is the atomic density matrix, $2\gamma_a$ and $2\gamma_b$ are radiative spontaneous transition probabilities per unit time for a single atom to change from level $|2\rangle$ to $|1\rangle$ and from $|3\rangle$ to $|2\rangle$, respectively, $G = (G_{21}^2 + G_{32}^2)^{1/2}$ and $tg\alpha = G_{32}/G_{21}$, where G_{21} and G_{32} are the Rabi frequencies for the atomic transitions $|2\rangle \longrightarrow |1\rangle$ and $|3\rangle \longrightarrow |2\rangle$, respectively, $J_{k\ell}$ $(k,\ell = 1,2,3)$ are the collective angular momenta of the atoms which have the following form in the Schwinger representation (Schwinger 1965)

$$J_{k\ell} = C_k^+ C_\ell \qquad (k,\ell = 1,2,3),$$

where the operators C_k and C_k^+ obey the boson commutation relation

$$[C_k, C_\ell^+] = \delta_{k\ell}$$

and can be treated as the annihilation and creation operators for the atoms populating the level $|k\rangle$.

Further we investigate only the case of the intense external fields when the following relation is fulfilled

$$G \gg N\gamma_{21}, N\gamma_{32} . \tag{2}$$

After the canonical transformation

$$\begin{aligned} C_3 &= -2^{-1/2}\sin\alpha\, Q_1 + \cos\alpha\, Q_2 + 2^{-1/2}\sin\alpha\, Q_3 , \\ C_2 &= 2^{-1/2} Q_1 + 2^{-1/2} Q_3 , \\ C_1 &= -2^{-1/2}\cos\alpha\, Q_1 - \sin\alpha\, Q_2 + 2^{-1/2}\cos\alpha\, Q_3 , \end{aligned} \tag{3}$$

and using the secular approximation (Agarwal et al. 1978, Bogolubov et al. 1985), i.e., ignoring the part of the Liouville operator L containing rapidly oscillating terms with frequencies nG (n=1,2,3,4), one can find a stationary solution of the master equation in the form (Bogolubov et al. 1985)

$$\tilde{\rho} = Z^{-1} \sum_{P=0}^{N} x^P \sum_{M=0}^{P} |P,M\rangle\langle M,P| , \tag{4}$$

where

$$x = \gamma_b \cos^2\alpha / \gamma_a \sin^2\alpha ,$$

$$Z = [(N+1)x^{N+2} - (N+2)x^{N+1} + 1]/(x-1)^2 ,$$

$\tilde{\rho} = U\rho U^+$ with U being a unitary operator representing the canonical transformation (3) ; $|P,M\rangle$, an eigenstate of the operators $R = R_{11} + R_{33}$, R_{11} and of the operator of the total number of atoms, $\hat{N} = R_{11} + R_{22} + R_{33}$ where $R_{k\ell} = Q_k^+ Q_\ell$ $(k,\ell = 1,2,3)$ are the collective angular momenta of "dressed" atoms. The operators Q_k, Q_k^+ satisfy the boson commutation relation

$$[a_k, a_l^+] = \delta_{kl} \quad , \tag{5}$$

so

$$[R_{kl}, R_{k'l'}] = R_{kl'}\delta_{k'l} - R_{k'l}\delta_{kl'} \quad . \tag{6}$$

Applying the stationary density matrix (4) one calculates the statistical moments

$$\langle R\rangle = Z^{-1}\left[N(N+1)X^{N+3} - 2N(N+2)X^{N+2} + (N+1)(N+2)X^{N+1} - 2X\right]\Big/(X-1)^3 \quad , \tag{7}$$

$$\langle R^2\rangle = Z^{-1}\left[N^2(N+1)X^{N+4} - N(3N^2+6N-1)X^{N+3} + (N+2)\cdot(3N^2+3N+2)X^{N+2} - (N+1)^2(N+2)X^{N+1} + 4X^2 + 2X\right]\Big/(X-1)^4 \quad , \tag{8}$$

where $\langle\cdots\rangle$ indicates the mean value in the stationary state (4).

