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I.V.Barashenkov, A.D.Gocheva, V.G.Makhankov,
I.V.Puzynin

ON THE STABILITY OF SOLITARY WAVES OF THE (Φ^3-Φ^5)-NONLINEAR SCHRÖDINGER EQUATION UNDER NON-VANISHING BOUNDARY CONDITIONS

1985

1. Introduction

The so-called ($\psi^3 - \psi^5$)-nonlinear Schrödinger equation,

$$i\psi_t + \Delta\psi + \alpha\psi + \psi|\psi|^2 - \psi|\psi|^4 = 0 \quad (1)$$

emerges independently in several branches of nonlinear physics. It serves as a semiclassical limit for the equations of nuclear fluid dynamics with Skyrme's forces [1]. The authors of ref. [2] have applied it to investigate stationary light beams in self-focusing nonlinear medium. Eq.(1) is also the Hartree-Fock approximation to the Schrödinger equation for 2-body attractive and 3-body repulsive contact interactions [3]. The energy functional corresponding to (1),

$$H = \int d\vec{x} \left\{ |\vec{\nabla}\psi|^2 - \alpha|\psi|^2 - \tfrac{1}{2}|\psi|^4 + \tfrac{1}{3}|\psi|^6 \right\} \quad (2)$$

may in addition be interpreted as Landau's expansion for free energy H in powers of order parameter ψ and its gradient. This expansion is assumed as the base of the phenomenological theory of phase transitions [4-6]. The expansion (2) in which terms up to the third order in $|\psi|^2$ are kept, arises, for example, in the description of ferroelectric transition in KDP (KH_2PO_4) [6] and phase separation in He^3 - He^4 mixtures [7].

Recently [8] exact soliton-like solutions to Eq.(1) in one-dimensional space have been found under non-vanishing boundary conditions of the form

$$\psi(x,t) \to \psi^{\pm} = const \quad \text{as} \quad x \to \pm\infty .$$

In the present communication we examine their stability.

It is convenient to use the following form of the ($\psi^3 - \psi^5$)-nonlinear Schrödinger equation:

$$i\varphi_t + \varphi_{xx} - \tau_0(2A+\tau_0)\varphi + 2(A+2\tau_0)|\varphi|^2\varphi - 3|\varphi|^4\varphi = 0. \quad (3)$$

Suppose that $\psi(x,t)$ is a solution of Eq.(1). The corresponding solution of Eq.(3) is then given by the linear substitution

$$\varphi(x,t) = b\,\psi(\sqrt{3}\,b^2 x,\ 3b^4 t) \quad ,$$

where

$$f^2 = \frac{2}{3}(A + 2\varkappa_0),$$

$$A/\varkappa_0 = -2 - \frac{3}{4}\frac{1}{\alpha}\left(1 - \sqrt{1+4\alpha}\right), \qquad (4)$$

$\varkappa_0$ and A being real constants, $\varkappa_0 > 0$. It is clear that ψ may be transformed into φ only at $\alpha \geqslant -1/4$ and A $> -2\varkappa_0$.

The energy functional H and the conserved "number of particles" N for Eq. (3) are, respectively,

$$H = \int dx \left\{ |\varphi_x|^2 + (|\varphi|^2 - \varkappa_0)^2 (|\varphi|^2 - A) \right\} \qquad (5)$$

and

$$N = \int dx \left\{ |\varphi|^2 - \varkappa_0 \right\}. \qquad (6)$$

We consider the solutions to Eq. (3) with the following spatial asymptotics:

$$\varphi(x,t) \to \varphi^{\pm} = const \quad \text{as} \quad x \to \pm\infty, \qquad (7)$$

where $|\varphi^{\pm}|^2 = \varkappa_0$ may be physically interpreted as an equilibrium density of a medium filling the entire space. The integrals (5) and (6) will converge provided the field φ approaches its asymptotic values sufficiently fast. The value of $\varkappa_0 > 0$ may be chosen completely arbitrarily since according to (4) the solution φ to Eq. (3) is characterized not by a pair (A, $\varkappa_0$) but through a single quantity A/$\varkappa_0$. In this note we report the results of computations which were performed for $\varkappa_0$ = 1.

