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CRITICAL CURRENTS IN JOSEPHSON JUNCTIONS WITH MICROINHOMOGENEITIES ATTRACTING SOLITONS

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In long Josephson junctions (LJJ) with local inhomogeneities, in which the maximum value of the Josephson current, $j(x)$, locally decreases, there exist stable static distributions of the magnetic flux, $\Phi(x) = \varphi(x)\cdot(\Phi_0/2\pi)$ *, or fluxon bound states, localized near the inhomogeneities (as distinct from microshorts, they attract solitons and antisolitons). The bound states were theoretically studied in a series of papers/1-6/ , where the problem of observing these states in experiments was also touched upon. The most direct way is provided by the technique of scanning a focused low-power laser beam, which allows one to find the Josephson current distribution along LJJ by measuring changes, due to local heating, of the maximum total current at zero voltage (see eq.(6) below). This method has been recently applied/7/ to studying Josephson current distributions in homogeneous LJJ in external magnetic fields/8/ . A less direct but more traditional approach is to measure maximum (critical) values of the external(bias) current γ in the applied magnetic field h .

In this letter we will demonstrate a dramatic effect of the attractive microinhomogeneity on the critical current $\gamma_c(h)$ for low

*Here $\varphi(x)$ is the usual Josephson phase variable, Φ_0 is the magnetic flux quantum, x is the distance normalized to the Josephson penetration length λ_J ; we will also normalize time t to the inverse of the Josephson frequency ω_J and Josephson current to its maximum value. The magnetic field is normalized to $h_0 = \Phi_0/4\pi\lambda_L\lambda_J$, where λ_L is the London penetration depth.

values of h . There the soliton (fluxon) and antisoliton (antifluson) bound states on inhomogeneity give rise to a cross-shaped structure ("soliton cross", see fig.1 below), and for long enough

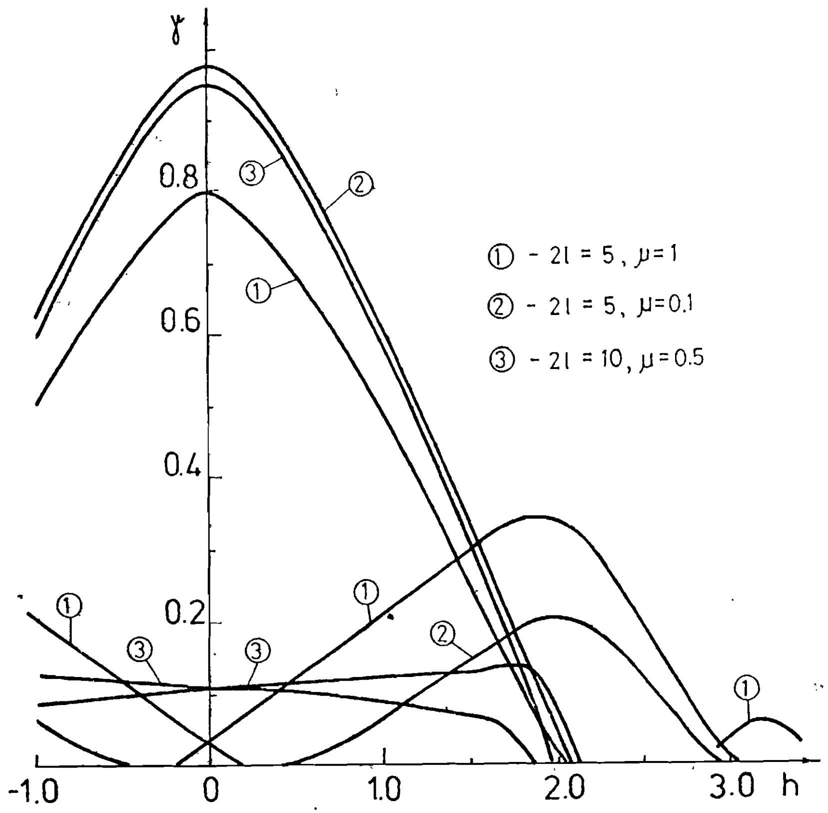

Fig. 1. The dependence of the critical bias current γ_c on the applied magnetic field h for different states in overlap LJJ with an inhomogeneity placed in the middle of the junction ($x_0 = 0$).

junctions the corresponding branches exist in a fairly large interval of h ($|h| \lesssim (1.5 \div 2)$, in our units). For higher values of $|h|$ the effects of inhomogeneity are gradually dying out.

