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CONCEPT OF QUASI-AVERAGES AND SPONTANEOUS SYMMETRY BREAKING

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Twenty six yeas ago N.N.Bogolubov formulated his, fundamental for the theory of macroscopic system, principle - the so-called idea of quasi-averages [1]. The formulation of this principle was based on investigation in the theory of superfluidity and superconductivity which resulted in the construction of microscopic theories of these physical phenomena [2-5].

The general situation for these macroscopic systems is a degeneration of vacuum states, therefore to define this state uniquely, one must take away the degeneration, that is one has to break the symmetry using one of possible ways,for instance,introducing into the Hamiltonian an infinitesimal source [1,6].

The subsequent development of statistical mechanics and quantum field theory showed an extreme fruitfulness of the concept of quasi-averages and the related notion of spontaneous symmetry breaking.

The main idea of the concept of quasi-averages is the nonuniqueness of choosing spaces of states for macroscopic systems. This nonuniqueness has stimulated a number of works on the problem of nonequivalent representations of operator algebras [7] which has led to working out powerful algebraic methods [8-10] in the theory of systems with spontaneously broken symmetry.

The fact that the symmety of a Hamiltonian may be different from that of a vacuum has been clearly understood first in the theory of condensed matter, and then strongly influenced quantum field theory and elementary particle physics. This influence has been emphasized by Weinberg in his Nobel lecture [11].

It is just the notion of symmetry breakdown which has yielded the fundamental basis for constructing the unified theory of electromagnetic and weak interactions.

Formulating the concept of quasi-averages Bogolubov [6] has proved the so-called theorem on $1/q^2$-singularities.According to this theorem,if the continuous symmetry of a system is broken, elementary excitations appear having an energy that goes to zero in the long-wave limit $q \to 0$. In other words, gapless excitations appear, photons being examples. Analogous results have been obtained in quantum field theory by Goldstone [12] and Higgs [13].

The Bogolubov concept of quasi-averages is the main concept in the contemporary theory of phase transitions. Several trends based on the Bogolubov approach may be mentioned in statistical mechanics. Among them the method of variational inequalities and the method of approximating Hamiltonians have been found to

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be very successful [14,15]. These methods let to define in the thermodynamic limit asymptotically exact thermodynamic potentials, correlation functions and Green functions for various model systems in the theory of superconductivity [14], magnetism [16], radiation [17], etc. The existence of a long-range order in statistical systems of one and two dimensions has also been examined [18,19]. Consistent use of the concept of quasi-averages has let to develop a microscopic approach for describing heterophase states in the systems with symmetry breakdowns [20-22].

The concept of quasi-averages has found a wide application in the theory of nuclei and of nuclear matter [23].

In the literature one usually discusses different technical tricks connected with the procedure of quasi-averaging, while the ideological aspect of this concept remains often unclear. In the present paper we analyse the principal meaning of the Bogolubov concept of quasi-averages which is a clue for solving the problem of nonuniqueness of states in the case of degenerate macroscopic systems. Our main thesis is as follows. The Bogolubov concept of quasi-averages shows the way to choose a space of states corresponding to a particular thermodynamic phase. For clearness the consideration will be given with the help of the Ising model. We shall also present a short reviews of essential methods of quasi-averaging, i.e., the method of infinitesimal sources [1,6], the method of nonequivalent commutation relations [24,25], the method of boundary conditions for Green functions [26]. Finally we shall illustrate the ideas of quasi-averaging applying them to a system with heterophase states [20-22].

