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SQUEEZING OF LIGHT VIA NONDEGENERATE FOUR-WAVE MIXING IN A SYSTEM OF THREE-LEVEL ATOMS

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A squeezed state is a nonclassical state of light having the property that its noise in one electric-field quadrature is less than that of a coherent state (Walls 1983). A number of nonlinear optical systems susceptible to produce a squeezed state have been analysed theoretically. These include the degenerate parametric oscillator (Milburn and Walls 1981, Lugiato and Strimi 1982 b) , Collet and Gardiner 1984), four-wave mixing (Yuen and Shapiro 1979, Bondurant et al. 1984, Reid and Walls 1985 a, Reid et al. 1984, Klauder et al. 1986), resonance fluorescence (Walls and Zoller 1981, Mandel 1982, Loudon 1984a, Lakshmi and Agarwall 1984, Anantha et al. 1984), optical bistability (Lugiato and Strini 1982a; Reid and Walls 1985b), two-photon transition (Loudon 1984b, Savage and Walls 1986, Bogolubov et al. 1986) and others. Slusher et al. (1985) have recently reported observing a squeezed state in four-wave mixing experiment in an atomic beam of sodium.

In this letter, we present the squeezed-state generation via nondegenerate four-wave mixing in a system of three-level atoms. The collective atomic effects, cavity damping and the effects of atomic and field reservoirs are taken into account. For the case of a large number of atoms the system can give perfect squeezing.

The N three-level atoms concentrated in a region small compared to the wavelength of all the relevant radiation modes interact with two cavity modes $\vec{E}_1, \vec{E}_2$ and with two pumping waves E_3 , E_4 with frequencies $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 , respectively (Fig.1). The pumping fields $\vec{E}_3$ and $\vec{E}_4$ are assumed intense and can be treated classically. For simplicity the pumping field $\vec{E}_3$ is assumed to be in resonance with the level separation $\omega_2 - \omega_1 = \omega_{21}$ $(\hbar \equiv 1)$ and the pumping field $\vec{E}_4$ is assumed to be in resonance with $\omega_3 - \omega_2 = \omega_{32}$. Let a_1, a_1^+ and a_2, a_2^+ be the annihilation and creation operators of the modes $\vec{E}_1$ and $\vec{E}_2$, respectively.

The coherence part of the hamiltonian in the rotating wave approximation and interaction picture is

$$H_{coh} = \Delta_1 a_1^+ a_1 + \Delta_2 a_2^+ a_2 + G_{21}(J_{21} + J_{12}) + G_{32}(J_{32} + J_{23}) + g_1(J_{21} a_1 + a_1^+ J_{12}) + g_2(J_{32} a_2 + a_2^+ J_{23}), \tag{1}$$

where $\Delta_1 = \Omega_1 - \omega_{21}$, $\Delta_2 = \Omega_2 - \omega_{32}$

$$J_{ij} = \sum_{k=1}^{N} |i\rangle_k {}_k\langle j| \qquad (i,j = 1,2,3)$$

are the collective angular momenta of atoms

$$G_{21} = -\vec{d}_{21}\vec{E}_3, \quad G_{32} = -\vec{d}_{32}\vec{E}_4 \quad ; \quad g_1 = -\vec{d}_{21}\cdot\frac{\vec{E}_1}{|\vec{E}_1|}$$

and $g_2 = -\vec{d}_{32}\cdot\vec{E}_2/|\vec{E}_2|$, here $\vec{d}$ is the electric dipole operator for the atom. Let the fields $\vec{E}_1$ and $\vec{E}_2$ be initially in a coherent state. After Agarwal (1974), considering the operators a_1, a_1^+ and a_2, a_2^+ in Hamiltonian (1) as c-numbers and using the markovian approximation, one finds the master equation for the atomic system as:

$$\frac{\partial \rho}{\partial t} = -i\,[H_{coh}, \rho] + \left.\frac{\partial \rho}{\partial t}\right|_A = L\rho, \tag{2}$$

where the dissipative term for the atoms is

$$\left.\frac{\partial \rho}{\partial t}\right|_A = -\gamma_{21}(J_{21}J_{12}\rho - 2J_{12}\rho J_{21} + \rho J_{21}J_{12}) - \gamma_{32}(J_{32}J_{23}\rho - 2J_{23}\rho J_{32} + \rho J_{32}J_{23}),$$