The mathematical model for LJJ is the following.

$$\varphi_{xx} - V_\varphi(\varphi;x) - \gamma = \varphi_{tt} + \alpha\varphi_t - \beta\varphi_{xtt}\,; \quad \varphi_x(\pm\ell,t) = h_\pm, \quad (1)$$

where $\varphi = \varphi(x,t)$ is the time-dependent phase defined above, $-\ell \le x \le \ell$ .The subscripts denote partial derivatives, α is the quasiparticle loss, β is the surface resistance, $h = \frac{1}{2}(h_+ + h_-)$ is the external magnetic field, and $j = (h_+ - h_-)$ is the external current flowing through the junction edges $x = \pm\ell$,while γ is distributed along the whole junction.For a general inhomogeneous LJJ we have $V_\varphi(\varphi;x) = j(x)\cdot\sin\varphi$. With $j(x) \equiv 1$ and constant parameters α, β, γ equation (1) represents the standard ($\alpha\beta\gamma$) model of LJJ/9/ .

For simplicity we consider here one microinhomogeneity in $j(x)$ which can be approximated by the δ -function ($\alpha\beta\gamma\delta$ -model):

$$j(x) = 1 - \mu\,\delta(x - x_0)\,, \quad 0 \le \mu \le 1. \qquad (2)$$

A better description of the real LJJ can be obtained with some smeared $j(x)$, e.g., $j(x) = j_{sm}(x) \equiv \tanh^2\left[\frac{2}{\mu}(x - x_0)\right]$. However, for $\mu \lesssim (0.5 \div 1)$, $\mu \ll 2\ell$, the solutions of eq.(1) with $j(x)$ given by eq.(2) or with $j = j_{sm}$ are qualitatively equivalent provided that $\varphi(x,t)$ does not change rapidly in the interval $|x - x_0| \lesssim \mu/2$.

The right-hand side of eq.(1) determines only the time evolution of φ and vanishes for static solutions $\varphi(x)$ considered here. We will be interested only in stable states $\varphi(x)$ for which the linear boundary value problem (see/2-4/)

$$-\psi^{(n)}_{xx} + V_{\varphi\varphi}(\varphi;x)\,\psi^{(n)} = \lambda_n\,\psi^{(n)}(x)\,; \quad \psi^{(n)}_x(\pm\ell) = 0 \qquad (3)$$

has a positive definite spectrum of eigenvalues ($0 < \lambda_0 \le \lambda_1 \le \lambda_2 \le \ldots$). In this case a perturbed solution will return to $\varphi(x)$:

$$\varphi(x,t) = \varphi(x) + e^{-\alpha t/2} \sum_{n=0}^{\infty} \psi^{(n)}(x) \sin(\omega_n t + \alpha_n), \qquad (4)$$

where $\omega_n^2 = \lambda_n - \frac{1}{4}\alpha^2$, and we disregard the β -term in eq.(1).

The eigenvalues λ_n depend on the parameters $p = \{\gamma, h, j, \ell, \mu, x_0, ...\}$, i.e., $\lambda_n = \lambda_n(p)$. If $\lambda_n(p) = 0$, we have a bifurcation. In particular, let $\lambda_0(p)$ be the lowest eigenvalue for a stable solution $\varphi(x;p)$; when $\lambda_0(p) = 0$, the solution $\varphi(x;p)$ loses its stability. The equation $\lambda_0(p) = 0$ defines a hypersurface in the parameter space, we call it the bifurcation surface (BS), and the values of p belonging to BS are called critical, $p = p_c$. To find BS for different stable states is an important and in general very difficult problem (see /1-5/). However, it is easy to obtain an equation for a hyperplane tangent to BS at any bifurcation point p_c :