Consider now the Ising model with the N-particle Hamiltonian

$$H_N = -\frac{1}{4} \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z , \qquad (1)$$

in which $J_{ij} = J_{ji} \geq 0$ and σ_i^z is the z-component of the Pauli operator connected with the lattice site i. The Hamiltonian (1) is invariant with respect to the group Z_2 and is defined on the space of states

$$\mathcal{H}_N = \bigotimes_{i=1}^{N} C_1^2 . \qquad (2)$$

The vectors of this space are direct products of spin functions $\psi_{i+} = \binom{1}{0}$ and $\psi_{i-} = \binom{0}{1}$. We shall use the following obvious notation for the vectors of space (2). The vector with all spins up will be written as

$$\psi_+^N = \bigotimes_{i=1}^{N} \psi_{i+} \equiv \underset{1}{\uparrow} \underset{2}{\uparrow} \dots \underset{N}{\uparrow} . \qquad (3)$$

Correspondingly, the vector with all spins down will have the form

$$\psi_-^N = \bigotimes_{i=1}^{N} \psi_{i-} \equiv \overset{1}{\downarrow} \overset{2}{\downarrow} \dots \overset{N}{\downarrow} . \qquad (4)$$

Let us stress that in this notation the row of arrows does not necessarily mean a linear chain but it is simply one of possible enumerations of sites in an arbitrary lattice. In addition to vectors (3) and (4) one can construct an example of the vector with disoriented spins

$$\psi_0^N = \uparrow \downarrow \uparrow \dots \downarrow . \qquad (5)$$

All vectors (3)-(5) are translationally invariant.

An elementary excitation in the Ising model is the overturn of spin. This overturn is usually called the spin flip. Therefore, the corresponding excitation may be called the flippon.

As is evident, a flippon excitation above ψ_+^N is described by the function

$$\psi_+^N(i) = \uparrow \uparrow \dots \uparrow \downarrow \uparrow \dots \uparrow . \qquad (6)$$

In the same manner one can construct many-flippon functions $\psi_+^N(i_1 \dots i_n)$. Flippon excitations above ψ_-^N and ψ_0^N can be also considered using the same procedure.

Flippon excitations above the ordered vectors ψ_+^N and ψ_-^N mean the appearance of disordered clusters with inverse spins, and flippon excitations above the disordered vector ψ_0^N lead to the formation of ordered clusters. As is obvious, any two functions of space (2) can be transformed to each other by a sequence of flippon excitations.

Let us now clarify the question which of functions of space (2) pretends to the role of the vacuum vector. The latter is by definition a function from which one is able to construct all other functions by means of flippon excitations with a nonnegative energy. If we denote a vacuum by $|0\rangle$, a one-flippon function by $|i\rangle$ and many-flippon functions by $|i_1 \dots i_n\rangle$, the flippon energy should be defined as

$$\epsilon(i) = \langle i | H_N | i \rangle - \langle 0 | H_N | 0 \rangle , \qquad (7)$$

analogously, the energy of an n-flippon excitation is

$$\epsilon(i_1 \dots i_n) = \langle i_1 \dots i_n | H_N | i_1 \dots i_n \rangle - \langle 0 | H_N | 0 \rangle . \qquad (8)$$

The set of quantities (8) is the collective spectrum of the considered system.

Choosing as a vacuum either ψ_+^N or ψ_-^N for the flippon energy above an ordered state we get

$$\epsilon_+(i) = \epsilon_-(i) = J , \quad J \equiv \frac{1}{N} \sum_{ij} J_{ij} . \qquad (9)$$

As far as the flippon energy (9) is positive, the vectors ψ_+^N and ψ_-^N are vacuum ones.

If we choose as $|0\rangle$ the vector ψ_0^N, then flippon energies will be nonpositive. Therefore we shall call ψ_0^N the pseudovacuum. Generally speaking, in the Ising model with a finite N any function of space (2) can serve as a pseudovacuum.

When calculating thermodynamic functions (potentials, average values, Green functions, etc.) one has to take traces over a basis in a chosen space. The system with a finite N has only one unitary irreducible basis. Such a system has only one thermodynamic state and no phase transitions. The order parameter of the Ising model (1), that is the average spin, is zero due to the Z_2-symmetry of the system:

$$\langle\sigma_i^z\rangle_N \equiv \operatorname*{Tr}_{\mathcal{H}_N} \sigma_i^z e^{-H_N/\Theta} \Big/ \operatorname*{Tr}_{\mathcal{H}_N} e^{-H_N/\Theta} = 0 \quad (\Theta,\ N<\infty),$$

where Θ is the temperature in energy units.

