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SPECTRUM OF SQUEEZING
# IN COLLECTIVE RESONANCE
FLUORESCENCE

Squeezed states of the radiation field that may have potential application in low-noise precision measurement and detection, have become the subject of extensive theoretical[1-13] and experimental works.[14,15] . The potential applications of the single-mode[16] and two-mode[17] squeezed states in the detectors of the gravity waves have been considered.

The squeezing in the resonance fluorescence has been investigated in [7-13] . In particular, the spectrum of squeezing in a one-atom fluorescent field has been calculated by Walls and coworkers in ref. [12] , where the possible maximum squeezing was considerably less than a perfect squeezing and absent for the case of intense external field.

In our previous work [13] , we have shown the existence of a large squeezing in the mixture of two sidebands of the collective resonance fluerescence in the case of intense external field when squeezing is absent for the whole fluorescent field and for the separate spectral line of Mollow's triplet.

In the present paper we wish to give a spectral analysis of the squeezing in the mixture of two sidebands of the collective fluorescent field. The calculation based on the experimental scheme with the use of the Fabry-perot spectrum analysers is discussed too.

We consider the resonant fluorescence from N two-level atoms of the Dicke model interacting with a monochromatic driving field with the frequency ω_L and an emitted field (Fig.1). In treating the external field classically and using the Markov and rotating waves approximation with respect to the coupl-
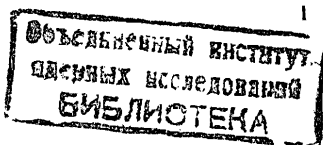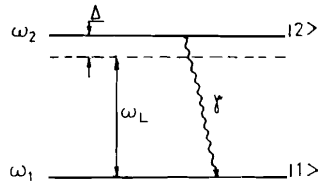
Fig. 1. Two - level atoms interacting with the external field and with the emitted field.

ing of the system with the vacuum field, one can find a master equation for the reduced density matrix ϱ for the system alone in the form [18]

$$\frac{\partial \varrho}{\partial t} = -i \left[\frac{\Delta}{2} (J_{22} - J_{11}) + G (J_{12} + J_{21}) , \varrho \right]$$
$$- \frac{\gamma}{2} \left(J_{21} J_{12} \varrho - J_{12} \varrho J_{21} + H.C. \right) \equiv L \varrho , \qquad (1)$$

where γ is the transition rate $|2\rangle \rightarrow |1\rangle$ for spontaneous radiation; $\Delta = \omega_{21} - \omega_L$ is the frequency detuning of a resonance; $G = -d_{21} E_0$ is the matrix element of the driving field and atomic interaction, J_{kj} (k , j = 1,2) are the collective angular momenta of atoms having the following form in the Schwinger representation [19]

$$J_{kj} = C_k^+ C_j \qquad (k,j = 1, 2) ,$$

where C_k obey the boson commutation relation

$$[C_k , C_j] = \delta_{kj} .$$

Further, we investigate only the case of an intense driving field or a large deturning Δ , so that

$$\Omega = \left(\frac{1}{4} \Delta^2 + G^2 \right)^{1/2} \gg N\gamma . \qquad (2)$$

As in the work [13] , we make the canonical transformation permitting us to move from bare two-level atoms to atoms "dressed" by external field

$$C_1 = \cos\varphi \, Q_1 + \sin\varphi \, Q_2 , \qquad (3)$$
$$C_2 = -\sin\varphi \, Q_1 + \cos\varphi \, Q_2 ,$$

where $\mathrm{tg}\, 2\varphi = 2G/\Delta$ and Q_k satisfy the boson commutation relation

$$[Q_k , Q_j] = \delta_{kj} ,$$

so that the collective angular momenta for the dressed atoms R_{kj} (k , j = 1,2) satisfy the commutation relation

$$[R_{kj} , R_{k'j'}] = R_{kj'} \delta_{k'j} - R_{k'j} \delta_{kj'} . \qquad (4)$$

After making the canonical transformation (3) and in the case when the condition (2) is fulfiled, one can use the secular approximation [20,13] , i.e. neglect the part of the Liouville operator appeared in eq.(1) which contains rapidly oscillating terms at frequencies 2Ω and 4Ω , and find the stationary solution of the master equation in the form [13]

$$\tilde{\varrho} = U \varrho U^+ = Z^{-1} \sum_{N_1 = 0}^{N} X^{N_1} |N_1\rangle\langle N_1| , \qquad (5)$$

where U is a unitary operator representing the canonical transformation (3) ,

$$X = \mathrm{ctg}^4\varphi , \quad Z = (X^{N+1} - 1)/(X-1) ,$$

$|N_1\rangle$ is an eigenstate of the operators R_{11} , $\hat{N} = R_{11} + R_{22}$.