where terms $2\gamma_{ij}$ are transition rates caused by the atomic reservoirs from level $|i\rangle$ to $|j\rangle$. As was done in the works by Schwinger (1965), Bogolubov et al. (1985), we introduce the Schwinger representation for angular momentum

$$J_{ij} = c_i^+ c_j \qquad (i,j = 1,2,3),$$

where c_i obey the boson commutation relation

$$[c_i, c_j^+] = \delta_{ij}. \tag{3}$$

Further, we shall consider only the case of intense pumping fields $\vec{E}_3$ and $\vec{E}_4$ so that

$$G = (G_{21}^2 + G_{32}^2)^{1/2} \gg N\gamma_{ij},\ g_{1,2}\cdot|\vec{E}_{1,2}| \tag{4}$$

and it is possible to develop an approximation scheme that allows us to obtain analytic results.

After performing the canonical transformation

$$\begin{aligned} c_3 &= -2^{-1/2}\sin\alpha\, Q_1 + \cos\alpha\, Q_2 + 2^{-1/2}\sin\alpha\, Q_3 \\ c_2 &= 2^{-1/2} Q_1 + 2^{-1/2} Q_3 \\ c_1 &= -2^{-1/2}\cos\alpha\, Q_1 - \sin\alpha\, Q_2 + 2^{-1/2}\cos\alpha\, Q_3 \end{aligned} \tag{5}$$

with $tg\,\alpha = G_{32}/G_{21}$, one finds that the Liouville operator appearing in equation (2) splits into two components L_0 and L_1. The component L_0 is slowly varying in time whereas L_1 contains rapidly oscillating terms at frequencies G, $2G$ and $4G$. For intense pumping fields $\vec{E}_{3,4}$ so that condition (4) is fulfilled, it is reasonable to make the secular approximation (Agarwal et al. 1978, Bogolubov et al. 1985), i.e. to retain only a slowly varying part. Correction to the results obtained in this fashion will be of order

$$(N\gamma_{ij}/G)^2 \quad \text{or} \quad (g_{1,2}|\vec{E}_{1,2}|/G)^2 .$$

Making the secular approximation, one finds a stationary solution of the master equation (Bogolubov et al. 1985)

$$\tilde{\rho} = Z^{-1}\sum_{R=0}^{N} X^R \sum_{M=0}^{R} |R, M><M, R| , \tag{6}$$

where

$$X = \frac{\gamma_{32}\cos^2\alpha}{\gamma_{21}\sin^2\alpha}$$

$$Z = \frac{(N+1)X^{N+2} - (N+2)X^{N+1} + 1}{(X-1)^2}$$

$\tilde{\rho} = U\rho U^+$, where U is the unitary operator representing the canonical transformation (5). $|R, M>$ is an eigenstate of the operators $R = R_{11} + R_{33}$; R_{11} and of the operator of the total number of atoms

$$\hat{N} = J_{11} + J_{22} + J_{33} = R_{11} + R_{22} + R_{33} .$$

Here

$$R_{ij} = Q_i^+ Q_j \qquad (i, j = 1, 2, 3)$$

Using solution (6) one can calculate the statistical moments. In particular, we find

$$\langle R\rangle_S = \frac{N(N+1)X^{N+3} - 2N(N+2)X^{N+2} + (N+1)(N+2)X^{N+1} - 2X}{(X-1)((N+1)X^{N+2} - (N+2)X^{N+1} + 1)} \tag{7}$$

$$\langle R^2\rangle_S = [N^2(N+1)X^{N+4} - N(3N^2+6N-1)X^{N+3} + (N+2)\cdot (3N^2+3N-2)X^{N+2} - (N+1)^2(N+2)X^{N+1} + 4X^2 + 2X]/[(X-1)^2((N+1)X^{N+2} - (N+2)X^{N+1} + 1)] , \tag{8}$$

where $\langle A\rangle_S$ indicates the expectation value of operator A over the stationary state (6). Now we return to **Hamiltonian** (1). Following the laser theory by Haken (1970), one may obtain a quantum Langevin equation for cavity modes $\vec{E}_1$, $\vec{E}_2$ in the form