In the case of an infinite system ($N \to \infty$) the situation is not so trivial. Let us introduce the definition

$$\psi_{\dots}(i_1 \dots i_n) = \lim_{N\to\infty} \psi^N(i_1\dots i_n). \qquad (10)$$

As far as the number of flippons can go to infinity we are able to construct different countable bases: $\{\psi_+(i_1\dots i_n)\}$, $\{\psi_-(i_1\dots i_n)\}$ and $\{\psi_0(i_1\dots i_n)\}$ each of which generates the corresponding separable Hilbert space: $\mathcal{H}_+$, $\mathcal{H}_-$ or $\mathcal{H}_0$.

In this way in the thermodynamic limit we have a set of spaces, but not only one as for finite system. Three of spaces, that is $\mathcal{H}_+$, $\mathcal{H}_-$ and $\mathcal{H}_0$, have translationally invariant pseudovacua. Besides these spaces there exist an infinite number of other spaces with translationally noninvariant pseudovacua. It may be stressed that all spaces mentioned are mutually orthogonal.

When there are several spaces, one can choose, for averaging, different possible bases defining the corresponding traces. For instance, for the energy one may get

$$E_\alpha(\Theta) = \lim_{N\to\infty} \operatorname*{Tr}_{\mathcal{H}_\alpha} H_N e^{-H_N/\Theta} \Big/ N \operatorname*{Tr}_{\mathcal{H}_\alpha} e^{-H_N/\Theta}, \qquad (11)$$

where $\alpha = +, -, 0$.

Consider the properties of spaces $\mathcal{H}_+$, $\mathcal{H}_-$ and $\mathcal{H}_0$. The space $\mathcal{H}_+$ is Z_2-nonsymmetric. If when constructing the basis $\psi_+(i_1\dots i_n)$ according to eq.(10) the maximal $n<\infty$ while $N\to\infty$, then $\langle\sigma_i^z\rangle_+ = 1$ at all temperatures. And if $n < N/2$, then at zero temperature again $\langle\sigma_i^z\rangle_+ = 1$, where $\langle\dots\rangle_+$ is the averaging with traces over the space $\mathcal{H}_+$. However, in the latter case ($n < N/2$) the rise of temperature excites flippons, and the average spin becomes less than unity: $\langle\sigma_i^z\rangle_+ < 1$ at $\Theta > 0$.

The vacuum ψ_- is also Z_2-noninvariant but is invariant with respect to the group of translations over the lattice. If constructing basis (10) we take $n<\infty$, then $\langle\sigma_i^z\rangle_- \equiv -1$ and if $n<N/2$, then $\langle\sigma_i^z\rangle_- \geq -1$.

The pseudovacuum ψ_0 is invariant with respect to the group of translations as well as to the group Z_2. Therefore $\langle\sigma_i^z\rangle_0 \equiv 0$ at all temperatures. It is possible to say that the space $\mathcal{H}_0$ corresponds to disordered states as opposed to the spaces $\mathcal{H}_+$ and $\mathcal{H}_-$ corresponding to ordered states.

It is clear that flippon excitations above ψ_+ or ψ_- lead to a partial disordering of spins, while flippon excitations above ψ_0 yield a partial spin ordering.

Thus, the question arises how the system can choose one or another thermodynamic state. The answer is that the system chooses such a state that is thermodynamically more profitable than others. This means that the Gibbs potential G (or the free energy) is to be minimal. The thermodynamic state corresponding to a particular phase can be characterized by an order parameter. In our case the order parameter is the average spin $\langle\sigma_i^z\rangle$. As we have seen, there are two possibilities when $\langle\sigma_i^z\rangle \neq 0$ and $\langle\sigma_i^z\rangle \equiv 0$. The system prefers that possibility for which the potential $G\{\langle\sigma_i^z\rangle\}$ is less. For example, below a temperature Θ_c, called the Curie temperature, the Ising system can be in the ordered state having $\langle\sigma_i^z\rangle \neq 0$, which means that

$$G\{\langle\sigma_i^z\rangle \neq 0\} < G\{\langle\sigma_i^z\rangle \equiv 0\}. \qquad (12)$$

Above Θ_c there exists only one solution for the order parameter $\langle\sigma_i^z\rangle \equiv 0$, and the system is in paramagnetic state.

