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COHERENT TRAPPING OF POPULATIONS IN A THREE-LEVEL TWO-MODE MODEL WITH MULTIPHOTON TRANSITIONS

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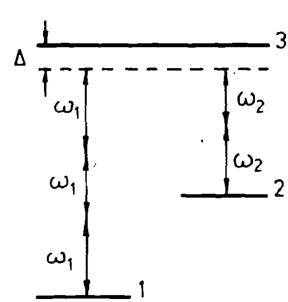
Much attention has recently been paid to the study of model problems describing an interaction of an atom having a finite number of levels with an electromagnetic field (see, e.g., review[1]). First of all, this is due to a possibility of deriving exact solutions and describe rigorously some nonlinear effects that should prove of interest owing to the development of quantum electronics.

Some strict results were obtained for an important and interesting object - a three-level atom interacting with a finite number of modes of the electromagnetic field. For a model problem describing such a system a new integral of motion could have been established[2] and the Heisenberg equation could have been integrated for the operators of level populations and photon numbers. In particular, it has been shown that the quantum revival and collapse are possible in the loss-free three-level two-mode model[5], the theoretical investigations under suitable conditions photon antibunching and anticorrelation between the modes[16].

In our recent paper[6] we have studied a special case of a multiphoton two-mode processes on a three-level atom for which we have calculated the level populations, photon numbers, the photon characteristic distribution functions, the statistical moments of photon numbers and the correlations of modes. In the present paper we shall continue studying such a system by the formalism of dressed states[7-10].



The three-level atom model considered here is shown in the figure (for the case $m_1 = 3$, $m_2 = 2$). Let the upper level 3 be coupled with level 1 (level 2) due to the interaction with the field in mode 1 (mode 2) via m_1-photon (m_2-photon) transition. The model Hamiltonian of the system under consideration is

$$H = H_A + H_F + H_{AF} \cdot \qquad (1)$$

Here H_A and H_F describe a free atom and a free field, respectively, and H_{AF} describes the atom-field interaction in the dipole and rotating wave approximations

$$H_A = \sum_{j=1}^{3} \hbar \Omega_j R_{jj} \; ; \qquad H_F = \sum_{\alpha=1}^{2} \hbar \omega_\alpha a_\alpha^+ a_\alpha \; ; \tag{2}$$

$$H_{AF} = \sum_{\alpha=1}^{2} \hbar g_\alpha \left(R_{3\alpha} a_\alpha^{m_\alpha} + R_{\alpha 3} a_\alpha^{+m_\alpha} \right).$$

The operator $R_{jj} = |j\rangle\langle j|$ describes the population of level j. The operator $R_{ij} = |i\rangle\langle j|$ corresponds to the atomic transition from level j to level i ($i \neq j$). The operators a_α (a_α^+) describe near-resonant mode α of the radiation field in the cavity. The ω_α's are the mode frequencies and the detuning parameter Δ can be defined as

$$\Omega_3 - \Omega_\alpha - m_\alpha \omega_\alpha = \Delta , \tag{3}$$

where $\hbar \Omega_j$ is the corresponding level energy and g_α is the coupling constant.

We denote by $|i; n_1, n_2\rangle$ an eigenstate vector of the free Hamiltonian $H_A + H_F$, where $|i\rangle$ is an atomic eigenstate vector corresponding to level i, and by $|n_1, n_2\rangle$ a Fock state with n_1 photons in mode 1 and n_2 photons in mode 2. This vector describes the so-called undressed state of the system /7-10/. The eigenstates of the full Hamiltonian are easily found by solving the Schrödinger equation

$$H\psi = E\psi .$$

Their expressions in terms of the undressed states $|i; n_1, n_2\rangle$ are given by

$$\psi_{+;n_1,n_2} = \frac{\lambda_1(n_1)}{\sqrt{\lambda_+^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |1; n_1+m_1, n_2\rangle + \frac{\lambda_2(n_2)}{\sqrt{\lambda_+^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |2; n_1, n_2+m_2\rangle + \frac{\lambda_+(n_1,n_2)}{\sqrt{\lambda_+^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |3; n_1, n_2\rangle ,$$

$$\psi_{-;n_1,n_2} = \frac{\lambda_1(n_1)}{\sqrt{\lambda_-^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |1; n_1+m_1, n_2\rangle + \frac{\lambda_2(n_2)}{\sqrt{\lambda_-^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |2; n_1, n_2+m_2\rangle - \frac{\lambda_-(n_1,n_2)}{\sqrt{\lambda_-^2(n_1,n_2) + \lambda_0^2(n_1,n_2)}} |3; n_1, n_2\rangle \tag{4}$$