Now we calculate the "dressed" atomic correlation functions $\langle R_{12}(\tau)R_{21}(0)\rangle$, $\langle R_{21}(\tau)R_{12}(0)\rangle$, $\langle R_{12}(0)R_{21}(\tau)\rangle$ and $\langle R_{21}(0)R_{12}(\tau)\rangle$ which are useful for a subsequent calculation of the spectrum of squeezing.

By applying the quantum fluctuation regression theorem (Lax 1968) and the secular approximation one finds the equations for "dressed" atomic correlation functions take the form

$$\frac{d}{d\tau}\langle R_{12}(\tau)R_{21}(0)\rangle = -iG\,\langle R_{12}(\tau)R_{21}(0)\rangle - \langle\Gamma_-(\tau)R_{12}(\tau)R_{21}(0)\rangle \quad , \tag{9}$$

$$\frac{d}{d\tau}\langle R_{21}(0)R_{12}(\tau)\rangle = -iG\,\langle R_{21}(0)R_{12}(\tau)\rangle - \langle R_{21}(0)R_{12}(\tau)\Gamma_+(\tau)\rangle \quad , \tag{10}$$

$$\frac{d}{d\tau}\langle R_{21}(\tau)R_{12}(0)\rangle = iG\,\langle R_{21}(\tau)R_{12}(0)\rangle - \langle\Gamma_+(\tau)R_{21}(\tau)R_{12}(0)\rangle \quad , \tag{11}$$

$$\frac{d}{d\tau}\langle R_{12}(0)R_{21}(\tau)\rangle = iG\,\langle R_{12}(0)R_{21}(\tau)\rangle - \langle R_{12}(0)R_{21}(\tau)\Gamma_-(\tau)\rangle \quad , \tag{12}$$

where

$$\Gamma_-(\tau) = \frac{\gamma_b}{2} + \frac{\gamma_a}{2}(1+\sin^2\alpha) + \frac{1}{2}(\gamma_a\sin^2\alpha - \gamma_b\cos^2\alpha)(N-2R(\tau)) , \tag{13}$$

$$\Gamma_+(\tau) = \frac{\gamma_b}{2}(1+2\cos^2\alpha) + \frac{\gamma_a}{2}\cos^2\alpha + \frac{1}{2}(\gamma_a\sin^2\alpha - \gamma_b\cos^2\alpha)(N-2R(\tau)) \quad . \tag{14}$$

After Gompagno and Persico (1982) we factorize

$$\begin{aligned}
\langle\Gamma_-(\tau)R_{12}(\tau)R_{21}(0)\rangle &= \langle\Gamma_-\rangle\cdot\langle R_{12}(\tau)R_{21}(0)\rangle \quad ,\\
\langle\Gamma_+(\tau)R_{21}(\tau)R_{12}(0)\rangle &= \langle\Gamma_+\rangle\cdot\langle R_{21}(\tau)R_{12}(0)\rangle \quad ,\\
\langle R_{21}(0)R_{12}(\tau)\Gamma_+(\tau)\rangle &= \langle\Gamma_+\rangle\cdot\langle R_{21}(0)R_{12}(\tau)\rangle \quad ,\\
\langle R_{12}(0)R_{21}(\tau)\Gamma_-(\tau)\rangle &= \langle\Gamma_-\rangle\cdot\langle R_{12}(0)R_{21}(\tau)\rangle \quad .
\end{aligned} \tag{15}$$

Using the stationary solution (4) it is easy to show that in the case of a large number of atoms the factorization (15) gives a smaller error with higher order than $N^{-1/2}$ in the calculation of the steady-state spectrum. By applying the factorization (15) equations (9–12) yield the "dressed" atomic correlation functions in the form

$$\begin{aligned}
\langle R_{12}(\tau)R_{21}(0)\rangle &= e^{(-iG-\langle\Gamma_-\rangle)\tau}\langle R_{12}R_{21}\rangle \quad ,\\
\langle R_{21}(\tau)R_{12}(0)\rangle &= e^{(iG-\langle\Gamma_+\rangle)\tau}\langle R_{21}R_{12}\rangle \quad ,\\
\langle R_{21}(0)R_{12}(\tau)\rangle &= e^{(-iG-\langle\Gamma_+\rangle)\tau}\langle R_{21}R_{12}\rangle \quad ,\\
\langle R_{12}(0)R_{21}(\tau)\rangle &= e^{(iG-\langle\Gamma_-\rangle)\tau}\cdot\langle R_{12}R_{21}\rangle \quad .
\end{aligned} \tag{16}$$