The choice of a stable thermodynamic state of a macroscopic system can be done by using the Bogolubov concept of quasi-averages [1,6]. The general meaning of the concept is to choose in one or another way such a state that minimizes the free energy. There are several methods of quasi-averaging which we are going to discuss below.

An obvious method of quasi-averaging has been suggested by Bogolubov [1,6]. This is the method of external sources. To choose one of possible states, it is enough to add, to the Hamiltonian H_N, an external field, obtaining the Hamiltonian

$$H_N(B) = -\frac{1}{4}\sum_{ij} J_{ij}\sigma_i^z\sigma_j^z - B\sum_i \sigma_i^z. \qquad (13)$$

The additional term here reduces the Z_2-symmetry of Hamiltonian (1) choosing by this, when averaging, only the vectors of either the space $\mathcal{H}_+$ or $\mathcal{H}_-$ according to the sign of the field B. Then the quasi-average for σ_i^z is defined as

$$\prec\sigma_i^z\succ \equiv \lim_{B\to 0}\lim_{N\to\infty}\langle\sigma_i^z\rangle_{H_N(B)}. \qquad (14)$$

Quasi-averages of other operators can be defined in the same manner.

The widely known mean-field approximation is also a kind of quasi-averaging. Actually, in this approximation one replaces an initial Hamiltonian, let us take e.g. H_N, for a mean-field Hamiltonian that is in our case

$$H_N^{MF} = -\frac{1}{4}\sum_{ij} J_{ij}\,(2\sigma_i^z - \langle\sigma_i^z\rangle_{MF})\langle\sigma_i^z\rangle_{MF}. \tag{15}$$

The latter, as is evident, is not invariant with respect to the Z_2-transformations. The symmetry is broken by the choice of sign for the mean spin $\langle\sigma_i^z\rangle_{MF}$ found by averaging with Hamiltonian (15).

Note that for a special type of interactions, the so-called long-range interactions, Hamiltonian (15) asymptotically exactly (as $N\to\infty$) describes all thermodynamic characteristics of the considered system [14,15,22].

In general case the procedure of quasi-averaging using infinitesimal sources can be formulated as follows. Let the Hamiltonian H_N be symmetric with respect to the group G, while a state of a system below a transition point be characterized by the subgroup $G_1 < G$. To define quasi-averages, break the symmetry of the initial Hamiltonian including in it an infinitesimal source:

$$H_N \to H_N(\nu) = H_N + \nu\Gamma,$$

where Γ is an operator lowering the Hamiltonian symmetry to G_1. The quasi-average of any operator $\mathcal{O}$ is by definition

$$\prec\mathcal{O}\succ = \lim_{\nu\to 0}\ \lim_{N\to\infty} \mathrm{Tr}\,\mathcal{O}\, e^{-H_N(\nu)/\Theta} / \mathrm{Tr}\, e^{-H_N(\nu)/\Theta}. \tag{16}$$

Stress that the order of limits here as well as in (14) must not be changed. Note that in some specific cases the limit $\nu\to 0$ can be absent [27]. This situation may be illustrated using the BCS-Bogolubov superconductivity model with the Hamiltonian

$$H_N = \sum_k \epsilon_k a_k^+ a_k - \sum_{k'} J_{kk'}\, a_k^+ a_{-k}^+ a_{-k'} a_{k'}, \tag{17}$$

in which the combination of momentum and spin is marked by k. Hamiltonian (17) is invariant with respect to the group U(1). The symmetry can be broken by an additional infinitesimal source:

$$H_N \to H_N(\nu) = H_N - \sum_k (\nu a_{-k} a_k + \nu^* a_k^+ a_{-k}^+).$$

Let $\nu = |\nu| e^{i\phi}$, then $H_N(\nu)$ can be written as $H_N(|\nu|)$ with the help of the gauge transformation $a_k \to a_k e^{i\phi/2}$. Defining now the average of $\mathcal{O} = a_k^+ a_{-k}^+$ one obtains

$$\prec a_k^+ a_{-k}^+\succ = \lim_{\nu\to 0}\ \lim_{N\to\infty} \left(\frac{\nu^*}{|\nu|}\langle a_k^+ a_{-k}^+\rangle\right).$$

As is obvious, the limit $\nu\to 0$ for the latter expression is not correctly defined.

However, it is possible to define gausi-averages invoking no external sources [24,25]. To show this, let us pass to a quasi-spin representation [28,29]:

$$a_k^+ a_k = \frac{1}{2}(1-\sigma_k^z), \quad a_{-k} a_k = \sigma_k^+.$$

Then (17) becomes

$$H_N = \frac{1}{2}\sum_k \epsilon_k (1-\sigma_k^z) - \sum_{kk'} J_{kk'}\sigma_k^- \sigma_{k'}^+. \tag{18}$$

The U(1)-symmetry of the model means the invariance of eq.(18) with respect to spin rotations characterized by the angle ϕ in the xy-plane. The vacua corresponding to different fixed angles ϕ and ϕ' are mutually orthogonal [7,30]. Therefore in each space $\mathcal{H}_\phi$ corresponding to a vacuum with a fixed angle ϕ one can realize a representation of canonical commutation relations for the algebra of local observables, representation with different fixed angles ϕ and ϕ' being unitary nonequivalent [7]. Quasi-averages can be defined by means of new operators [24,25]

$$\beta_k = u_k^2\sigma_k^+ + v_k^2\sigma_k^- LL^+ + u_k v_k (2\sigma_k^-\sigma_k^+ - 1)L, \tag{19}$$

where

$$L = \frac{1}{N}\sum_k \sigma_k^- \Big/ \frac{1}{N}\sum_k \sigma_k^- \rangle, \quad u_k^2 + v_k^2 = 1.$$

The definition of quasi-averages is as follows:

$$\prec\sigma_k^-\succ = \lim_{N\to\infty}\langle\sigma_k^- L^+\rangle. \tag{20}$$

As is clear, the operator L^+ fixes a vacuum. In the thermodynamic limit L becomes a unitary operator.

Another method of quasi-averaging which is very convenient in practice is connected with boundary conditions for Green-function equations [20,31]. These equations can be presented in several forms: as an infinite hierachical Bogolubov chain, in a variational Schwinger form, or as Dyson integral equations. In any case these equations are strongly nonlinear and contain integral operators. Such equations have, generally speaking, an infinite number of solutions. When the Hamiltonian of a considered system is invariant with respect to a group of transformations containing nontrivial subgroups, then among a manifold of solutions for Green functions one can separate classes of solutions connected with partucular subgroups of the total group of transformations. As far as the connection exists between almost any thermodynamic phase and symmetry properties of systems, we are choosing a particular thermodynamic phase when we separate solutions with a fixed type of symmetry. Thus, it is just the fixation of a concrete symmetry of Green functions narrowing the set of possible solutions to equations of motion and choosing those of them that correspond to a particular thermodynamic phase which plays the role of a boundary condition.