$$\begin{aligned} \dot{a}_1(t) &= (-i\Delta_1 - \varkappa_1)a_1(t) - ig_1 J_{12}(t) + F_1(t) \\ \dot{a}_2(t) &= (-i\Delta_2 - \varkappa_2)a_2(t) - ig_2 J_{23}(t) + F_2(t) , \end{aligned} \tag{9}$$

where $\varkappa_1$, $\varkappa_2$ and $F_1(t)$, $F_2(t)$ are cavity damping constants and noise operators for the modes $\vec{E}_1$ and $\vec{E}_2$, respectively. The noise operators $F_\lambda(t)$ $(\lambda = 1, 2)$ obey the relations (Haken 1970)

$$\begin{aligned} &\langle F_\lambda(t)\rangle_H = \langle F_\lambda^+(t)\rangle_H = 0 \\ &\langle F_\lambda^+(t)F_{\lambda'}^+(t')\rangle_H = \langle F_\lambda(t)F_{\lambda'}(t')\rangle_H = 0 \\ &\langle F_\lambda^+(t)F_{\lambda'}(t')\rangle_H = n_{th,\lambda}(T)\cdot 2\varkappa_\lambda\, \delta(t-t')\,\delta_{\lambda\lambda'} \\ &\langle F_\lambda(t)F_{\lambda'}^+(t')\rangle_H = (n_{th,\lambda}(T)+1)\cdot 2\varkappa_\lambda\, \delta(t-t')\,\delta_{\lambda\lambda'} , \end{aligned} \tag{10}$$

where $\langle\cdots\rangle_H$ indicates the thermal average over the states of heatbath ; $n_{th,\lambda}(T)$ is the number of thermal quanta at a temperature T for a field mode $\vec{E}_\lambda$. Using the canonical transformation (5) one finds

$$J_{12} = \frac{1}{2}\cos\alpha\,(R_{33} - R_{11}) + \frac{1}{2}\cos\alpha\,(R_{31} - R_{13}) - \frac{1}{\sqrt{2}}\sin\alpha\,(R_{21} + R_{23}) \tag{11}$$

$$J_{23} = \frac{1}{2}\sin\alpha\,(R_{33} - R_{11}) + \frac{1}{2}\sin\alpha\,(R_{13} - R_{31}) + \frac{1}{\sqrt{2}}\cos\alpha\,(R_{12} + R_{32}) . \tag{12}$$

It is easy to see that (Bogolubov et al. 1985) in the secular approximation one can write

$$R_{12}(t) = e^{-iGt}\tilde{R}_{12}(t), \quad R_{21}(t) = e^{iGt}\tilde{R}_{21}(t)$$

$$R_{23}(t) = e^{-iGt}\tilde{R}_{23}(t), \quad R_{32}(t) = e^{iGt}\tilde{R}_{32}(t)$$

$$R_{13}(t) = e^{-2iGt}\tilde{R}_{13}(t), \quad R_{31}(t) = e^{2iGt}\tilde{R}_{31}(t),$$

where $\tilde{R}_{ij}(t)$ are slowly varying in time.

As follows from calculations, a substantial squeezing in the mixture of two modes $\vec{E}_1$ and $\vec{E}_2$ can be obtained if modes $\vec{E}_1$ and $\vec{E}_2$ are located near fluorescence spectra at frequencies $\omega_{21} \pm G$ and $\omega_{32} \mp G$, respectively (Fig.1), i.e.

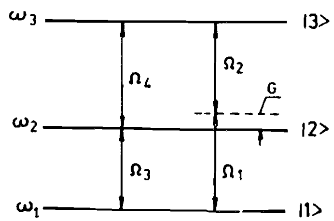
$$|\delta_1|, \ |\delta_2| \ll G, \tag{13}$$

where $\delta_1 = \Delta_1 - G, \quad \delta_2 = \Delta_2 + G$

or

$$|\Delta_1 + G|, \ |\Delta_2 - G| \ll G. \tag{14}$$

Fig. 1. Three-level atoms interacting with two pumping fields $\vec{E}_3, \vec{E}_4$ and with two cavity modes $\vec{E}_1$ and $\vec{E}_2$.