The values $\langle\Gamma_+\rangle$ and $\langle\Gamma_-\rangle$ are the spectral widths of the spectrum bands centered at frequencies $\Omega_2 \pm G$ and $\Omega_1 \pm G$ respectively (Bogolubov et al 1985). As is easily seen from the relations (13-14), in the case of $\chi = 1$ the spectral widths $\langle\Gamma_+\rangle$ and $\langle\Gamma_-\rangle$ are the same as in the single-atom spectrum. For the case of $\chi \neq 1$ and a large number of atoms N the spectral widths $\langle\Gamma_+\rangle$ and $\langle\Gamma_-\rangle$ are proportional to N and are approximately equal. With the use of the commutation relations (5-6) the statistical moments $\langle R_{12}R_{21}\rangle$ and $\langle R_{21}R_{12}\rangle$ can be written in the form

$$\langle R_{12}R_{21}\rangle = \langle R_{32}R_{23}\rangle = \frac{1}{2}(N+1)\langle R\rangle - \frac{1}{2}\langle R^2\rangle \quad , \tag{17}$$

$$\langle R_{21}R_{12}\rangle = \langle R_{23}R_{32}\rangle = \frac{1}{2}(N-2)\langle R\rangle - \frac{1}{2}\langle R^2\rangle \quad , \tag{18}$$

where the quantities $\langle R\rangle$ and $\langle R^2\rangle$ can be found in (7-8).

III. Spectrum of squeezing in fluorescent fields

In this section we discuss the squeezing in the fluorescent field in the collective double resonant process and give a spectral picture of squeezing. In the radiation zone, the positive frequency parts of the electric fields $E_a^{(+)}(\vec{x},t)$ and $E_b^{(+)}(\vec{x},t)$, corresponding to the lower and upper atomic transition $|2\rangle \to |1\rangle$ and $|3\rangle$ to $|2\rangle$ have the form (Mandel 1982, Walls and Zoller 1981, Collet et al 1984)

$$E_a^{(+)}(\vec{x},t) = E_{a,free}^{(+)}(\vec{x},t) + \psi_a(\vec{x})J_{12}(t-\lambda/c)\cdot \exp\left[-i\Omega_1(t-\lambda/c)\right] \quad , \tag{19}$$

$$E_b^{(+)}(\vec{x},t) = E_{b,free}^{(+)}(\vec{x},t) + \psi_b(\vec{x})J_{23}(t-\lambda/c)\cdot \exp\left[-i\Omega_2(t-\lambda/c)\right] \quad , \tag{20}$$

where $\psi_a(\vec{x})$ and $\psi_b(\vec{x})$ are geometrical factors, $\vec{x}$ is the vector of the observation point, $\lambda = |\vec{x}|$.
With the use of the canonical transformation (3) the atomic collective angular momenta $J_{12}(t)$ and $J_{23}(t)$ have the structure

$$J_{12}(t) = \frac{1}{2}\cos\alpha\, D_3(t) + \frac{1}{2}\cos\alpha\left(\tilde{R}_{31}(t)e^{2iGt} - \tilde{R}_{13}(t)e^{-2iGt}\right) - \frac{1}{\sqrt{2}}\sin\alpha\left(\tilde{R}_{23}(t)e^{-iGt} + \tilde{R}_{21}(t)e^{-iGt}\right) \tag{21}$$

$$J_{23}(t) = \frac{1}{2}\sin\alpha\, D_3(t) + \frac{1}{2}\sin\alpha\left(\tilde{R}_{31}(t)e^{2iGt} - \tilde{R}_{13}(t)e^{-2iGt}\right) + \frac{1}{\sqrt{2}}\cos\alpha\left(\tilde{R}_{12}(t)e^{-iGt} + \tilde{R}_{32}(t)e^{iGt}\right), \tag{22}$$

where $D_3 = R_{33} - R_{11} = R - 2R_{11}$,

$$R_{31}(t) = \tilde{R}_{31}(t)e^{2iGt}, \quad R_{13}(t) = \tilde{R}_{13}(t)e^{-2iGt},$$
$$R_{21}(t) = \tilde{R}_{21}(t)e^{iGt}, \quad R_{12}(t) = \tilde{R}_{12}(t)e^{-iGt},$$
$$R_{32}(t) = \tilde{R}_{32}(t)e^{iGt}, \quad R_{23}(t) = \tilde{R}_{23}(t)e^{-iGt}.$$