$$\langle \Delta\gamma\ \psi_c^{(0)} \rangle + \langle \Delta V_\varphi(\varphi_c;x)\ \psi_c^{(0)} \rangle = \Delta h \langle V_{\varphi\varphi}(\varphi_c;x)\ x\ \psi_c^{(0)} \rangle +$$

$$+ \frac{\Delta j}{2\ell} \left\{ \langle \psi_c^{(0)} \rangle + \frac{1}{2} \langle V_{\varphi\varphi}(\varphi_c;x)\ x^2\ \psi_c^{(0)} \rangle \right\}. \qquad (5)$$

Here $\langle ... \rangle \equiv \int_{-\ell}^{\ell} dx (...)$, Δ denotes a change in parameters, e.g. $\Delta\gamma = \gamma_c' - \gamma_c$; $\varphi_c = \varphi(x;p_c)$, $\psi_c^{(0)}$ is the solution $\psi^{(0)}$ of eq.(3), corresponding to φ_c, i.e., $\psi_c^{(0)} = \psi^{(0)}(x;p_c)$.

Consider, for example, $\Delta V_\varphi = -\mu_1 \delta(x-x_1) \sin\varphi_0$, where μ_1 is small, and $-\ell < x_1 < \ell$. With $\Delta h = \Delta j = 0$ eq. (5) gives

$$\Delta\gamma = \mu_1 \sin\varphi_c(x_1)\ \psi_c^{(0)}(x_1) / \langle \psi_c^{(0)}(x) \rangle, \qquad (6)$$

where φ_c and $\psi_c^{(0)}$ correspond to $\mu_1 = 0$. This formula is relevant to scanning overlap-geometry LJJ with a weak focused laser beam/7,10/, the effect of heating of LJJ at $x = x_1$ being represented by the μ_1 -term (with $\Delta h = \Delta\gamma = 0$ one obtains from eq.(6) a similar relation for in-line geometry). The importance of this formula lies in

that it accounts for all the so-called nonlocal effects of heating/10/ in terms of fairly simple and usually qualitatively known function $\psi_c^{(0)}(x_1)$, a detailed discussion will be given elsewhere.

We can also use eq.(6) to compare $\gamma_c(h)$ curves for homogeneous and inhomogeneous LJJ. Consider the simplest stable state in a homogeneous junction for $h_\pm = 0$, i.e., $\varphi = \varphi_0 = -\arcsin\gamma$. For this solution $\lambda_0(\gamma) = \sqrt{1-\gamma^2}$, and the corresponding critical value of γ is $\gamma_c = 1$. Regarding the μ -term in eq.(2) as a perturbation, ΔV_φ, and using eq.(6) we obtain $\Delta\gamma \equiv \gamma_c - 1 = -\mu/2\ell$.

For homogeneous junctions this state is the only stable state available at $h = 0$ (in what follows we discuss only overlap geometry for which $h_+ = h_- = h$).

If $\mu > 0$ and the junction is long enough, there exists a bound state of a slightly deformed fluxon localized at $x = x_0$. For $2\ell \gg 1$ we can approximate the fluxon centered at $x = \xi$ as $\varphi \approx \varphi_s(x-\xi)$, where $\varphi_s(x) \equiv 2\,\mathrm{ar}\cos(-\tanh x)$. Let $x_0 = 0$, then for $\gamma = 0$ and $h = 0$ the fluxon is located at $x = 0$, i.e., $\xi = 0$. When $\gamma \neq 0$, we can determine $\xi(\gamma)$ by minimizing the energy of the fluxon

$$\mathcal{E}(\xi) = \int_{-\ell}^{\ell} dx \left[\frac{1}{2} \left(\frac{d\varphi_s}{dx} \right)^2 + V(\varphi_s;x) + \gamma\varphi_s \right], \quad h_\pm = 0. \qquad (7)$$