$$\psi_{0,n_1,n_2} = \frac{\lambda_2(n_2)}{\lambda_0(n_1,n_2)} |1, n_1+m_1, n_2\rangle - \frac{\lambda_1(n_1)}{\lambda_0(n_1,n_2)} |2; n_1, n_2+m_2\rangle$$

and also by

$$\psi_{1;\tilde{n}_1,n_2} = |1; \tilde{n}_1, n_2\rangle \qquad \text{with } \tilde{n}_1 \leq m_1 - 1$$

$$\psi_{2;n_1,\tilde{n}_2} = |2; n_1, \tilde{n}_2\rangle \qquad \text{with } \tilde{n}_2 \leq m_2 - 1$$

where we use the relations

$$R_{ij}|j\rangle = |i\rangle \; ; \qquad a^m|n\rangle = \sqrt{\frac{n!}{(n-m)!}}\, |n-m\rangle ;$$

$$a^+ a|n\rangle = n|n\rangle \; ; \qquad a^{+m}|n\rangle = \sqrt{\frac{(n+m)!}{n!}}\, |n+m\rangle , \tag{5}$$

and the following notation is introduced

$$\lambda_\pm = \lambda \pm \frac{\Delta}{2} \; ; \qquad \lambda = \sqrt{\lambda_0^2(n_1,n_2) + \frac{\Delta^2}{4}} ,$$

$$\lambda_0^2(n_1,n_2) = \lambda_1^2(n_1) + \lambda_2^2(n_2) \; ; \tag{6}$$

$$\lambda_i(n_i) = g_i \sqrt{\frac{(n_i+m_i)!}{n_i!}} , \quad (i = 1,2) .$$

Here $\lambda_\pm$ and 2λ are the frequencies of the two-photon Rabi oscillations in the system.

The eigenenergies $E_{\nu;n_1,n_2}$ (ν = 0, $\pm$,1,2) of the full Hamiltonian H that correspond to the eigenstates $|\psi_{\nu;n_1,n_2}\rangle$ are found to be

$$E_{0;n_1,n_2} = \hbar(\Omega - \Delta) \; ;$$

$$E_{\pm;n_1,n_2} = \hbar\Omega \pm \hbar\lambda_\mp , \tag{7}$$

where $\Omega = \Omega_3 + n_1\omega_1 + n_2\omega_2$

and

$$E_{1;\tilde{n}_1,n_2} = \hbar(\Omega_1 + \tilde{n}_1\omega_1 + n_2\omega_2) \qquad \text{with } \tilde{n}_1 \leq m_1 - 1$$

$$E_{2;n_1,\tilde{n}_2} = \hbar(\Omega_2 + n_1\omega_1 + \tilde{n}_2\omega_2) \qquad \text{with } \tilde{n}_2 \leq m_2 - 1 .$$

We now proceed to calculate the probabilities for the multiphoton transitions of the atom. Let us denote by $\varphi(t)$ the wave function of the total "atom + field" system in the Schrödinger picture. Then, the probability of finding the atom on its j-th level at time t as a result of the transition $i \to j$ initiated by n_1 photons in mode 1 and n_2 photons in mode 2 of the field is defined by the formula .

$$P(t; i\to j) = \sum_{n_1' n_2'} \left|\langle \varphi_{i;n_1,n_2}(t) | j; n_1', n_2'\rangle\right|^2. \tag{8}$$

Here, the initial condition

$$\varphi_{i;n_1,n_2}(0) = |i; n_1, n_2\rangle \tag{9}$$

has been assumed. By expanding $\varphi_{i;n_1,n_2}(0)$ in terms of the dressed eigenstates (4), we can easily find the time dependent wave functions $\varphi_{i;n_1,n_2}(t)$. They read