With the use of the condition (13) and secular approximation equations (9) reduce to

$$\dot{\tilde{a}}_1(t) = (-i\delta_1 - \varkappa_1)\tilde{a}_1(t) + i\frac{g_1}{\sqrt{2}}\sin\alpha\,\tilde{R}_{23}(t) + \tilde{F}_1(t) \tag{15}$$

$$\dot{\tilde{a}}_2(t) = (-i\delta_2 - \varkappa_2)\tilde{a}_2(t) - i\frac{g_2}{\sqrt{2}}\cos\alpha\,\tilde{R}_{32}(t) + \tilde{F}_2(t), \tag{16}$$

where $a_1(t) = e^{-iGt}\tilde{a}_1(t), \quad F_1(t) = e^{-iGt}\tilde{F}_1(t)$

$$a_2(t) = e^{iGt}\tilde{a}_2(t), \quad F_2(t) = e^{iGt}\tilde{F}_2(t).$$

For simplicity we consider only the case of $n_{th,\lambda}(T) = 0$, i.e. the temperature $T = 0$. In this case, as is easily seen from relations (10) and eqs. (15-16), the noise operators $\tilde{F}_{1,2}(t)$ cannot affect the normally ordered variance of signal modes $\vec{E}_1$ and $\vec{E}_2$ but they give commutators $[a_1, a_1^+]$ and $[a_2, a_2^+]$ additional values equal to $1 - e^{-2\varkappa_1 t} \xrightarrow[t\to\infty]{} 1$ and $1 - e^{-2\varkappa_2 t} \xrightarrow[t\to\infty]{} 1$, respectively (Haken 1974).

Missing the noise operator, one may obtain a stationary solution of eqs. (15-16) in the form

$$\tilde{a}_1 = \frac{ig_1}{\sqrt{2}}\sin\alpha\frac{\tilde{R}_{23}}{i\delta_1 + \varkappa_1}; \quad \tilde{a}_2 = \frac{-ig_2}{\sqrt{2}}\cos\alpha\frac{\tilde{R}_{32}}{i\delta_2 + \varkappa_2} \tag{17}$$

we shall consider the normally-ordered variable of fluctuation in the in-phase (b_1) and out-of-phase components (b_2) of the mixture of signal modes a_1 and a_2

$$b_1 = \frac{1}{2}(b^+ + b) \quad ; \quad b_2 = \frac{-i}{2}(b^+ - b),$$

where $\quad b = a_1 + a_2 \quad ; \quad b^+ = a_1^+ + a_2^+.$

By using solution (17) and steady-state density matrix (6), one finds the normally-ordered variance of fluctuation of the operators b_1 and b_2 in the form

$$\langle:(\Delta b_{1,2})^2:\rangle = \frac{1}{4}\Big\{\frac{g_1^2}{\varkappa_1^2}\sin^2\alpha\langle R_{32}R_{23}\rangle_s + \frac{g_2^2}{\varkappa_2^2}\cos^2\alpha\langle R_{23}R_{32}\rangle_s$$
$$\pm \frac{g_1 g_2}{\varkappa_1\varkappa_2}\sin\alpha\cos\alpha\left(\langle R_{32}R_{23}\rangle_s + \langle R_{23}R_{32}\rangle_s\right)\Big\}. \tag{18}$$

where

$$\langle R_{23}R_{32}\rangle_s = \frac{1}{2}(N-2)\langle R\rangle_s - \frac{\langle R^2\rangle_s}{2} + N \tag{19}$$

$$\langle R_{32}R_{23}\rangle_s = \frac{1}{2}(N+1)\langle R\rangle_s - \frac{1}{2}\langle R^2\rangle_s. \tag{20}$$

Here $\langle R\rangle_s$ and $\langle R^2\rangle_s$ can be found in relations (7-8).