By using solution (5), one can calculate the statistical moment $\langle R_{11}^n \rangle$, where $\langle B \rangle$ is the mean value of an operator B insteady-state (5). In particular, we find

$$\langle R_{11} \rangle = Z^{-1} (N X^{N+2} - (N+1) X^{N+1} + X) / (X-1)^2 \qquad (6)$$

$$\langle R_{11}^2 \rangle = Z^{-1} \left(N^2 X^{N+3} - (2N^2 + 2N + 1) X^{N+2} + (N+1)^2 X^{N+1} - \right.$$
$$\left. - X^2 - X \right) / (X-1)^3 . \qquad (7)$$

In the case of a large number of atoms, by application of the quantum fluctuation regression theorem and the method of factorization by Compagno and Persico [21] , one finds the sta-

tionary atomic correlation functions $\langle R_{12}(\tau) R_{21}(0) \rangle$, $\langle R_{21}(\tau) R_{12}(0) \rangle$, $\langle R_{12}(0) R_{21}(\tau) \rangle$ and $\langle R_{21}(0) R_{12}(\tau) \rangle$ in the form

$$\langle R_{12}(\tau) R_{21}(0) \rangle = e^{(-2i\Omega - \Gamma_-)\tau} \langle R_{12} R_{21} \rangle , \qquad (8)$$

$$\langle R_{21}(0) R_{12}(\tau) \rangle = e^{(-2i\Omega - \Gamma_+)\tau} \langle R_{21} R_{12} \rangle , \qquad (9)$$

$$\langle R_{12}(0) R_{21}(\tau) \rangle = e^{(2i\Omega - \Gamma_-)\tau} \langle R_{12} R_{21} \rangle , \qquad (10)$$

$$\langle R_{21}(\tau) R_{12}(0) \rangle = e^{(2i\Omega - \Gamma_+)\tau} \langle R_{21} R_{12} \rangle , \qquad (11)$$

where

$$\Gamma_- = \gamma \left[\sin^2\varsigma + \sin^2\varsigma \cdot \cos^2\varsigma + \frac{1}{2}(\sin^2\varsigma - \cos^2\varsigma)(N - 2\langle R_{11} \rangle) \right] , \qquad (12)$$

$$\Gamma_+ = \gamma \left[\sin^2\varsigma \cdot \cos^2\varsigma + \cos^2\varsigma + \frac{1}{2}(\sin^2\varsigma - \cos^2\varsigma)(N - 2\langle R_{11} \rangle) \right] \qquad (13)$$

are the spectral widths of the two sidebands centered at the frequencies $\omega_L - 2\Omega$ and $\omega_L + 2\Omega$, respectively.

Using the commutation relation (4), one can write the statistical moments $\langle R_{21} R_{12} \rangle$ and $\langle R_{12} R_{21} \rangle$ in the form

$$\langle R_{21} R_{12} \rangle = -\langle R_{11}^2 \rangle + (N-1)\langle R_{11} \rangle + N , \qquad (14)$$

$$\langle R_{12} R_{21} \rangle = -\langle R_{11}^2 \rangle + (N+1)\langle R_{11} \rangle , \qquad (15)$$

where statistical moments $\langle R_{11} \rangle$ and $\langle R_{11}^2 \rangle$ can be found in (6-7). Further, we consider the normally-ordered variance in the mixture of two frequency components of two sidebands. In the radiation zone, the positive frequency part of the electric field has the form [8,12]

$$E^{(+)}(\vec{x}, t) = E^{(+)}_{free}(\vec{x}, t) + \psi(\vec{x}) \cdot J_{12}(t - \lambda/c) \cdot e^{-i\omega_L(t-\lambda/c)} , \qquad (17)$$

where $\psi(\vec{x})$ is a geometrical factor, $\lambda = |\vec{x}|$.

