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ON THE LOWER CRITICAL DIMENSIONALITY OF THE CLASSICAL RANDOM AXIS MODEL

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The effects of quenched random impurities is a subject which has recently attracted much theoretical and experimental interest. In particular great attention has been devoted to the investigation of the random axis model (RAM) which was originally proposed by Harris et al.[1] to describe the magnetic properties of amorphous alloys containing rare-earth ions with asymmetric charge distributions. The original spin Hamiltonian describing the model is of the form:

$$H = -J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j - D \sum_j (\vec{n}_j \cdot \vec{S}_j)^2 , \qquad (1)$$

where $J > 0$ is a nearest-neighbour ferromagnetic exchange interaction between classical n-component unit spins $\{\vec{S}_j\}$ and $\{\vec{n}_j\}$ are random n-component unit direction vectors which are assumed to be distributed independently from site to site. In (1), fluctuations in the anisotropy strength are ignored.

A renormalization group analysis [2] showed that the Heisenberg fixed point of the pure spin model was unstable against randomness in $d = 4-\epsilon$ dimensions and no new fixed point could be found. This result and a successive $\epsilon = d-4$ expansion [3] suggested that four-dimension is the lower critical dimensionality d_{cL} for ferromagnetism in the RAM. Pelcovits et al.[4] presented a variety of independent evristic arguments yielding to the conclusion that the random uniaxial anisotropy will destroy long-range order below four dimensions for n-vector models with $n \geq 2$. Analogous conclusions follow from a domain argument [5] similar to the one used in the case of random-field problem [6]. On the other hand, it seems that calculation [3] does not exclude the possibility of a partially aligned state below four dimensions, where a spontaneous magnetization seems to exist but with large transverse fluctuations. Furthermore, numerical studies [5,7] of the RAM have come to varying conclusions, some arguing for a ferromagnetic, other for a spin-glass phase at low temperature for $d \leq 4$.

A reliable answer to this question is lacking at the present and a rigorous proof of the absence of ferromagnetism (and therefore an exact calculation of d_{cL}) in the RAM is needed. An attempt in this direction was made by Pelcovits [3] which extended to the RAM the Schuster proof [8] of the destruction of long-range order for the random field model with $n \geq 2$ below four dimensions. However this demonstration cannot be considered rigorous due to the use of the replica trick in combination with Bogolubov inequalities. Indeed, it involves the analytical continuation $m \to 0$ (m is the replica index) in an inequality which may change direction as m passes through one [3,8].

In this note we present an extension of the Rice method [9] for the destruction of long-range order in pure systems with continuous symmetry as a reliable approach (in our opinion) to calculate d_{cL} for ferromagnetism in the RAM. According to the basic idea of the Rice method, d_{cL} is here determined as the dimensionality below which the large-distance behaviour of the two-point correlation function $G(\vec x_1 - \vec x_2)$ for the Ginzburg-Landau-Wilson (GLW) order parameter $\psi(\vec x)$ becomes "inconsistent" with the general Yang's criterion [10] for off-diagonal-long-range-order ($G(\vec x_1 - \vec x_2) \to$ a finite constant as $|\vec x_1 - \vec x_2| \to \infty$). Thus, one is essentially interested in examining $G(\vec x_1 - \vec x_2)$ in the limit as $|\vec x_1 - \vec x_2| \to \infty$.

For formal simplicity this will be realized for $n = 2$ (i.e., X-Y - like models), being straightforward the extension to the general case $n \geq 2$. In this case, the RAM can be described by the GLW effective Hamiltonian:

$$\mathcal{H}\{\psi, a\} = \int d^dx \{ c_0 |\nabla \psi(\vec x)|^2 + r_0 |\psi(\vec x)|^2 + \frac{u_0}{2} |\psi(\vec x)|^4 - \frac{1}{4}[a(\vec x)\psi^*(\vec x) + a^*(\vec x)\psi(\vec x)]^2 \}, \qquad (2)$$

where $\psi(\vec x)$ is a complex field and the complex random variable $a(\vec x) = a_1(\vec x) + i a_2(\vec x)$, describing the impurity effect, is governed by a Gaussian probability distribution $\mathcal{P}\{a\}$ with Fourier component averages:

$$[a(\vec k)]_{av} = [a^*(\vec k)]_{av} = 0, \qquad [a^*(\vec k)\, a(\vec k')]_{av} = \Delta \delta_{\vec k, \vec k'}. \qquad (3)$$

The random-axis anisotropy term in the continuous version (2) of the RAM for $n = 2$ differs slightly from the expresseion (1) where the strength of the anisotropy is constant and only the direction is random. In general, a random strength of the anisotropy as in (2) is more realistic [11] but the difference will not change the results in any qualitative way.