In the secular approximation $\tilde{R}_{k\ell}(t)$ $(k,\ell = 1,2,3)$ are slowly varying "dressed" atomic operators.

In subsequent calculations we drop out the free parts $E_{a,free}^{(+)}(\vec{x},t)$ $E_{b,free}^{(+)}(\vec{x},t)$ in relations (19-20) which do not affect the normally ordered variance of fluorescent fields $E_a^{(+)}(\vec{x},t)$ and $E_b^{(+)}(\vec{x},t)$. For the stationary limit the delayed time contribution has been also ignored (Cresser 1983, Collet et al 1984).

After the works (Aspect et al 1980, Apanasevich et al 1979, Bogolubov et al 1985) we can consider the operators

$$-\frac{1}{2}\cos\alpha\,\tilde{R}_{12}(t),\ -\frac{1}{\sqrt{2}}\sin\alpha\,\tilde{R}_{23}(t),\ \frac{1}{2}\cos\alpha\,D_3(t),\ -\frac{1}{\sqrt{2}}\sin\alpha\,\tilde{R}_{21}(t)$$

and $\frac{1}{2}\cos\alpha\,\tilde{R}_{31}(t)$ as amplitude operators for the sources of the spectrum bands centered at the frequencies $\Omega_1 - 2G, \Omega_1 - G, \Omega_1, \Omega_1 + G$ and $\Omega_1 + 2G$; and for simplicity we denote these operators by L_{-2}, L_{-1}, L_0, L_1 and L_2 , respectively. Analogously, the operators $-\frac{1}{2}\sin\alpha\,\tilde{R}_{13}(t)$, $\frac{1}{\sqrt{2}}\cos\alpha\,\tilde{R}_{12}(t)$, $\frac{1}{2}\sin\alpha\,D_3(t)$ $\frac{1}{\sqrt{2}}\cos\alpha\,\tilde{R}_{32}(t)$ and $\frac{1}{2}\sin\alpha\,\tilde{R}_{31}(t)$ can be considered as amplitude-operators for the sources of the spectrum bands centered at the frequencies Ω_2-2G, Ω_2-G, Ω_2, Ω_2+G and Ω_2+2G and for simplicity we denote these operators U_{-2}, U_{-1}, U_0, U_1 and U_2 , respectively.

In the case of intense external fields when condition (2) is fulfilled, it is easy to show (Bogolubov et al. 1987) that squeezing is absent for all separate spectral bands L_k and U_k $(k=0,\pm1,\pm2)$, for the whole fluorescent fields corresponding to the lower and upper atomic transitions. The squeezing exists only in the mixture of two spectrum bands L_1 and U_{-1} or L_{-1} and U_1 ; moreover, the degree of squeezing in the mixture of L_1 and U_{-1} and in the mixture of L_{-1} and U_1 is the same.

Further, we analyse a spectral picture of squeezing in the mixture of two spectrum bands L_1 and U_{-1} (or L_{-1} and U_1). With the use of the relations (19-22) and the secular approximation one finds the Fourier transform of the field $E_a^{(+)}(\vec{x},t)$ at a frequency ν_1 located near Ω_1-G and field $E_b^{(+)}(\vec{x},t)$ at a frequency ν_2 located near Ω_2+G in the form

$$\tilde{E}_a^{(+)}(\vec{x},\Omega_1-G+\varepsilon_1) = -\frac{1}{\sqrt{2}}\sin\alpha\;\Psi_a(\vec{x})\,\tilde{R}_{21}(\varepsilon_1)\;, \qquad (23)$$

$$\tilde{E}_b^{(+)}(\vec{x},\Omega_2+G-\varepsilon_2) = \frac{1}{\sqrt{2}}\cos\alpha\;\Psi_b(\vec{x})\,\tilde{R}_{12}(\varepsilon_2)\;,$$

where

$$\varepsilon_1 = \nu_1-(\Omega_1-G)\;,\quad \varepsilon_2 = \Omega_2+\Omega-\nu_2\;.$$