Consider an example of a system with the Hamiltonian

$$H = \int \psi^+(\vec{r}) \left(-\frac{\nabla^2}{2m} - \mu\right)\psi(\vec{r})\, d\vec{r} + \frac{1}{2}\int \psi^+(\vec{r})\, \psi^+(\vec{r}')\, \phi(\vec{r}-\vec{r}')\, \psi(\vec{r}')\, \psi(\vec{r})\, d\vec{r}\, d\vec{r}', \tag{21}$$

in which $\psi(\vec{r}) \equiv \psi(\vec{r}, t)$ is a field operator in the Heisenberg representation. If we would like to describe a crystalline phase, we should impose a periodicity condition

$$G(\vec{r}, \vec{r}', t - t') = G(\vec{r} + \vec{a}, \vec{r}' + \vec{a}, t - t'), \tag{22}$$

where $\vec{a}$ is a lattice vector. And if want to consider a uniform liquid phase, then we have to use the condition

$$G(\vec{r}, \vec{r}', t - t') = G(\vec{r} - \vec{r}', 0, t - t'). \tag{23}$$

In this way equations of motion for Green functions give solutions corresponding to thermodynamic phases which we are interested in [20,31].

Let us now proceed to the problem of a microscopic description of heterophase states. Such states have been experimentally observed in variuos systems. For instance an admixture of normal electrons in superconductors [32] can be treated as a kind of heterophase fluctuations. In some ferro- and antiferromagnets there are paramagnetic nuclei [33,34]. In ferroelectrics paraelectric fluctuations exist [35]. A striking example of a heterophase mixture is the usual water containing fluctuating nuclei of ice [36] in the temperature range from 0°C up to +4°C. Nuclear matter with coexisting multiquark clusters can be also interpreted as a type of a heterophase system [37].

All methods of quasi-averaging we have described above deal with possibility of separating pure phases. How could we depict a mixture of several phases?

Return to the Ising model (1). Let the ordered phase be described by the space $\mathcal{H}_+$. This phase is characterized by a nonzero order parameter

$$\langle \sigma_i^z \rangle_+ \equiv \langle \sigma_{i+}^z \rangle \neq 0. \tag{24}$$

On the contrary, the disordered phase should correspond to the trivial order parameter

$$\langle \sigma_i^z \rangle_0 \equiv \langle \sigma_{i0}^z \rangle \equiv 0. \tag{25}$$

Taking into account that heterophase fluctuations may occur in any place of a system one has to average over all possible

positions of these fluctuations as well as over their shapes and sizes [20,21]. For each fixed configuration of fluctuations the regions of the ordered phase correspond to the local symmetry breaking [38]. Supposing that the system for $N \to \infty$ is ergodic [39] and averaging over heterophase configurations [20,21] we may replace in the Hamiltonian the sums of the type $\sum_{i=1}^{N_\alpha}$, where $\alpha = +, 0$, by the sums $w_\alpha \sum_{i=1}^{N}$ with the factor $w_\alpha = N_\alpha / N$. N_α being the average number of particles in the corresponding phase. As a result, the Hamiltonian of the heterophase system takes the form [40-42]

$$H = H_+ \oplus H_0, \quad H_\alpha = N\left(\frac{A}{2} w_\alpha^2\right) - \frac{w_\alpha^2}{4} \sum_{ij} J_{ij}\, \sigma_{i\alpha}^z \sigma_{j\alpha}^z, \tag{26}$$

in which the parameter A describes the competition between direct and exchange interactions of particles [43,44]. By definition the condition

$$w_+ + w_0 = 1 \tag{27}$$

is valid. So, only one of the concentrations w_α is an independent function, and it is convenient to use the notation

$$w \equiv w_+, \quad w_0 = 1 - w. \tag{28}$$

The concentration of the ordered phase w should be found by minimizing the free energy of the system.

To demonstrate consequences of the heterophase approach, we shall below consider the case of the long-range interaction [14,42] when exact solutions can be obtained in the thermodynamic limit. In such a case Hamiltonian (26) is thermodynamically equivalent to the approximating one

$$H_{app} = H_+^{app} \oplus H_0^{app}, \quad H_\alpha^{app} = \frac{N}{2} w_\alpha^2 A - \frac{J}{4} w_\alpha^2 \sum_i (2\sigma_{i\alpha}^z - C_\alpha) C_\alpha, \tag{29}$$

where

$$J \equiv \sum_j J_{ij}, \quad C_\alpha \equiv \langle \sigma_{i\alpha}^z \rangle.$$