The solution we are interested in reads [8]

$$\varphi_s(x,t) = e^{i\vartheta_0} \frac{i + th\{\sqrt{a}(\xi - \xi_0)\}\, tg\,\mu/2}{[1 + th^2\{\sqrt{a}(\xi - \xi_0)\}\, tg^2 \mu/2]^{1/2}} \cdot$$

$$\cdot \left[\frac{A\varkappa_0 + v^2/2 + \varkappa_0\sqrt{A^2+v^2}\, ch\{2\sqrt{a}(\xi - \xi_0)\}}{2\varkappa_0 - A + \sqrt{A^2+v^2}\, ch\{2\sqrt{a}(\xi - \xi_0)\}} \right]^{1/2}, \qquad (8)$$

where $\xi = x - vt$, $\cos\mu = (A\varkappa_0 + v^2/2)/(\varkappa_0\sqrt{A^2+v^2})$, $a = (c^2 - v^2)/4 > 0$, and $c = 2[\varkappa_0(\varkappa_0 - A)]^{1/2}$. c stands for the velocity of acoustic waves in the medium. The formula (8) describes a solitary wave of rarefaction (a "hole") propagating with velocity $v < c$. At the spatial infinities we have

$$\varphi^{\pm} = \pm e^{i\vartheta_0} \left[\frac{tg\,\mu/2 \pm i}{tg\,\mu/2 \mp i} \right]^{1/2} \sqrt{\varkappa_0}. \qquad (9)$$

For A < 0 the solution (8) is kink-shaped while at 0 < A < $\varkappa_0$ it looks like a one-dimensional bubble. In the propounded communication we confine ourselves to the latter case as soon as it admits the most obvious generalization to higher spatial dimensions.

At 0 < A < $\varkappa_0$ the static (v = 0) bubble looks like

$$\varphi_s(x) = \frac{\sqrt{\varkappa_0}\, ch(\frac{f}{2}x)}{\{\varkappa_0/A + sh^2(\frac{f}{2}x)\}^{1/2}}, \qquad (10)$$

where we have used the translational and U(1) invariances of Eq. (3) to fix $\vartheta_0 = x_0 = 0$. It is fitting to note that the transition from the travelling wave (8) to the standing one (10) is nothing but the selection of a particular solution from one-parameter family. There exists no any reference frame transformation which could transfer the moving bubble (8) to the rest frame under the fixed boundary conditions (7). In other words, if we used the Galilean invariance of Eq. (3) and by virtue of the transformation

$$\varphi(x,t) \to \tilde{\varphi}(x,t) = e^{-i\frac{v}{2}(x + \frac{v}{2}t)} \varphi(x + vt, t)$$

passed to the co-ordinate frame in which the soliton is motionless ($\partial|\tilde{\varphi}|^2/\partial t = 0$), the constant asymptotics (7) would acquire the factor of $\exp\{-ivx/2 - iv^2t/4\}$. This means simply that in this frame the medium surrounding the bubble would itself move with the velocity ($-v$).

From the above given considerations it follows that, in contrast to the case of vanishing boundary conditions, the stability analysis for the travelling soliton (8) cannot be reduced to the one for the standing wave (10). For each pair of quantities ($A/\varkappa_0$, v) the problem should be treated independently. In this note we restrict ourselves to the solution (10), i.e., to the case of v = 0.

2. Linear Stability Analysis

Let us linearize Eq. (3) with respect to small (at least initially) perturbation $\delta\varphi$(x,t) in the vicinity of the soliton (10) and assume that localized $\delta\varphi$s exist which grow exponentially with time. More precisely, we shall seak for the separable $\delta\varphi$ of the form

$$\delta\varphi(x,t) = \{f(x) + ig(x)\}\, e^{\nu t}, \qquad (11)$$

where $f(x)$, $g(x)$ and ν are real, and

$$f(x) \to 0\,, \quad g(x) \to 0 \quad \text{as} \quad |x| \to \infty\,. \tag{12}$$