The mixture of the two frequency components $\tilde{E}_a^{(+)}(\vec{x},\nu_1)$ and $\tilde{E}_b^{(+)}(\vec{x},\nu_2)$ is defined as

$$\tilde{E}_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{\sqrt{2}}\left[\tilde{E}_a^{(+)}(\vec{x},\Omega_1-G+\varepsilon_1)+\tilde{E}_b^{(+)}(\vec{x},\Omega_2+G-\varepsilon_2)\right],$$

$$\tilde{E}_M^{(-)}(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{\sqrt{2}}\left[\tilde{E}_a^{(-)}(\vec{x},\Omega_1-G+\varepsilon_2)+\tilde{E}_b^{(-)}(\vec{x},\Omega_2+G-\varepsilon_1)\right].$$

After Collet et al.(1984) the quadrature phase components for the mixture $\tilde{E}_M^{(\pm)}(\vec{x},\varepsilon_1,\varepsilon_2)$ are defined as:

$$\tilde{E}_\theta(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{2}\left[\tilde{E}_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2)e^{i\theta}+\tilde{E}_M^{(-)}(\vec{x},\varepsilon_1,\varepsilon_2)e^{-i\theta}\right] \qquad (24)$$

which for $\theta=0$ and $\theta=\frac{\pi}{2}$ coincide with the in-phase ($\tilde{E}_1(\vec{x},\varepsilon_1,\varepsilon_2)$) and out-phase ($\tilde{E}_2(\vec{x},\varepsilon_1,\varepsilon_2)$) components, respectively. By using the stationary atomic correlation functions (16) one finds

$$\langle\tilde{R}_{21}(\varepsilon_1)\tilde{R}_{12}(\varepsilon_2)\rangle = \delta(\varepsilon_1-\varepsilon_2)\cdot\langle R_{21}R_{12}\rangle\cdot\frac{2\langle\Gamma_+\rangle}{\varepsilon_1^2+\langle\Gamma_+\rangle^2}\;, \qquad (25)$$

$$\langle\tilde{R}_{12}(\varepsilon_2)\tilde{R}_{21}(\varepsilon_1)\rangle = \delta(\varepsilon_1-\varepsilon_2)\langle R_{12}R_{21}\rangle\cdot\frac{2\langle\Gamma_-\rangle}{\varepsilon_1^2+\langle\Gamma_-\rangle^2}\;. \qquad (26)$$

The normally ordered variance of the quadrature phase component $\tilde{E}_\theta(\varepsilon_1,\varepsilon_2)$ can be found using the relations (23-24) and (25-26)

$$\langle:(\Delta\tilde{E}_\theta(\varepsilon_1,\varepsilon_2))^2:\rangle = \delta(\varepsilon_1-\varepsilon_2)\cdot S_\theta(\varepsilon_1) \qquad (27)$$

with

$$S_\theta(\varepsilon_1) = \frac{1}{2}\left(\gamma_a\sin^2\alpha-\cos2\theta\sin\alpha\cos\alpha\,(\gamma_a\gamma_b)^{1/2}\right)\cdot\langle R_{12}R_{21}\rangle\cdot\frac{\langle\Gamma_-\rangle}{\varepsilon_1^2+\langle\Gamma_-\rangle^2}+\frac{1}{2}\left(\gamma_b\cos^2\alpha-\cos2\theta\,\sin\alpha\cos\alpha\,(\gamma_a\gamma_b)^{1/2}\right)\cdot\frac{\langle\Gamma_+\rangle}{\varepsilon_1^2+\langle\Gamma_+\rangle^2}\;, \qquad (28)$$

where we identify $S_\phi(\varepsilon_1)$ with the phase sensitive normally ordered spectrum of the operator $\tilde{E}_\phi(\varepsilon_1, \varepsilon_2)$.

