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ELASTIC PROCESSES IN THE DICKE MODEL ON CRYSTAL

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The influence of the lattice vibrations on thermodynamic characteristics of a two-level system described by the Dicke model has widely been discussed during the last decade. In this connection we should like to note the paper by Thompson[1] in which a modification of the Dicke Hamiltonian allowing for one-phonon scattering processes has been studied. In fact, these processes can be treated as a correction to a dipole-photon interaction of the type

$$\sum_k \lambda_k \left\{ 1 + \frac{1}{\sqrt{N}} \left(K_q b_q + K_q^* b_q^+ \right) \right\} \qquad , \quad \lambda_k \equiv \sqrt{\frac{2\pi\hbar\rho}{\omega_k}}\, \omega d_{+-} \,, \tag{1}$$

where b_k and b_k^+ are the phonon operators, ω is the transition resonance frequency, d_{+-} is the matrix element of the dipole moment operator $(d_{+-} = \langle + | \hat{d} | - \rangle)$, and the coefficients K_q and K_q^* are determined from the expression for the Brillouin energy[2]

$$-\sum_{f=1}^{N} \hat{d}_f \, E(x_f) (u_f \nabla)\, \rho \, .$$

Here $E(x_f)$ is the transverse electromagnetic field at point x_f, u_f is emitter's shift from site f and ρ is density. Taking into account only a finite number of modes of the phonon field, Thompson[1] has shown that the lattice vibrations may change the kind of the equilibrium phase transition in the dipole sybsystem from the second to the first one.

The dynamics of the superradiance momentum has been studied in ref.[3] allowing for one-phonon scattering processes (1). It was shown that such processes hasten radiation.

Another mechanism of coupling of the dipole and phonon subsystems due to a direct interactions of dipoles of the ferroelectric type has been discussed in refs.[4-6]. It was also pointed a possibility of changing the kind of a transition. A generalized version of the Dikke model proposed in refs.[4-6] has further been used while discussing a possibility of generation of the electromagnetic superradiance in ferroelectrics and pyromagnetics[7-9] (see also ref.[10]).

In a number of papers[11-13] the influence of the lattice vibrations on the properties of the dipole subsystem was based on the expansion over shifts of the u_f parameter of the dipole-photon coupling λ_k in the Dicke model for the interaction of dipoles with the electromagnetic radiation the wave length of which is of an order of the lattice constant. In this case it became possible to establish the change of the kind of a phase transition and to calculate the contribution of one-phonon scattering processes to thermodynamic characteristics of the dipole subsystem. However, it should be emphasized that "modifications" of the Dicke model proposed in refs.[11-13] do not reproduce the physical situation described by the model as, first, the parameter λ_k is independent of the positions of atoms (see, for instance, ref.[14]) and second, the Dicke Hamiltonian is derived under the assumption that the sizes of the region containing radiating atoms are less than the radiation wave length.

An important class of processes related with the lattice vibrations are as is known elastic processes in which momentum transferred by a photon to an atom under radiation is transferred to a crystal as a whole. The contribution due to these processes may be taken into account by using the Debye-Waller factor (see, for instance, ref.[16]) whose introduction leads to renormalization of the dipole-photon coupling parameter to

$$\lambda \rightarrow \tilde{\lambda} = \lambda e^{-2w} . \tag{2}$$

where

$$w = W(\Theta) = \frac{3}{2} \frac{\hbar^2 K^2 \Theta^2}{M \Theta_D^3} \int_0^{\Theta_D/\Theta} \left(\frac{1}{e^z - 1} + \frac{1}{2} \right) z \, dz \, .$$

Here K is the scattering vector length, M is the mass of an atom and Θ_D is the Debye temperature.

Now we perform renormalization (2) in the standard Dicke model (see, for instance, ref.[17]) and use the method developed in ref.[18]. As a result, for the parameter ξ defining the value of spontaneous polarization in the dipole subsystem one can derive the following equation:

$$\xi = \frac{\lambda^2 e^{-4w} \xi}{\hbar\omega \sqrt{\hbar^2\omega^2 + \frac{4\lambda^4 e^{-8w}}{\hbar^2\omega^2} \xi^2}} \, th \frac{\sqrt{\hbar^2\omega^2 + \frac{4\lambda^4 e^{-8w}}{\hbar^2\omega^2} \xi^2}}{2\Theta} , \tag{3}$$

where λ is the dipole-photon coupling parameter (1) for only one resonance mode of radiation in the system.