Putting $\varphi(x,t) = \varphi_s(x) + \delta\varphi(x,t)$ in Eq.(3) and keeping only up to first order in $\delta\varphi$ gives the eigenvalue problem

$$\begin{aligned} U f(x) &= -\nu g(x) \\ L g(x) &= \nu f(x), \end{aligned} \tag{13}$$

where U (upper) and L (lower) operators are defined by

$$U = -\frac{d^2}{dx^2} + \tau_0(2A+\tau_0) - 6(2\tau_0+A)\varphi_s^2(x) + 15\varphi_s^4(x)\,, \tag{14}$$

$$L = -\frac{d^2}{dx^2} + \tau_0(2A+\tau_0) - 2(2\tau_0+A)\varphi_s^2(x) + 3\varphi_s^4(x)\,.$$

If the problem (12)-(13) has a solution $z \equiv (\nu, f, g)$ then $z' \equiv (-\nu, f, -g)$ is a solution again. Therefore the existence of a real eigenvalue $\nu \neq 0$ will irrespective of its sign indicate exponential instability of the bubble (10).

It is worth noting the following fact. One easily verifies that

$$L\varphi_s(x) = 0$$

(this equality coincides with Eq.(3) for real static solutions). It ensues from here that for a discrete eigenvalue $\nu \neq 0$

$$\int_{-\infty}^{\infty} \varphi_s(x) f(x)\, dx = 0 \tag{15}$$

(multiply the lower equation in (13) by $\varphi_s(x)$ and integrate). Eq.(15) coincides with the condition for the infinitesimal deviation $\delta\varphi(x,t)$ (11) to keep the integral (6) undisturbed i.e., to obey $N[\varphi_s] = N[\varphi_s + \delta\varphi]$. In this way any perturbation of the form (11), where ν $(\nu \neq 0)$ is a discrete eigenvalue and $\{f(x), g(x)\}$ is the corresponding eigenvector normalized to a sufficiently small number, does not change the number of particles in the system. We therefore study the so-called Q-stability [9].

3. Numerical Solution of the Eigenvalue Problem

We solved the problem (13) on the finite interval $[-R, R]$, $R \gg 1$ and the boundary conditions (12) had been replaced by

$$f(\pm R) = g(\pm R) = 0\,. \tag{16}$$

The normalization condition had been chosen as

$$\int_{-R}^{R} \{f^2(x) + g^2(x)\}\, dx - 1 = 0. \tag{17}$$

In numerical studies of the set (13), (16), (17) we utilized the iterative scheme based on the modified continuous analogue of Newton's method [12]. The differential operators were approximated by finite-difference ones with accuracy of order $O(h^4)$ on a uniform mesh, h being the step. The integrals were calculated by means of Simpson's formula of the same accuracy. The scheme, therefore, allows one to find the difference solution to an accuracy of $O(h^4)$. A three-point difference operator is inverted in each iteration instead of five-point one, and this special feature simplifies greatly the finding of iteration corrections. Also one is permitted to avoid vast data arrays in the case when the eigenfunctions decay slowly as $|x| \to \infty$ and R ought to be chosen sufficiently large.

To analyze numerically convergence of the solution $z_{R,h,k} \equiv \{\nu, f(x), g(x)\}_{R,h,k}$ of the set (13), (16), (17) (k being a number of iterations) to the solution $z = \{\nu, f(x), g(x)\}$ of the initial problem (13), (12) and to evaluate accuracy of the former we used the following inequality

$$\|z - z_{R,h,k}\| \leq \|z - z_R\| + \|z_R - z_{R,h}\| + \|z_{R,h} - z_{R,h,k}\|\,. \tag{18}$$

Here

$\|\cdot\|$ is the mesh analogue of the norm in C;

$z_{R,h}$ stands for the exact solution to the difference problem approximating the set (13), (16), (17) to accuracy of $O(h^4)$.

The error $\theta_R = \|z - z_R\|$, $R \gg 1$ was estimated numerically by comparing the solutions obtained under fixed h and varying R. Our calculations indicate that it is small comparing to other items in the right-hand side of inequality (18). The error $\theta_k = \|z_{R,h} - z_{R,h,k}\| \sim \delta_k$[13] ($\delta_k$ stands for discrepance of the solution) is small since the criterium of the iteration process termination was $\delta_k \leq \varepsilon$, $\varepsilon \ll 1$. Hence, at $R \gg 1$ and $\varepsilon \ll 1$ the error $\theta_h = \|z_R - z_{R,h}\|$ originating from the replacement of the set (13), (16), (17) by the difference problem prevails for the approximate solution $z_{R,h,k}$. The order of convergence of the mesh solution $z_{R,h}$ to the solution z_R was verified by computing the ratios