In relations (27-28), for simplicity, we have dropped the argument $\vec{x}$ (the position of the detector). Moreover, we have followed the usual convention and have renormalized the correlation functions to the total flux (Mandel 1982, Collet et al. 1984). The integration of the spectrum $S_\phi(\varepsilon_1)$ over all frequencies gives the following expression for the normally ordered variance of mixture of two spectrum bands L_1 and U_{-1} (or L_{-1} and U_1):

$$\langle : (\Delta E_\phi)^2 : \rangle = \frac{1}{4}\gamma_a \sin^2\alpha \langle R_{12} R_{21} \rangle + $$

$$+ \frac{1}{4}\gamma_b \cos^2\alpha \langle R_{21} R_{12} \rangle - \frac{1}{4}\cos 2\phi\, (\gamma_a \gamma_b)^{1/2} \cdot$$

$$\cdot \sin\alpha \cdot \cos\alpha \, (\langle R_{12} R_{21} \rangle + \langle R_{21} R_{12} \rangle) \quad ,$$

thus, squeezing in the mixture of two spectrum bands L_1 and U_{-1}, in agreement with the work by Bogolubov et al. (1987). In fig.2, the spectrum $S_1(\varepsilon_1)$ (i.e. when $\phi = 0$) is plotted as a function of $\varepsilon_1 / N\gamma_a$ for the case of $ctg^2\alpha = 0.8$ and $\gamma_b/\gamma_a = 1$. The peak degree of squeezing occurs at the point $\varepsilon_1 = 0$, i.e. when the two frequency components $\tilde{E}_a^{+}(\nu_1)$ and $\tilde{E}_b^{(+)}(\nu_2)$ are located at the frequencies $\Omega_1 + G$ and $\Omega_2 - G$ (or $\Omega_1 - G$ and $\Omega_2 + G$), respectively. The peak squeezing $S_1(\varepsilon_1 = 0)$ as a function of the parameter $ctg^2\alpha$ for the case of $\gamma_b/\gamma_a = 1$ (solid curves) and $\gamma_b/\gamma_a = 1.8$ (dashed curves) is plotted in fig.3. For the case of $X = 1$ we have

$$\langle R_{12} R_{21} \rangle = \langle R_{21} R_{12} \rangle \qquad \text{and}$$

as a result, $S_\phi(\varepsilon_1) \geqslant 0$, thus squeezing is absent in this

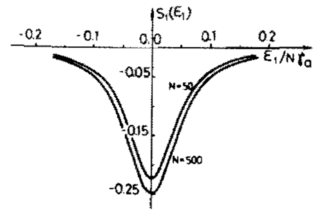

Fig. 2. Spectrum $S_1(\mathcal{E}_1)$ as a function of $\mathcal{E}_1/N\gamma_a$ for the case of $ctg^2\alpha = 0.8$, $\gamma_b/\gamma_a = 1$.

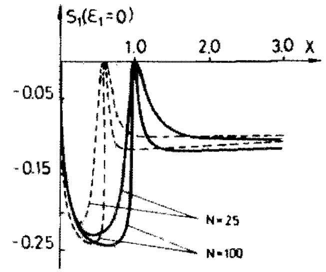

Fig. 3. Peak of spectral squeezing $S_1(\mathcal{E}_1 = 0)$ as a function of the parameter $ctg^2\alpha$ for the case of $\gamma_b/\gamma_a = 1$ (solid curves) and $\gamma_b/\gamma_a = 1.8$ (dashed curves).

case (see fig.3). Squeezing is also absent for the case of $X = 0$ and $X \to \infty$. As is seen from figs. 2-3, a large squeezing occurs, and for the case of $X < 1$ and $N \gg 1$ the squeezing tends to a limited value $S_1 = -0.25$ (Perfect squeezing). We note that though the degree of squeezing for the mixture of two frequency components $\tilde{E}_a^{(+)}(\Omega_1 + G)$ and $\tilde{E}_b^{(+)}(\Omega_2 - G)$ (or $\tilde{E}_a^{(+)}(\Omega_1 - G)$ and $\tilde{E}_b^{(+)}(\Omega_2 + G)$) is large and nearly perfect squeezing is possible, as mentioned above, the squeezing is absence for any separate spectrum bands L_k, U_k ($k = 0, \pm 1, \pm 2$) and any separate frequency components.