Consider now the ground state of the system. From eq.(3) for the spontaneous polarization at $\Theta = 0$ we have

$$\xi_0 = \frac{\sqrt{\lambda^4 e^{-8w_0} - \hbar^4\omega^4}}{2\lambda^2 e^{-4w_0}}\,, \qquad (4)$$

where the quantity

$$w_0 = \lim_{\Theta\to 0} W(\Theta) = \frac{3}{8}\,\frac{\hbar^2 K^2}{M\Theta_D^2}$$

defines the contribution due to zero lattice vibrations. It should be noted that the spontaneous polarization in the ground state (4) is less than the relevant value in the Dicke model neglecting elastic mechanisms

$$\xi_0/\xi_0^{(D)} < 1\,, \qquad \xi_0^{(D)} \equiv \frac{\sqrt{\lambda^4 - \hbar^4\omega^4}}{2\lambda^2}\,.$$

Then, the zero spontaneous polarization in the ground state is possible only under the condition

$$w_0 < \frac{1}{2}\ln\frac{\lambda}{\hbar\omega}\,. \qquad (5)$$

The dependence of ξ_0 on w_0 at fixed $\lambda/\hbar\omega$ is shown in fig. 1.

In the case of arbitrary temperatures eq.(3) with expression for $W(\Theta)$ allows one to determine the temperature dependence of the spontaneous polarization ξ for different values of the parameter

$$\beta = \frac{3}{2}\,\frac{\hbar^2 K^2}{M\Theta_D^3}\,.$$

The results of the relevant calculations shown in fig. 2 indicate that an increase in the vibrational contribution (parameter β) results in the lowering of the critical temperature and decreasing of the spontaneous polarization in the dipole subsystem. The dependence of Θ_c on β is shown in fig. 3.

In the theory of equilibrium phase transition the Dicke model is characterized by the strong coupling condition /17/

$$\lambda > \hbar\omega$$

which is the condition for a phase transition with the zero temperature Θ_c^{D}. In the case under consideration the critical temperature is defined by

$$\left(\frac{\hbar\omega}{\lambda}\right)^2 e^{4w(\Theta_c)} = \operatorname{th}\frac{\hbar\omega}{2\Theta_c}\,.$$

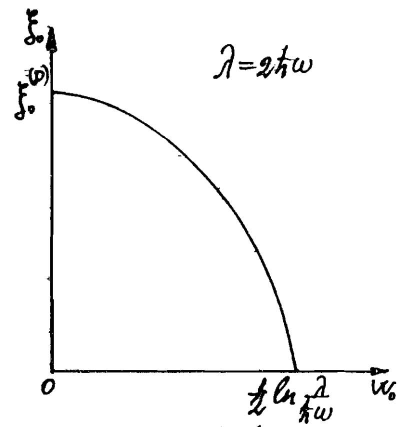

Fig.1

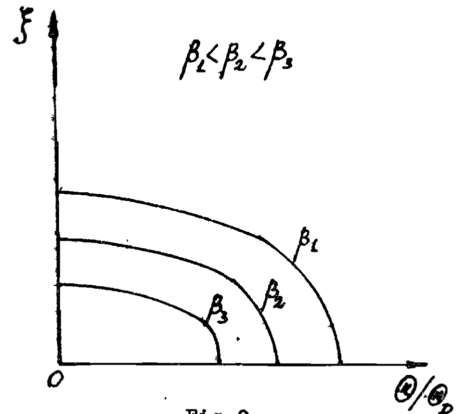

Fig.2

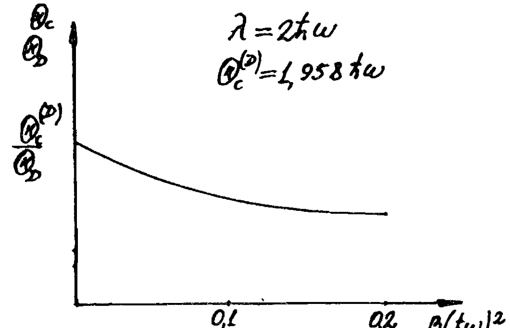

Fig.3

A condition necessary for the zero solution of this equation is inequality (5) which plays here the role of the strong coupling condition.

Thus, the consideration of elastic processes in describing equilibrium properties of the Dicke model on a crystal leads to the following results:

1. The presence of the lattice vibrations causes decrease in the value of the spontaneous polarization and lowering of the critical temperature.

2. The standard condition of strong coupling in the Dicke model is changed by inequality (5) imposing additional limitations on the choice of parameters λ and ω.

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Алискендеров Э.И., Шумовский А.С. E17-86-72
Учет упругих процессов в модели Дикке на кристалле

Исследовано влияние упругих процессов на кристалле на термодинамические характеристики двухуровневой системы дикковского типа. Получен аналог условия сильной связи, учитывающей параметры кристалла.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Aliskenderov E.I., Shumovsky A.S. E17-86-72
Elastic Processes in the Dicke Model on a Crystal

The influence of elastic processes in a crystal on thermodynamic characteristics of a two-level system of the Dicke-type is studied. An analog of the strong coupling condition allowing for the crystal parameters is obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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