With the use of the canonical transformation (3), the atomic collective angular moment $J_{12}(t)$ has the structure

$$J_{12}(t) = \sin\varsigma \cos\varsigma (R_{22}(t) - R_{11}(t)) + \cos^2\varsigma \, \tilde{R}_{12}(t) e^{-2i\Omega t} - \sin^2\varsigma \, \tilde{R}_{21}(t) e^{2i\Omega t} , \qquad (18)$$

where

$$R_{12}(t) = \tilde{R}_{12}(t) e^{-2i\Omega t} , \quad R_{21}(t) = \tilde{R}_{21}(t) e^{2i\Omega t} .$$

In the secular approximation $\tilde{R}_{12}(t)$ and $\tilde{R}_{21}(t)$ are the slowly varying "dress" atomic operators.

In the following calculations we drop the free part $E^{(+)}_{free}$ in relation (17) which does not affect the normally ordered variance of fluorescent field. The delayed time contribution has been ignored too [22] in the stationary limit.

By applying the relations (17-18) and the secular approximation, one finds the Fourier transform of the field at two frequencies ν_1 and ν_2 located on the two sidebands in the form

$$\tilde{E}^{(+)}(\vec{x}, \omega_L + 2\Omega - \varepsilon_1) = \psi(\vec{x}) \cos^2\varsigma \, \tilde{R}_{12}(\varepsilon_1) , \qquad (19)$$

$$\tilde{E}^{(+)}(\vec{x}, \omega_L - 2\Omega + \varepsilon_2) = -\psi(\vec{x}) \cdot \sin^2\varsigma \, \tilde{R}_{21}(\varepsilon_2) ,$$

where

$$\varepsilon_1 = \omega_L + 2\Omega - \nu_1 ,$$
$$\varepsilon_2 = -(\omega_L - 2\Omega) + \nu_2 .$$

The in-phase (E_1) and out-of-phase (E_2) components of the mixture of two frequency components on the two sidebands of fluorescent field are defined as

$$\tilde{E}_1(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{2}\left[\tilde{E}_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2) + \tilde{E}_M^{(-)}(\vec{x},\varepsilon_1,\varepsilon_2)\right], \quad (20)$$

$$\tilde{E}_2(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{-i}{2}\left[\tilde{E}_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2) - \tilde{E}_M^{(-)}(\vec{x},\varepsilon_1,\varepsilon_2)\right],$$

where

$$E_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{\sqrt{2}}\left[\tilde{E}^{(+)}(\vec{x},\omega_L+2\Omega-\varepsilon_1) + \tilde{E}^{(+)}(\vec{x},\omega_L-2\Omega+\varepsilon_2)\right],$$

$$E_M^{(+)}(\vec{x},\varepsilon_1,\varepsilon_2) = \frac{1}{\sqrt{2}}\left[\tilde{E}^{(-)}(\vec{x},\omega_L+2\Omega-\varepsilon_2) + \tilde{E}^{(-)}(\vec{x},\omega_L-2\Omega+\varepsilon_1)\right].$$

By using the stationary atomic correlation functions (8-11), one finds

$$\langle\tilde{R}_{21}(\varepsilon_1)\tilde{R}_{12}(\varepsilon_2)\rangle = \delta(\varepsilon_1-\varepsilon_2)\langle R_{21}R_{12}\rangle_s \cdot \frac{2\Gamma_+}{\varepsilon_1^2+\Gamma_+^2}, \quad (21)$$

$$\langle\tilde{R}_{12}(\varepsilon_2)\tilde{R}_{21}(\varepsilon_1)\rangle = \delta(\varepsilon_1-\varepsilon_2)\langle R_{12}R_{21}\rangle_s \cdot \frac{2\Gamma_-}{\varepsilon_1^2+\Gamma_-^2}. \quad (22)$$

By applying relations (19-22), the normally-ordered variance of the operators $\tilde{E}_{1,2}(\varepsilon_1,\varepsilon_2)$ is

$$\langle :(\Delta\tilde{E}_{1,2}(\varepsilon_1,\varepsilon_2))^2: \rangle = \delta(\varepsilon_1-\varepsilon_2) \cdot S(\varepsilon_1), \quad (23)$$

where

$$S_{1,2}(\varepsilon_1) = \frac{\gamma}{2}\left\{(\cos^4\varsigma \mp \sin^2\varsigma\cdot\cos^2\varsigma)\cdot\langle R_{21}R_{12}\rangle \cdot \right.$$

$$\left. \frac{\Gamma_+}{\varepsilon_1^2+\Gamma_+^2} + (\sin^4\varsigma \mp \sin^2\varsigma\cdot\cos^2\varsigma)\langle R_{12}R_{21}\rangle\cdot\frac{\Gamma_-}{\varepsilon_1^2+\Gamma_-^2}\right\}. \quad (24)$$