$$p_i = \left| \left(z^{(i)}_{R,h_1,k_1} - z^{(i)}_{R,h_2,k_2}\right) / \left(z^{(i)}_{R,h_2,k_2} - z^{(i)}_{R,h_3,k_3}\right) \right|$$

on a sequence of condensing grids with steps $h_1=h$, $h_2=h/2$, $h_3=h/4$ (i.e., the relations $\log_2 p_i \sim 4$ were checked). Here

$$Z^{(1)}_{R,k_j,k_j} = \nu_{R,k_j,k_j} ;$$

by $Z^{(i)}_{R,k_j,k_j}$ $(i>1)$ we denote the eigenfunction values in nodes of the mesh with step h.

To evaluate the absolute error of the solution $Z_{R,h,k}$ we used the following expansions [14]

$$\nu_{k,h} = \nu_R + ch^4 + o(h^4) ;$$

$$f_{R,h}(x) = f_R(x) + r_1(x)h^4 + o(h^4) ;\ g_{R,h}(x) = g_R(x) + r_2(x)h^4 + o(h^4),$$

where c is a constant and $r_i(x)$ does not depend on h.

For A = 0.1 and A = 0.5 the accuracy of the solution was numerically investigated: under fixed h (h = 0.2) for varying R (R =40.; 50.; 80.) and under fixed R (R = 50.) for varying h (h = 0.4; 0.2; 0.1). The convergence to zero of the integral (15) values computed by Simpson's rule was verified as well.

The problem (13), (16), (17) was solved for A = 0.1; 0.2;...; 0.9 on the interval [-50., 50.] with the step h = 0.2. The accuracy ε was taken to be 10^{-8}. On the grounds of the above estimates the approximation parameters R = 50. and h = 0.2 were selected in such a way, that the absolute error of the found solution was less than 10^{-3}.

4. Results and Discussion

We solved the problem (13) at 9 equidistant points picked out from the interval (0,1). For each considered A (A = 0.1; 0.2;...; 0.9) the bound state solution with $\nu \neq 0$ was found. Thus the soliton (10) is unstable even against the infinitesimal perturbations not disturbing the total number of particles N (i.e., "Q-unstable" [9]) in the whole band $0 < A/\tau_0 < 1$. The corresponding growth rates (eigenvalues ν) are given in Table I. The curve ν (A) is illustrated in Fig.I. The eigenfunctions f(x) and g(x) are tabulated (Table II) and plotted (Fig. III) for three values of A (A = 0.1; 0.5; 0.9) typifying the whole band. For convenience of a reader the shape of the bubble (10) is reproduced in Fig. II for the same As.

It must be stressed once again that our instability conclusion relates to the bubble at rest only. Concerning travelling ones it

has been shown in ref. [8] that for transonic excitations of small (but finite) amplitude the ($\varphi^3 - \varphi^5$)-nonlinear Schrödinger equation (3) is reduced to Korteveg-de Vries equation. In this reduction transonic weak rarefaction bubble (eq. (8) at $v \lessapprox c$) is transformed just into the KdV soliton. The latter is well-known to be stable at any velocity [10-11]. Thus some critical velocity v_{cr} seems to exist such that the bubble (8) is stable at $v > v_{cr}$ and unstable at $v < v_{cr}$.

In conclusion let us remark that the growth rate ν characterizes solely a linear stage of decay of the bubble. This stage is defined by requirement that the perturbation can still be considered small. Thus the quantity $1/\nu$ should not be identified with the bubble's lifetime.