IV. Physical spectrum of squeezing

In this section we discuss a possible experimental scheme using the Fabry-Perot frequency analysers. We investigate the squeezing of the mixture of two fields $E_{D,a}^{(+)}(t)$ and $E_{D,b}^{(+)}(t)$ frequency-filtered from fluorescent fields by two identical (without losses) Fabry-Perot analysers which have the filter frequencies equal to

$$\omega_a^{(f)} = \Omega_1 - G - \delta \ , \quad \omega_b^{(f)} = \Omega_2 + G + \delta \tag{29}$$

(or $\omega_a^{(f)} = \Omega_1 + G - \delta \ , \quad \omega_b^{(f)} = \Omega_2 - G + \delta$

After the works (Cresser 1983, Collet et al 1974) the fields $E^{(+)}_{D,a}(t)$ and $E^{(+)}_{D,b}(t)$ can be written as

$$E^{(+)}_{D,a}(\delta, t) = \int_{-\infty}^{+\infty} J_1(t-t')\, e^{-i\omega_a^{(f)}(t-t')} \cdot E_a^{(+)}(t')\, dt' \tag{30}$$

$$= \int_{-\infty}^{+\infty} e^{i(\Omega_1 - G + \varepsilon_1)t} \cdot \tilde{J}_a(\varepsilon_1 + \delta) . \tilde{E}_a^{(+)}(\Omega_1 - G + \varepsilon_1)\, d\varepsilon_1$$

$$E^{(+)}_{D,b}(t) = \int_{-\infty}^{+\infty} J_2(t-t')\, e^{-i\omega_b^{(f)}(t-t')} E_b^{(+)}(t')\, dt' =$$

$$= \int_{-\infty}^{+\infty} e^{i(\Omega_2 + G - \varepsilon_2)t}\, \tilde{J}_b^{*}(\varepsilon_2 + \delta_2)\, \tilde{E}_b^{(+)}(\Omega_2 + G - \varepsilon_2)\, d\varepsilon_2$$

where

$$J_a(\tau) = \theta(\tau)\ (2\Gamma_a)^{1/2} . \ e^{-\Gamma_a \tau} \ ,$$

$$J_b(\tau) = \theta(\tau)\ (2\Gamma_b)^{1/2} . \ e^{-\Gamma_b \tau} \ ,$$

with Γ_a, Γ_b being the filter bandwidths; and $\tilde{J}_a(\varepsilon_1 + \delta)$ and $\tilde{J}_b(\varepsilon_2 + \delta)$, the Fourier transforms of $J_a(\tau)$ and $J_b(\tau)$, respectively.

We define an electric field operator with phase θ or the mixture of $E^{(+)}_{D,a}(t)$ and $E^{(+)}_{D,b}(t)$ as (24)

$$E_\theta(t) = \frac{1}{2}\left[E^{(+)}_{MD}(\delta, t)\, e^{i\theta} + E^{(-)}_{MD}(\delta, t)\, e^{-i\theta} \right] ,$$

where

$$E^{(+)}_{MD}(\delta, t) = \frac{1}{\sqrt{2}} \left[E^{(+)}_{D,a}(\delta, t) + E^{(+)}_{D,b}(\delta, t) \right] ,$$

$$E^{(-)}_{MD}(\delta, t) = \frac{1}{\sqrt{2}} \left[E^{(-)}_{D,a}(\delta, t) + E^{(-)}_{D,b}(\delta, t) \right] .$$