With V given by eq.(2) we find that $d\mathcal{E}/d\xi = 0$, when $2\pi\gamma = 4\mu \sinh\xi / \cosh^3\xi$. The critical value of the bias current is determined from the equation $d\gamma/d\xi = 0$ which is satisfied for $\xi = \xi_c$, $\tanh\xi_c \equiv 1/\sqrt{3}$. Taking into account, in this simple approximation, the boundary conditions $\varphi_x(\pm\ell) = h$ we can arrive at the following approximation for critical values of γ :

$$2\pi\gamma_c^{\pm}(h) \simeq 8\mu/3\sqrt{3} \pm 8\sqrt{2}\, e^{-\ell} h + O(h^2, e^{-2\ell}), \qquad (8)$$

where the upper (lower) sign corresponds to the bound soliton (antisoliton). A similar result can be obtained from eq.(5). This approximation is not valid for junctions of intermediate length $4 \lesssim 2\ell \lesssim 8$, moreover, for $2\ell < 4$, $\mu \leq 1$ there exists only one stable state in a zero magnetic field. Nevertheless, for LJJ of intermediate length the qualitative picture of the fluxon localized on the microinhomogeneity is correct, if μ is not too small. Therefore, $\gamma_c(h)$ has the corresponding brances which near $h = 0$ can be approximated by a linear function:

$\gamma_c^s(h) = \gamma_c^s(0) \pm \lambda^s h + \ldots$, i.e., we obtain the soliton-antisoliton cross.

Now we present the results of calculating $\gamma_c(h)$ numerically. We have solved equations (1), (3) with $j = j_{sm}$. All stable static solutions $\varphi(x)$ were obtained* in the interval $0 \leq h \leq 3$ ($h = h_+ = h_-$) together with the corresponding eigenvalues $\lambda_0(\gamma; h)$. Then the critical values of $\gamma(h)$ were found by using the condition $\lambda_0(\gamma_c; h) = 0$. For simplicity, we consider only symmetric geometry, i.e., $x_0 = 0$. For two typical values of ℓ the results are presented in Fig.1.

The highest branch, $\gamma_c^0(h)$ corresponds to the state which in the homogeneous limit is simply $\varphi = \varphi_0 = -\arcsin\gamma$. The calculated value of $\gamma_c^0(0)$ is equal to $(1 - \mu/2\ell)$ even for $\mu = 1$. The distribution of the magnetic field φ_x for this state is shown in Fig.2 for values of γ not very close to γ_c. If the Josephson current is given by eq.(2), one can easily obtain the following simple approximation for this state

$$\varphi_0(x) = -\arcsin\gamma - \frac{\mu\gamma}{2\varkappa}\cosh[\varkappa(|x|-\ell)]\left(\sinh\varkappa\ell - \frac{\mu\varkappa}{2}\cosh\varkappa\ell\right)^{-1}, \quad (9)$$

where $\varkappa^2 \equiv \sqrt{1-\gamma^2}$. The numerical solutions shown in Fig. 2 are de-

*We have used the continuum analog of the Newton method /11/. Some details are given in Ref. /12/.