According to eqs. (24) and (25)

$$C_+ \equiv C \equiv 0, \quad C_0 \equiv 0,$$

The free energy per spin is

$$f = A\left(w^2 - w + \frac{1}{2}\right) + \frac{w^2}{4} J C^2 - \Theta \ln\left(4\cosh\frac{w^2 C J}{2\Theta}\right);$$

minimizing it with respect to C and w we obtain

$$C = \tanh(w^2 JC/2\Theta), \quad w = A/2(A - \frac{J}{4}C^2). \qquad (30)$$

In addition to solutions given by eq.(30) we have those corresponding to pure phases, i.e., $w \equiv 1$ and $w \equiv 0$. The profitability of each kind of solutions must be checked by minimizing the free energy.

The model of heterophase ferromagnet has been analysed in detail earlier [40-42]. Here we list some of interesting results of this model.

1. Exact critical indices depend on constants entering into the Hamiltonian. Thus, for the critical indices corresponding to the specific heat (α) and to the order parameter (β) we have

$$\alpha = \begin{cases} 0, & A \neq 3J/2 \\ 1/2, & A = 3J/2 \end{cases}, \qquad \beta = \begin{cases} 1/2, & A \neq 3J/2 \\ 1/4, & A = 3J/2 \end{cases}.$$

2. When $A<0$, there exists a maximum in the specific heat below $\Theta_c = J/8$. A similar maximum occurs for the magnetization $M = wC$ below Θ_c. These anomalies can be due to the presence of a partial disorder that takes place below Θ_c, analogously, to the existence of a partial order that has been discovered [45] in magnets above Θ_c. The anomalies in the specific heat have also been observed in some magnets below Θ_c, and the interpretation of such anomalies has been given [46].

3. If $A < J/2$, but $A > 0$, the concentration of the ferromagnetic phase reaches the value $w=1$ at the temperature $\Theta_n < \Theta_c$. Below Θ_n solely the pure ferromagnetic phase remains, while above Θ_n the system is a mixture of ferro- and paramagnetic phases. At the point Θ_n a specific phase transition occurs, this transition can be called the nucleation as far as just at this point Θ_n the nuclei of the competing phase appear. At the nucleation point the jumps of the specific heat take place, which, probably, has been observed too [47].

4. When $A = 3J/2$ the order of the phase transition changes and for $0 < A < 3J/2$ the transition becomes of first order. The temperature of first-order phase transition lies in the interval (Θ_c, T_c) in which $T_c = J/2$ is the Curie temperature for the pure ferromagnetic phase, when $w \equiv 1$.

The existence of heterophase states and the peculiarities of their thermodynamic properties are directly connected with the presence, in the Hamiltonian, of the parameter A playing the role of disordering interaction. Without this interaction no heterophase states are possible. The competition between ordering and disordering interactions, that is between the parameters J and A, regulates the values of the phase concentrations of ferromagnetic and paramagnetic phases and thus defines thermodynamic properties of the system.

The considered approach may be applied to other systems with heterophase states. In particular, it has been used for describing superconductors [48,49] and crystals [26,50].

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Боголюбов Н.Н./мл./, Шумовский А.С., Юкалов В.И. E17-86-567
Концепция квазисредних и спонтанное нарушение симметрии

Рассмотрено основное содержание концепции квазисредних Боголюбова. На примере модели Изинга исследована проблема неоднозначности при выборе пространства состояний микроскопической системы. Проанализирован /основанный на идее квазисредних/ подход к описанию гетерофазных состояний.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bogolubov N.N., Jr., Shumovsky A.S., Yukalov V.I. E17-86-567
Concept of Quasi-Averages and Spontaneous Symmetry Breaking

The essential content of the Bogolubov idea of quasi-averaging is considered. The problem of nonuniqueness of the choice of the space of states for a microscopic system is investigated using as an example the Ising model. A microscopic description of heterophase states based on the concept of quasi-averages is examined.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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