For simplicity, we have dropped the argument $\vec{x}$ (the position of the detector) in equation (23-24) and have followed the usual convention after the works [7,12] . The integration of the spectrum of squeezing $S_{1,2}(\varepsilon_1)$ over all frequencies gives the expression for the normally-ordered variance of the mixture of two sidebands given in our previous work[13] . One can easily see

from relation (24) that squeezing is absent for the case of exact resonance $X = 1$.

The spectrum of squeezing $S_1(\varepsilon_1)$ as a function of $\frac{\varepsilon_1}{N\gamma}$ for the case of $X = 0.8$ and various numbers of atoms are plotted in fig. 2. The spectrum of squeezing $S_1(\varepsilon_1)$ has the Lorentzian form with an optimal degree of squeezing at the point $\varepsilon_1 = 0$, i.e. when two frequency components are located at the frequencies $\omega_L+2\Omega$ and $\omega_L-2\Omega$, respectively. The value $S_1(\varepsilon_1=0)$ as a function of the parameter X for various numbers of atoms is plotted in fig. 3. As is seen from figs. 2-3 the large squeezing is given. For the case of a large number of atoms the maximum squeezing tends to the limited value $S_1 = -0.25$ (perfect squeezing), as for the case of ideal parametric oscillators operating near the oscillation threshold[4] . We note that for the case of intense external field when the condition (2) is satisfied, the squeezing for the separate frequency component is absent. The maximum squeezing of the separate frequency component for the one-atom case [12] exists in a narrow band around $\nu=\omega_L$ for the case $\Omega = (3-\sqrt{7})\gamma/4$ and can reach the value -0.07 that is considerably less than the given about maximum squeezing of the mixture of two frequency component located near the two frequencies $\omega_L+2\Omega$ and $\omega_L-2\Omega$, respectively.
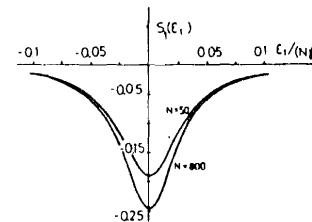
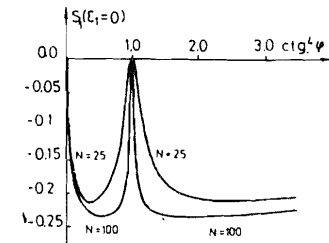Fig. 2. Spectrum of squeezing $S_1(\varepsilon_1)$ as a function of $\varepsilon_1/N\gamma$ for the case of $X = 0.8$.

Fig.3 Function $S_1(\varepsilon_1=0)$ is plotted against the parameter X .

In the simple interpretation, the large squeezing of two sidebands resonance fluorescence is connected with a strong correlation between the photons of the two sidebands (they are emitted by pairs)[23-25] and fluorescence process of two sidebands is considered as second-order nonlinear scattering. Finally, let us discuss the normally-ordered variance of the mixture of two fields $E_{D1}^{(+)}(t)$ and $E_{D2}^{(+)}(t)$ frequency filtered from fluorescence by two Fabry-Perot analysers which have the filter frequencies equal to

$$\omega_{D1} = \omega_L + 2\Omega \; ,$$
$$\omega_{D2} = \omega_L - 2\Omega \; . \tag{25}$$