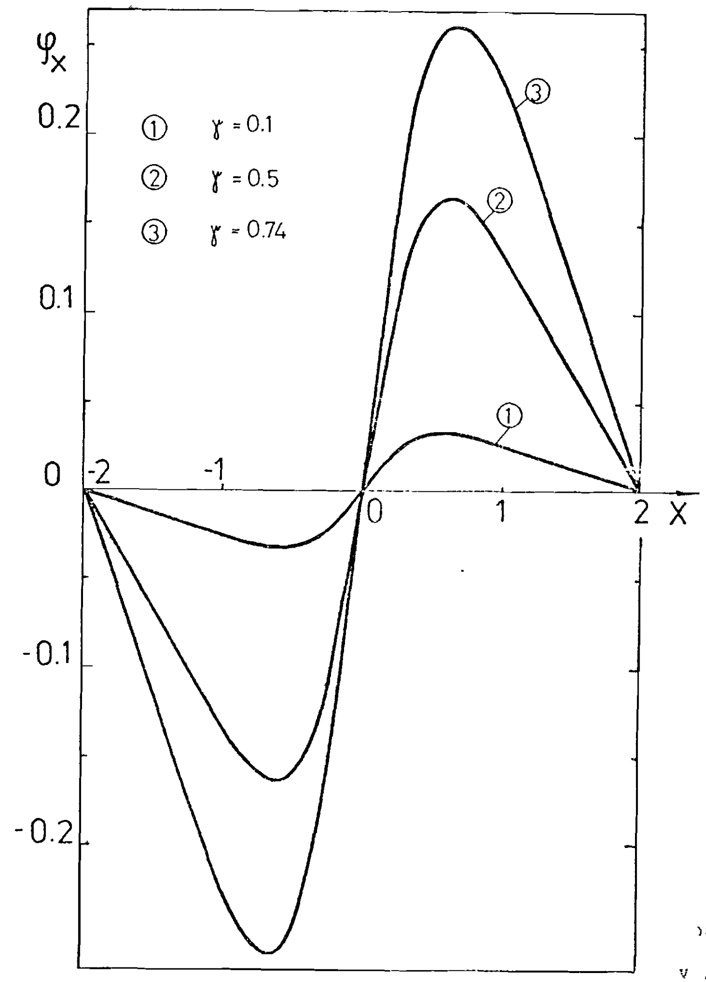

Fig. 2. The distribution of the magnetic field φ_x inside LJJ for the states with the maximum value of $\gamma_c(0)$. The inhomogeneity is described by $j = j_{sm} = \tanh^2[\frac{2}{\mu}(x - x_0)]$, $x_0 = 0$, $2\ell = 4$, $\mu = 1$.

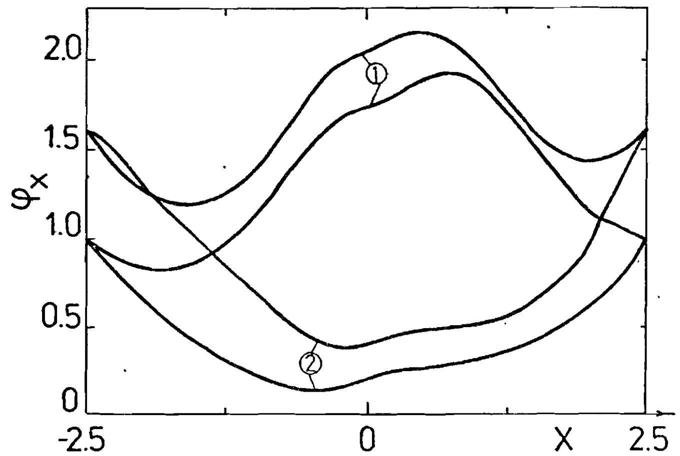

Fig. 3. The distribution of φ_x in the junction of length $2\ell = 5$ with $\mu = 1$, i.e. $j_{sm} = \tanh^2(2x)$. 1. Soliton localized on inhomogeneity. 2. The state with a maximum value of $\gamma_c(0)$.

rived with the smeared current, i.e., $j = \tanh^2(\frac{2x}{\mu})$ and they should not be compared with eq.(9) in the interval (-1,1). However, outside this interval eq.(9) gives rather a good approximation to exact solutions not very close to the critical ones.

These states exist also in the applied magnetic field h, up to $h = h_c \approx 2$, but are strongly deformed as can be seen in Fig.3. For large h the magnetic flux in such a state is localized near the edges of the junction. The other distribution of φ_x in this figure corresponds to the bound soliton state. It is also deformed due to the influence of the magnetic field and bias current. For small values of them it is well approximated by $2 \arccos(-\tanh(x - \xi))$ as discussed above. In the long junction ($2\ell = 10$) the soliton branch $\gamma_0^s(h)$ is well approximated by eq.(8) for $|h| \lesssim 1$.

There exist other states in the long junction. In fact, for $2\ell = 10$, $\mu = 1$ we find up to 5 stable states for large enough

values of h. The distributions of the magnetic field are presented in Fig. 4. Note that both the soliton and antisoliton states (1,5 in Fig.4) are stable for remarkably high values of $|h|$. Most prominent, qualitative effects of the microinhomogeneity are present for $|h| \lesssim 2$, and in general, for higher values of $|h|$ the states in LJJ become insensitive to the presence of inhomogeneity.