With the use of relations (23), (25-26), and (30-31) one finds the normally ordered variance for the quadrature phase component $E_\theta(\delta, t)$ in the form

$$S_{p,\theta}(\delta, \Gamma) = \langle : (\Delta E_\theta)^2 : \rangle = \frac{1}{2}\left(\gamma_b \cos^2\alpha - \cos 2\theta . \sin\alpha . \cos\alpha (\gamma_a \gamma_b)^{1/2}\right) . \langle R_{21} R_{12} \rangle . L_+(\delta, \Gamma) +$$
$$+ \frac{1}{2}\left(\gamma_a \sin^2\alpha - \cos 2\theta . \sin\alpha . \cos\alpha . (\gamma_a \gamma_b)^{1/2}\right) . \langle R_{12} R_{21} \rangle . L_-(\delta, \Gamma) \ , \tag{32}$$

where

$$L_+(\delta, \Gamma) = \frac{1}{\pi} . \int_{-\infty}^{+\infty} \frac{\Gamma . \langle \Gamma_+ \rangle\, d\varepsilon_1}{(\langle \Gamma_+ \rangle^2 + \varepsilon_1^2)(\Gamma^2 + (\delta + \varepsilon_1)^2)}$$

$$L_-(\delta, \Gamma) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\Gamma . \langle \Gamma_- \rangle\, d\varepsilon_1}{(\langle \Gamma_- \rangle^2 + \varepsilon_1^2)(\Gamma^2 + (\delta + \varepsilon_1)^2)}$$

Here for simplicity two filter bandwidhts are assumed to be equal $\Gamma_a = \Gamma_b = \Gamma$ and $e^{\pm(\Omega_1 + \Omega_2)t}$ is chosen to be equal to unity. The quantity $S_{p,\theta}(\delta, \Gamma)$ describes the squeezing in the quadrature phase component $E_\theta(\delta)$ and can be considered as a physical spectrum of squeezing in the mixture of the two spectrum bands L_1 and U_{-1} (or L_{-1} and U_1). The peak squeezing occurs for the case of $\delta = 0$ and has the form

$$S_{p,\theta}(\Gamma, \delta = 0) = \frac{1}{2}\left(\gamma_a \cos^2\alpha - \cos 2\theta . \sin\alpha . \cos\alpha . (\gamma_a \gamma_b)^{1/2}\right) . \frac{\langle R_{21} R_{12} \rangle}{\Gamma + \langle \Gamma_+ \rangle} + \frac{1}{2}\left(\gamma_b \sin^2\alpha - \cos 2\theta . \sin\alpha . \cos\alpha . (\gamma_a \gamma_b)^{1/2}\right) . \frac{\langle R_{12} R_{21} \rangle}{\Gamma + \langle \Gamma_- \rangle} . \tag{33}$$

It is easy to see from the relation (33) that in the limit case when $\Gamma \ll \langle \Gamma_+ \rangle, \langle \Gamma_- \rangle$ the peak squeezing $S_{P,\phi}(\Gamma, \delta = 0)$ coincides with the peak squeezing $S_{\phi}(\varepsilon_1 = 0)$ in the relation (28); and in this case the squeezing tends to a limit value $S_{P,\phi} = -1/4$ (Perfect squeezing) when $X < 1$ and the number of atoms is large.

In the case of $\Gamma \approx \langle \Gamma_+ \rangle, \langle \Gamma_- \rangle$ the quantity $S_{P,\phi}(\Gamma, \phi = 0)$ describes the squeezing of the mixture of two spectral bands L_1 and U_{-1} (or L_{-1} and U_1) and can reach the value -1/8 (50 per cent of squeezing), thus a maximum squeezing is in agreement with our previous work (Bogolubov et al. 1986).

To conclude, we note that in our case the collective effects increase the degree of squeezing (see figs. 2-3), and the described scheme can be used as a possible experimental scheme to observe spectral squeezing with a large degree of squeezing.

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Боголюбов Н.Н./мл./, Шумовский А.С., Чан Куанг E17-87-139
Спектр сжатия света в коллективном двойном оптическом резонансе

Исследовано спектральное сжатие флуоресцентного поля в коллективном двойном оптическом резонансе. Обсуждена возможная экспериментальная схема детектирования спектрального сжатия и показано условие получения почти стопроцентного сжатия.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Bogolubov N.N.Jr., Shumovsky A.S., Tran Quang E17-87-139
Spectrum of Squeezing in Collective Double Optical Resonance

The spectral squeezing in a fluorescent field of collective double resonant process is investigated. A possible experimental scheme with the use of the Fabry-Perot filters for observation of spectral squeezing is also discussed. The condition for a nearly perfect squeezing is shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987