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Table I
Growth rates (eigenvalues of the problem (13),(12))

A	ν
0.1	0.197
0.2	0.348
0.3	0.443
0.4	0.489
0.5	0.492
0.6	0.458
0.7	0.389
0.8	0.289
0.9	0.159

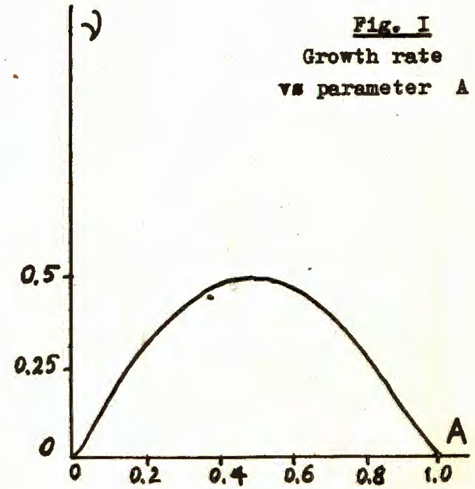


Fig. I
Growth rate vs parameter A

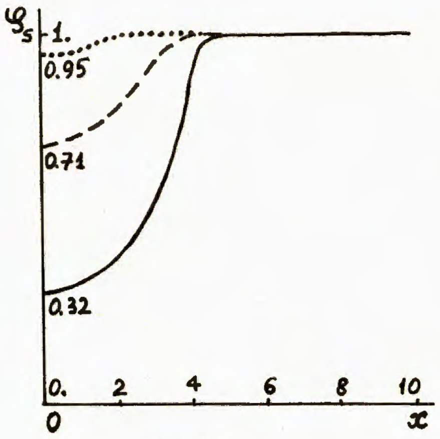


Fig. II

The bubble (10) at A = 0.1 (solid line), A = 0.5 (dashed line) and A = 0.9 (dotted line)

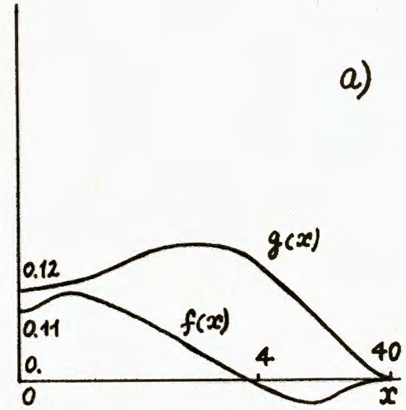
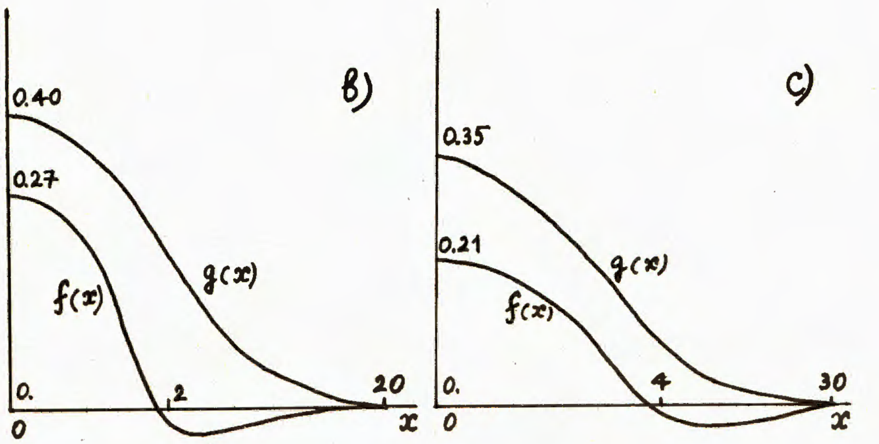


Fig. III

Eigenfunctions of the problem (13),(12) normalized to unity as in (17): a) A = 0.1; b) A = 0.5; c) A = 0.9



Due to $\varphi_s(x) = \varphi_s(-x)$, $f(x) = f(-x)$, $g(x) = g(-x)$ both the soliton and the eigenfunctions are plotted for $x \geqslant 0$ only.

Table II
Eigenfunctions of the problem (13), (12)
(R = 50, h = 0.2)