$$\begin{aligned}
&\varphi_{1;n_1+m_1,n_2}(t)\cdot \exp\left[i\left(\Omega-\tfrac{\Delta}{2}\right)t\right] = \\
&= |1; n_1+m_1, n_2\rangle \left\{ \frac{\lambda_1^2}{2\lambda_0^2\lambda}\left(\lambda_- e^{-i\lambda t} + \lambda_+ e^{i\lambda t}\right) + \frac{\lambda_2^2}{\lambda_0^2} e^{it\frac{\Delta}{2}}\right\} + \\
&+ |2; n_1, n_2+m_2\rangle\left\{\frac{\lambda_1\lambda_2}{2\lambda_0^2\lambda}\left(\lambda_- e^{-i\lambda t}+\lambda_+ e^{i\lambda t}\right) - \frac{\lambda_1\lambda_2}{\lambda_0^2} e^{it\frac{\Delta}{2}}\right\} - \\
&- i|3; n_1, n_2\rangle \frac{\lambda_1}{\lambda}\sin\lambda t\,; \\
&\varphi_{2;n_1,n_2+m_2}(t)\exp\left[i\left(\Omega-\tfrac{\Delta}{2}\right)t\right] = \\
&= |1; n_1+m_1, n_2\rangle\left\{\frac{\lambda_1\lambda_2}{2\lambda_0^2\lambda}\left(\lambda_- e^{-i\lambda t}+\lambda_+ e^{i\lambda t}\right) - \frac{\lambda_1\lambda_2}{\lambda_0^2} e^{it\frac{\Delta}{2}}\right\} + \\
&+ |2; n_1, n_2+m_2\rangle\left\{\frac{\lambda_2^2}{2\lambda_0^2\lambda}\left(\lambda_- e^{-i\lambda t}+\lambda_+ e^{i\lambda t}\right) + \frac{\lambda_1^2}{\lambda_0^2} e^{it\frac{\Delta}{2}}\right\} - \\
&- i|3; n_1, n_2\rangle \frac{\lambda_2}{\lambda}\sin\lambda t\,; \\
&\varphi_{3;n_1,n_2}(t)\exp\left[i\left(\Omega-\tfrac{\Delta}{2}\right)t\right] = -i|1; n_1+m_1, n_2\rangle\frac{\lambda_1}{\lambda}\sin\lambda t - \\
&- i|2; n_1, n_2+m_2\rangle\frac{\lambda_2}{\lambda}\sin\lambda t + |3; n_1, n_2\rangle\frac{1}{2\lambda}\left(\lambda_+ e^{-i\lambda t} + \lambda_- e^{i\lambda t}\right).
\end{aligned} \tag{10}$$

In particular, for one-photon transitions 1→3 and 2→3 we have

$$\begin{aligned}
P(t; 1\to 3) &= \frac{\lambda_1^2}{\lambda^2}\sin^2\lambda t\,, \\
P(t; 2\to 3) &= \frac{\lambda_2^2}{\lambda^2}\sin^2\lambda t\,,
\end{aligned} \tag{11}$$

and for two-photon processes we find

$$P(t; 1\to 2) = \frac{2\lambda_1^2\lambda_2^2}{\lambda_0^2\lambda}\left[\frac{1}{\lambda_+}\sin^2\frac{\lambda_+ t}{2} + \frac{1}{\lambda_-}\sin^2\frac{\lambda_- t}{2} - \frac{1}{2\lambda}\sin^2\lambda t\right] \tag{12}$$

which is in agreement with the result obtained by an exact solution of the Heisenberg equations [5].

The dressed states $\varphi_{0;n_1,n_2}$ (see the last eq. in (4)) are the coherent superpositions of only undressed states $|1; n_1+m_1, n_2\rangle$ and $|2; n_1, n_2+m_2\rangle$ but not $|3; n_1, n_2\rangle$. The existence of such dressed states uncoupled with the upper level 3 plays an important role in the mechanism of the population trapping effect [1,11-15] due to which the decay channels in multiphoton excitation can be turned off.

We call a state of the "atom+field" system, $|\Phi\rangle$ a coherent-trapping state if $|\Phi\rangle$ satisfies the following conditions:

1. $|\Phi\rangle$ is given by

$$|\Phi\rangle = |\Phi_A\rangle \otimes |\Phi_F\rangle, \tag{13}$$

where $|\Phi_A\rangle$ is a coherent-trapping atomic state and $|\Phi_F\rangle$ is a coherent-trapping field state.