As mentioned above, we are interested to calculate the random two-point correlation function:

$$G(\vec x) = [G(\vec x, \{a\})]_{av} = [\langle \psi(\vec x)\, \psi^*(0) \rangle]_{av} \qquad (4)$$

in the limit as $|\vec x| \to \infty$. With this in mind, we start by assuming that, at sufficiently low temperature, the pure system has long-range order (for $d > 2$) and the order parameter is given by $\psi = \psi_0 e^{i\phi_0}$, where $\psi_0 > 0$ (with $\psi_0^2 \approx -r_0/u_0$) and ϕ_0 is an arbitrary constant. Then, the quenched configurational average (4) can be evalued in terms of the Hamiltonian obtained from (2) by expanding $\psi(\vec x)$ about ψ_0 and keeping only leading terms in the small quantities. This is quite justified since the limit of weak disorder is assumed "a priori". Writing $\psi(\vec x)$ in terms of its modulus and phase as $\psi(\vec x) = (\psi_0 + \tilde\psi(\vec x))\, e^{i\phi(\vec x)}$, the Hamiltonian (2) reduces to:

$$\mathcal{H}\{\psi, a\} \simeq \text{const} + \mathcal{H}\{\tilde\psi\} + \mathcal{H}\{\phi, \{a_i\}\}, \qquad (5)$$

where the constant is inessential for our purposes, and:

$$\mathcal{H}\{\tilde\psi\} = \int d^dx [c_0(\ \tilde\psi(\vec x))^2 - 2r_0 \tilde\psi^2(\vec x)], \qquad (6)$$

$$\mathcal{H}\{\phi, \{a_i\}\} = \int d^dx [c_0 \psi_0^2 (\nabla \phi(\vec x))^2 - 2\psi_0^2 a_1(\vec x)\, a_2(\vec x)\, \phi(\vec x)]. \qquad (7)$$

Now, we are in a position to calculate the two-point correlation function $G(\vec x, \{a\}) = \langle \psi(\vec x)\psi^*(0) \rangle$ for a given random anisotropy field configuration $\{a\}$. From (5), the factorization:

$$G(\vec x, \{a\}) \simeq G_{\tilde\psi}(\vec x)\, G_\phi(\vec x, \{a_i\}) \qquad (8)$$

follows, where $G_{\tilde\psi}(\vec x)$ is characteristic of the pure system [9], such that $G_{\tilde\psi}(\vec x) \to \psi_0^2 = \text{const}$ in the limit as $|\vec x| \to \infty$ for $d > 2$, and:

$$G_\phi(\vec x, \{a_i\}) = \langle e^{i(\phi(\vec x) - \phi(0))} \rangle_{\mathcal{H}\{\phi, \{a_i\}\}} \qquad (9)$$

is the thermal average with respect to (7) for a given configuration of randomness. The calculation of (9) can be realized simply in terms of Fourier components. We find:

$$G_\phi(\vec x, \{a_i\}) = \exp\Big\{ \sum_{0<|\vec k|<\Lambda} \frac{1}{2c_0\psi_0^2 k^2} \Big[-\frac{1}{V}(1 - \cos \vec k \cdot \vec x) + 2i\psi_0^2 \frac{1}{V} \sum_{0<|\vec k'|<\Lambda} \big((\alpha_1(\vec k + \vec k')\,\alpha_2(\vec k') + \beta_1(\vec k + \vec k')\,\beta_2(\vec k'))(1 - \cos \vec k \cdot \vec x) + (\alpha_2(\vec k')\,\beta_1(\vec k + \vec k') - \alpha_1(\vec k + \vec k')\,\beta_2(\vec k')) \sin \vec k \cdot \vec x \big) \Big] \Big\}, \qquad (10)$$

where $\alpha_i(\vec k)$ and $\beta_i(\vec k)$ ($i = 1, 2$) are the real and imaginary parts of the Fourier transform $a_i(\vec k) = V^{-1/2} \int d^dx\, e^{-i\vec k \cdot \vec x} a_i(\vec x)$ of $a_i(\vec x)$, V is the volume of the system and a cutoff Λ in the wave-vector space has been assumed. For the random two-point correlation function (4), we have