In relation (18) and further, for simplicity, we take $\delta_1 = \delta_2 = 0$. The symbol $\langle\cdots\rangle$ indicates the expectation value over the states of heatbath and atomic steady-state (6). Taking into account the noise operators $F_{1,2}(t)$ one can find the commutator of the hermitian amplitude operators b_1 and b_2 as

$$\langle [b_1, b_2] \rangle = \frac{-i}{4}\left(\frac{g_1^2}{\varkappa_1^2}\sin^2\alpha - \frac{g_2^2}{\varkappa_2^2}\cos^2\alpha\right)\left(N - \frac{3}{2}\langle R\rangle_s\right) - i\,. \tag{21}$$

The factor of squeezing of the operators b_1 and b_2 can be defined as (Anantha et al. 1984)

$$F_{1,2} = \frac{\langle : (\Delta b_{1,2})^2 : \rangle}{\frac{1}{2}\,|\langle [b_1, b_2]\rangle|}\,. \tag{22}$$

We speak about squeezing if the factors F_1 or F_2 are less than zero. For the case of $X = 1$ (i.e. $\gamma_{32}/\gamma_{21} = G_{32}^2/G_{21}^2$) we have $\langle R_{32}R_{23}\rangle_s = \langle R_{23}R_{32}\rangle_s$ and it follows that

$$\langle : (\Delta b_{1,2})^2 : \rangle = \frac{1}{2}\langle R_{23}R_{32}\rangle_s \left(\frac{g_1 \sin\alpha}{\varkappa_1} \pm \frac{g_2\cos\alpha}{\varkappa_2}\right)^2 \geq 0$$

thus, squeezing is absent in this case. Squeesing also is absent for a separate mode $\vec{E}_1$ or E_2 (i.e. when $\frac{g_2}{\varkappa_2} \to 0$ or $\frac{g_1}{\varkappa_1} \to 0$). **The behaviour of the factor of squeezing F_2 as a function of the relation of the pumping field intensities $ctg^2\alpha$ when $\frac{g_1}{\varkappa_1} = \frac{g_2}{\varkappa_2} = \frac{\gamma_{32}}{\gamma_{21}} = 1$. and as a function of parameter $g_2/\varkappa_2$ when $ctg^2\alpha = 0.7$, $g_1/\varkappa_1 = 2$, $\gamma_{32}/\gamma_{21} = 1$ is plotted in fig.2 and fig.3, respectively. As is seen from fig.2 for the one atom case the squeezing is small. For the case of large number of atoms, as it seen from figs. 2-3, one can find the suitable values of parameters $ctg^2\alpha$, γ_{32}/γ_{21} $g_1/\varkappa_1$ and $g_2/\varkappa_2$ in which the substantial squeezing is presented.** In **fig.3, the 92% of squeezing is**

obtained for the case of N=1000. In the collective limit $N \to \infty$ the factor of squeezing tends to a limiting value $F_2 = -1$.

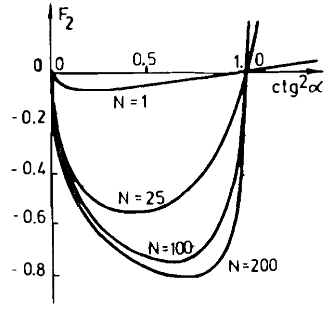


Fig. 2. Factor of squeezing F_2 as a function of the parameter ctg^2 for the case of $\frac{g_1}{\varkappa_1} = \frac{g_2}{\varkappa_2} = \frac{\gamma_{32}}{\gamma_{21}} = 1$

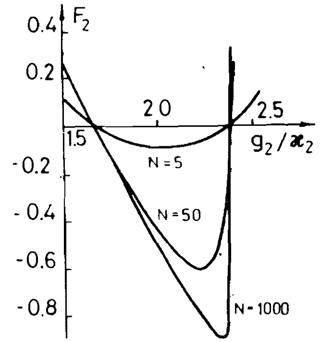


Fig. 3. Factor of squeezing F_2 as a function of the parameter $g_2/\varkappa_2$ for the case of $ctg^2\alpha = 0.7$; $\frac{g_1}{\varkappa_1} = 2$; $\frac{\gamma_{32}}{\gamma_{21}} = 1$.

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Сжатие света через невырожденное смешивание четырех волн в системе трех-уровневых атомов

Обсуждается генерация сжатого состояния света через вырожденное смешивание четырех волн в системе трех-уровневых атомов. Определено условие для получения большого сжатия света.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Squeezing of Light via Nondegenerate Four-Wave Mixing in a System of Three-Level Atoms

The generation of a squeezed state via nondegenerate four-wave mixing in a system of three-level atoms is discussed. The condition for large squeezing is discussed too.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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