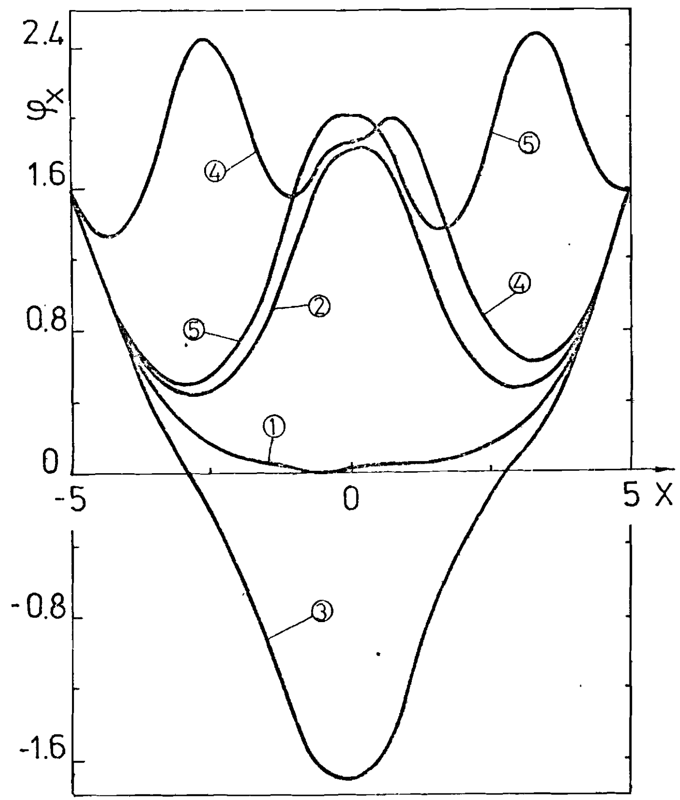

Fig. 4. The distribution of φ_x in the long junction, $2\ell = 10$, for different stable states. Values of λ_0 for these states are: $\lambda_0^1 = 0.61$, $\lambda_0^2 = 0.28$, $\lambda_0^3 = 0.20$, $\lambda_0^4 = 0.043$, $\lambda_0^5 = 0.080$.

Here we discussed in some detail only the symmetric overlap-geometry. For other types of geometries (e.g. for inline LJJ studied for a homogeneous case by Owen and Scalapino) as well as for asymmetric position of the inhomogeneity its influence on $\gamma_c(h)$ curves is qualitatively similar to that considered here.

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Филиппов А.Т. и др. E17-86-637

Критические токи джозефсоновских переходов с неоднородностями, притягивающими солитоны

Аналитически и численно изучается зависимость критического тока γ_c при нулевом напряжении в одномерном джозефсоновском переходе с притягивающими микронеоднородностями /МН/ от величины внешнего магнитного поля h. Главный качественный эффект в случае перекрывающейся геометрии перехода состоит в появлении характерной крестообразной структуры на графике $\gamma_c(h)$ в слабых магнитных полях, соответствующей солитону и антисолитону, локализованным на МН. Вместе с основным состоянием, существующем и в однородных переходах, они составляют три устойчивых состояния /тристабильность/, которые существуют в довольно широком интервале h.

Работа выполнена в Лаборатории вычислительной техники и автоматизации и Лаборатории теоретической физики ОИЯИ.

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Critical Currents in Josephson Junctions with Microinhomogeneities Attracting Solitons

Dependence of the critical zero-voltage current (γ_c) on the external magnetic field h in one-dimensional Josephson junctions with local attractive inhomogeneities is studied numerically and analytically. The main effect for the overlap geometry consists in forming near $\gamma_c(h)$ a cross-shaped structure in $h = 0$ due to the soliton and antisoliton localization on inhomogeneity. Together with the state existing in homogeneous junctions they form a collection of three stable states (tristability) in a fairly large interval of h.

The investigation has been performed at the Laboratory of Computing Techniques and Automation and at the Laboratory of Theoretical Physics, JINR.

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