2. The time evolution of $|\Phi\rangle$ is given by

$$|\Phi(t)\rangle = \exp(-iH_A t)\,|\Phi_A\rangle \otimes \exp(-iH_F t)\,|\Phi_F\rangle. \tag{14}$$

The coherent-trapping condition requires $|\Phi\rangle$ to be made of only the null dressed states $\varphi_{0;n_1,n_2}$

$$\begin{aligned}
|\Phi\rangle &= \sum_{n_1 n_2} C(n_1,n_2)\,|\varphi_{0;n_1,n_2}\rangle = \\
&= \sum_{n_1 n_2}\left\{\lambda_2(n_2)\,\mathcal{D}(n_1,n_2)\,|1; n_1+m_1, n_2\rangle - \lambda_1(n_1)\,\mathcal{D}(n_1,n_2)\,|2; n_1, n_2+m_2\rangle\right\},
\end{aligned} \tag{15}$$

where the notation $C(n_1,n_2)/\lambda_0(n_1,n_2) = \mathcal{D}(n_1,n_2)$

has been introduced. $\lambda_0(n_1,n_2)$, $\lambda_1(n_1)$ and $\lambda_2(n_2)$ are defined by relation (6). To fulfil condition (13), it is necessary that

$$\lambda_2(n_2)\,\mathcal{D}(n_1-m_1, n_2) = \lambda_1(n_1)\,\mathcal{D}(n_1, n_2-m_2)\,G_2/G_1\,. \tag{16}$$

Here G_1 and G_2 are C-numbers. From (4) and (14), we get the relation for determining $\mathcal{D}(n_1, n_2)$

$$\mathcal{D}(n_1 - m_1, n_2) = \mathcal{D}(n_1, n_2 - m_2)\, \frac{g_1 G_2}{g_2 G_1} \sqrt{\frac{(n_1+m_1)!\, n_2!}{n_1!\,(n_2+m_2)!}} \qquad (17)$$

Using (17) and the normalization conditions for $C(n_1, n_2)$

$$1 = \sum_{n_1, n_2} C^2(n_1, n_2) = \sum_{n_1, n_2} \mathcal{D}^2(n_1, n_2)\, \lambda_0^2(n_1, n_2)$$

we get an explicit expression for $\mathcal{D}(n_1, n_2)$

$$\mathcal{D}(n_1, n_2) = \frac{1}{\sqrt{G_1^2 + G_2^2}} \exp\left[-\frac{1}{2}\left(\frac{g_1}{G_1}\right)^{-\frac{2}{m_1}} - \frac{1}{2}\left(\frac{g_2}{G_2}\right)^{-\frac{2}{m_2}}\right] \times$$
$$\left(\frac{g_1}{G_1}\right)^{-\frac{n_1+m_1}{m_1}} \cdot \left(\frac{g_2}{G_2}\right)^{-\frac{n_2+m_2}{m_2}} \cdot \frac{1}{\sqrt{(n_1+m_1)!\,(n_2+m_2)!}} \qquad (18)$$

From (15) and (18) we find the factorization of the state

$$|\Phi_A\rangle = \frac{G_2}{\sqrt{G_1^2+G_2^2}}\, |1\rangle - \frac{G_1}{\sqrt{G_1^2+G_2^2}}\, |2\rangle ,$$
$$|\Phi_F\rangle = \exp\left[-\frac{1}{2}\left(Z_1^2 + Z_2^2\right)\right] \sum_{n_1, n_2} \frac{Z_1^{n_1} Z_2^{n_2}}{\sqrt{n_1!\, n_2!}}\, |n_1, n_2\rangle , \qquad (19)$$

where we have used the notation

$$Z_1 = \left(\frac{g_1}{G_1}\right)^{-\frac{1}{m_1}} , \qquad Z_2 = \left(\frac{g_2}{G_2}\right)^{-\frac{1}{m_2}} .$$

Thus, using eigenstate $\psi_{0; n_1, n_2}$ (superposition of the lower levels 1 and 2) we have obtained the functions of coherent-trapping of the two-mode three-level model with multiphoton transitions. It should be noted that the field states under the coherent-trapping condition are classified as coherent states.

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Алискендеров Э.И., Фам Ле Киен, Шумовский А.С. E17-86-455
Когерентное пленение населенностей в трехуровневой двухмодовой модели с многофотонными переходами

Исследована двухмодовая трехуровневая модель атома с многофотонными переходами и с учетом расстройки при помощи формализма одетых состояний. Вычислены вероятности переходов и населенности уровней, получены квантовомеханические аналоги частот Раби. Показана возможность существования когерентного пленения населенностей в такой модели и вычислены функции когерентного пленения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Aliskenderov E.I., Fam Le Kien, Shumovsky A.S. E17-86-455
Coherent Trapping of Populations in a Three-Level Two-Mode Model with Multiphoton Transitions

A two-mode three-level model atom with multiphoton transitions is studied taking into account detuning by the formalism of dressed states. Transition probabilities are calculated and quamtum mechanical analogs of the Rabi frequences are found. A possibility for the existence of coherent trapping of populations is shown in such a model and the coherent trapping states are found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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