	A = 0.1		A = 0.5		A = 0.9	
x	f(x)	g(x)	f(x)	g(x)	f(x)	g(x)
0.	0.107	0.117	0.271	0.400	0.209	0.354
2.0	$0.780 \cdot 10^{-1}$	0.230	$-0.160 \cdot 10^{-1}$	0.316	$0.762 \cdot 10^{-1}$	0.302
4.0	$-0.107 \cdot 10^{-1}$	0.244	$-0.451 \cdot 10^{-1}$	0.163	$-0.437 \cdot 10^{-1}$	0.201
6.0	$-0.110 \cdot 10^{-1}$	0.200	$-0.212 \cdot 10^{-1}$	$0.794 \cdot 10^{-1}$	$-0.513 \cdot 10^{-1}$	0.119
8.0	$-0.896 \cdot 10^{-2}$	0.162	$-0.101 \cdot 10^{-1}$	$0.385 \cdot 10^{-1}$	$-0.336 \cdot 10^{-1}$	$0.685 \cdot 10^{-1}$
10.0	$-0.727 \cdot 10^{-2}$	0.132	$-0.494 \cdot 10^{-2}$	$0.187 \cdot 10^{-1}$	$-0.195 \cdot 10^{-1}$	$0.389 \cdot 10^{-1}$
16.0	$-0.388 \cdot 10^{-2}$	$0.705 \cdot 10^{-1}$	$-0.569 \cdot 10^{-3}$	$0.216 \cdot 10^{-2}$	$-0.357 \cdot 10^{-2}$	$0.717 \cdot 10^{-2}$
22.0	$-0.207 \cdot 10^{-2}$	$0.376 \cdot 10^{-1}$	$-0.656 \cdot 10^{-4}$	$0.249 \cdot 10^{-3}$	$-0.659 \cdot 10^{-3}$	$0.132 \cdot 10^{-2}$
28.0	$-0.110 \cdot 10^{-2}$	$0.199 \cdot 10^{-1}$	$-0.756 \cdot 10^{-5}$	$0.287 \cdot 10^{-4}$	$-0.122 \cdot 10^{-3}$	$0.246 \cdot 10^{-3}$
34.0	$-0.573 \cdot 10^{-3}$	$0.104 \cdot 10^{-1}$	$-0.871 \cdot 10^{-6}$	$0.330 \cdot 10^{-5}$	$-0.226 \cdot 10^{-4}$	$0.456 \cdot 10^{-4}$
40.0	$-0.278 \cdot 10^{-3}$	$0.505 \cdot 10^{-2}$	$-0.100 \cdot 10^{-6}$	$0.380 \cdot 10^{-6}$	$-0.417 \cdot 10^{-5}$	$0.842 \cdot 10^{-5}$
46.0	$-0.961 \cdot 10^{-4}$	$0.174 \cdot 10^{-2}$	$-0.109 \cdot 10^{-7}$	$0.414 \cdot 10^{-7}$	$-0.694 \cdot 10^{-6}$	$0.140 \cdot 10^{-5}$
50.0	0.	0.	0.	0.	0.	0.

f(x) and g(x) being even, both eigenfunctions are given for positive arguments only.

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Установлена следующая стоимость подписки на 12 месяцев на издания ОИЯИ, включая пересылку, по отдельным тематическим категориям:

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Барашенков И.В. и др. E17-85-967

Об устойчивости уединенных волн $\phi^3-\phi^5$ нелинейного уравнения Шредингера при нетривиальных граничных условиях

Исследуется устойчивость солитоноподобного решения дырочного типа для уравнения

$$i\psi_t + \psi_{xx} + \alpha\psi + \psi|\psi|^2 - \psi|\psi|^4 = 0,$$

где $\psi(x,t) \to \psi^{\pm} = \mathrm{const}$ при $x \to \pm\infty$. В линейном приближении анализ устойчивости статического солитона сводится к численному решению задачи на собственные значения для системы двух связанных операторов типа Шредингера. Показано, что эта задача имеет локализованное решение при всех α, и следовательно неподвижный солитон всегда неустойчив. Вычислена зависимость инкремента неустойчивости от α.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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On the Stability of Solitary Waves of the $(\phi^3-\phi^5)$-Nonlinear Schrödinger Equation under Non-Vanishing Boundary Conditions

We examine the stability of the bubble-like soliton solution of the equation

$$i\psi_t + \psi_{xx} + \alpha\psi + \psi|\psi|^2 - \psi|\psi|^4 = 0,$$

where $\psi(x,t) \to \psi^{\pm} = \mathrm{const}$ as $x \to \pm\infty$. The analysis of the above equation linearized about the static bubble is reduced to an eigenvalue problem for two coupled Schrödinger operators. This problem is found to possess a bound state solution for all α. Consequently, there exist exponentially growing perturbations, and the motionless bubble appears to be always unstable. The dependence of the growth rate on α is numerically calculated.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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