$$G(\vec x) = G_{\tilde\psi}(\vec x)\, G_\phi(\vec x), \qquad (11)$$

where

$$G_\phi(\vec{x}) = [G_\phi(\vec{x}, \{a_i\})]_{av} . \qquad (12)$$

Since $\mathcal{P}\{a\} = \prod_{i=1}^{2} \mathcal{P}\{a_i\}$, making use of configurational averages (3), from (10) it follows that:

$$G_\phi(\vec{x}) = \exp\{-\lambda^{(P)}(\vec{x}) - \lambda^{(R)}(\vec{x})\} \equiv G_\phi^{(P)}(\vec{x})\, G_\phi^{(R)}(\vec{x}), \qquad (13)$$

where, in the thermodynamic limit:

$$\lambda^{(P)}(\vec{x}) = \frac{1}{2c_0\psi_0^2} \int \frac{d^d k}{(2\pi)^d} \frac{1 - \cos\vec{k}\cdot\vec{x}}{k^2} \qquad (14)$$

is the contribution already found for the pure system [9], and

$$\lambda^{(R)}(\vec{x}) = \frac{K_d \Lambda^d}{4dc_c^2}\Delta^2 \int \frac{d^d k}{(2\pi)^d} \frac{1 - \cos\vec{k}\cdot\vec{x}}{k^4} \qquad (15)$$

with $K_d = \pi^{-d/2} 2^{1-d} / \Gamma(d/2)$.

For our purpose we must investigate the asymptotic behaviour of $\lambda^{(R)}(\vec{x})$ for large $|\vec{x}|$. We find, for $\Lambda|\vec{x}| \gg 1$:

$$\lambda^{(R)}(\vec{x}) \simeq \frac{K_d \Lambda^d}{4dc_0^2}\Delta^2 \times \begin{cases} A(d)\,|\vec{x}|^{4-d}, & d < 4 \\ 2K_4 \ln(\Lambda|\vec{x}|), & d = 4, \end{cases} \qquad (16)$$

where A(d) is a positive constant whose explicit expression is inessential here. Then, from (11)-(13) and the result (16), one has:

$$G(x) \sim G_\phi^{(R)}(\vec{x}) \approx \exp\left[-\frac{K_d\Lambda^d}{4dc_0^2}\Delta^2 \times \begin{cases} A(d)\,|\vec{x}|^{4-d}, & d < 4 \\ 2K_4 \ln(\Lambda|\vec{x}|), & d = 4 \end{cases}\right] \qquad (17)$$

for $|\vec{x}| \to \infty$. In particular, for the physical case $d=3$, it results for large distances:

$$G(\vec{x}) \sim \exp\{-\frac{\Lambda^3\Delta^2}{96\pi^3 c_0^2}|\vec{x}|\}. \qquad (18)$$

Thus, $G(\vec{x}) \to 0$ for $|\vec{x}| \to \infty$ and, according to Yang's criterion, long-range order is impossible for $d \le 4$. This implies $d_{cL} = 4$ for the RAM without any ambiguity for $d = d_{cL}$ as, on the contrary it happens in the domain arguments.

In conclusion, we wish to emphasize that the extended Rice method, although not rigorous, is sufficiently general and physically quite reliable to yield the possibility to obtain, in a unified way and without ambiguities, a trustworthy estimate of the lower critical dimensionality for quenched random systems also when:

(i) long-range interactions and long-range correlated random p-fold anisotropies [12] are present;

(ii) quantum fluctuations are taken into account and the influence on criticality in the quantum regime has to be investigated.

A detailed investigation of these more general aspects of the problem will be the object of a future paper.

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О нижней критической размерности классической модели со случайной одноосной анизотропией

Метод Риса при описании разрушения дальнего порядка в чистых ферромагнитных системах с непрерывной симметрией распространен на модель со случайной одноосной анизотропией /система с примесями/ и применен при вычислении нижней пространственной критической размерности в этой модели без привлечения доменной аргументации и метода реплик.

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On the Lower Critical Dimensionality of the Classical Random Axis Model

The Rice method for the destruction of long-range order in pure systems with continuous symmetry is extended to the corresponding quenched random axis model for a reliable calculation of the lower critical dimensionality by avoiding domain arguments and the replica trick.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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