After the works [12,22], the fields $E_{D1}^{(+)}(t)$ and $E_{D2}^{(+)}(t)$ can be written as

$$E_{D1}^{(+)}(t) = \int_{-\infty}^{+\infty} J_1(t-t') e^{-i(\omega_L + 2\Omega)(t-t')} E^+(t') dt' =$$
$$= \int_{-\infty}^{+\infty} e^{i(\omega_L + 2\Omega - \varepsilon_1)t} \tilde{J}_1(-\varepsilon_1) \tilde{E}^{(+)}(\omega_L + 2\Omega - \varepsilon_1) \cdot d\varepsilon_1 \; , \tag{26}$$

$$E_{D2}^{(+)}(t) = \int_{-\infty}^{+\infty} J_2(t-t') e^{-i(\omega_L - 2\Omega)(t-t')} E^{(+)}(t') dt' =$$
$$= \int_{-\infty}^{+\infty} e^{i(\omega_L - 2\Omega + \varepsilon_2)t} \tilde{J}_2(\varepsilon_2) \cdot \tilde{E}^+(\omega_L - 2\Omega + \varepsilon_2) d\varepsilon_2 \; , \tag{27}$$

where
$$J_1(\tau) = \theta(\tau)(2\Gamma_1)^{1/2} e^{-\Gamma_1 \tau} \; ,$$

$$J_2(\tau) = \theta(\tau)(2\Gamma_2)^{1/2} e^{-\Gamma_2 \tau} \; ,$$

with Γ_1, Γ_2 are the filter bandwidths and $\tilde{J}_1(\varepsilon_1)$ and $\tilde{J}_2(\varepsilon_2)$ are the Fourier transform of $J_1(\tau)$ and $J_2(\tau)$, respectively. The in-phase ($E_1(t)$) and out-of-phase ($E_2(t)$)

components of the mixture of $E_{D1}^{(+)}(t)$ and $E_{D2}^{(+)}(t)$ can be defined as (20)

$$E_1(t) = \frac{1}{2}\left[E_{MD}^{(+)}(t) + E_{MD}^{(-)}(t) \right]$$
$$E_2(t) = \frac{-i}{2}\left[E_{MD}^{(+)}(t) - E_{MD}^{(-)}(t) \right] .$$

where

$$E_{MD}^{(+)}(t) = \frac{1}{\sqrt{2}}\left[E_{D1}^{(+)}(t) + E_{D2}^{(+)}(t) \right]$$

$$E_{MD}^{(-)}(t) = \frac{1}{\sqrt{2}}\left[E_{D1}^{(-)}(t) + E_{D2}^{(-)}(t) \right] .$$

By using the relations (19), and (21,22), one finds the normally-ordered variance of components $E_1(t)$ and $E_2(t)$ in the form:

$$\langle :(\Delta E_{1,2})^2: \rangle = \frac{\gamma}{2} \left\{ (\cos^4 \mathcal{G} \mp \sin^2 \mathcal{G} \cdot \cos^2 \mathcal{G}) \cdot \frac{\langle R_{21} R_{12} \rangle}{\Gamma_D + \Gamma_+} + (\sin^4 \mathcal{G} \mp \sin^2 \mathcal{G} \cdot \cos^2 \mathcal{G}) \cdot \frac{\langle R_{12} R_{21} \rangle}{\Gamma_D + \Gamma_-} \right\} , \tag{28}$$

where for simplicity two filter bandwidths are assumed to be equal, $\Gamma_1 = \Gamma_2 = \Gamma_D$.

It is easy to see from relation (28) that in the limited case of $\Gamma_D \ll \Gamma_+, \Gamma_-$, the value $\langle :(\Delta E_1)^2: \rangle$ coincides with the maximum squeezing S ($\varepsilon_1 = 0$) (see fig.3) and tends to the limited value $\langle :(\Delta E_1)^2: \rangle = -1/4$ for the case of large number of atoms. In the case when $\Gamma_D \sim \Gamma_+, \Gamma_-$ the value $\langle :(\Delta E_1)^2: \rangle$ describes the squeezing of the mixture of two sidebands and maximum squeezing can tend to the value $\langle :(\Delta E_1)^2: \rangle = -1/8$ when a number of atoms is large.

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Боголюбов Н.Н./мл./, Шумовский А.С.,   E17-87-102
Чан Куанг
Спектр сжатия в коллективной резонансной
флуоресценции

Представлен спектральный анализ сжатия в смеси двух крайних спектральных линий коллективной резонансной флуоресценции. Рассмотрено также влияние спектрофильтров Фабри-Перо. Для случая большого числа атомов сжатие может достигать предельного значения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Spectrum of Squeezing in Collective
Resonance Fluorescence

The spectral analysis of the squeezing in the mixture of two sidebands of collective resonance fluorescence is given. The scheme with use of the Fabry-Perot spectrum filters is also discussed. For the case of a large number of atoms the squeezing can reach the limited value.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.