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CRITICAL CURVES IN NONSTANDARD POTTS MODELS

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Consider the system on the two-dimensional torus lattice with p rows and q columns. Its state is described by a $p \times q$ matrix (s_{ik}), $s_{ik} \in Z_n$. We denote by Z_n the multiplicative cyclic group while Z'_n stays for its additive realization.

This is to be an arbitrary nonstandard Potts model [1] as suggested by Domb [2] hence the total energy of the system in a given state reads as follows

$$-\frac{E(s_{ik})}{kT} = a \sum_{i,k=1}^{p,q} (s_{ik}^{-1} a_{i,k+1} + s_{i,k+1}^{-1} a_{ik}) + \tag{1}$$
$$+ b \sum_{i,k=1}^{p,q} (s_{ik}^{-1} s_{i+1,k} + s_{i+1,k}^{-1} s_{ik}) .$$

The transfer matrix for the model can be represented in a convenient form [3] with use of generalized Pauli matrices [4] and introducing generalized cosh functions f_i, $i \in Z'_n$ [3]

$$f_i(x) = \frac{1}{n} \sum_{k=0}^{n-1} \omega^{-ki} \exp\{\omega^k x\}, \tag{2}$$

where x is an element of some associative algebra with unity and ω is the primitive n-th order root of unity. The crucial property of f_i's is their relation to the eigenvalues of the interaction matrix [3,5]

$$\sum_{i=0}^{n-1} f_i(a) f_{i-k}(a) = \frac{1}{n} \chi_k(2a), \qquad k \in Z'_n , \tag{3}$$

where $\chi_k(2a), k \in Z'_n$ (with $\chi_k = \chi_{-k}$) form the set of all eigenvalues of the circulant interaction matrix $W[\sigma_1]$:

$$W[\sigma_1] = \sum_{\ell=0}^{n-1} \exp\{2a \operatorname{Re} \omega^\ell\} \sigma_1^\ell = W[a]. \tag{4}$$

Here σ_1 denotes one of the three σ_1, σ_2, σ_3 generalized Pauli matrices [4,3,5]. Introducing finally the tensor products of matrices

$$X_k = 1 \times \dots \times 1 \times \sigma_1 \times \dots \times 1 , \qquad p \text{ terms} \tag{5}$$

$$Z_k = 1 \times \dots \times 1 \times \sigma_3 \times \dots \times 1 , \qquad p \text{ terms} \tag{6}$$

(Pauli matrices placed at the k-th site) one arrives at the following form of the transfer matrix M:

$$M = [g(a^*)]^p \exp\{b \sum_{k=1}^{p} (Z_k^{-1} Z_{k+1} + Z_{k+1}^{-1} Z_k)\} \exp a^* \sum_{k=1}^{p} (X_k + X_k^{-1}), \quad (7)$$

where $[g(a^*)]^n = \det W[a]$, and a^* is the dual parameter to be found from its defining relation

$$\det W[a^*] = n^n \det W[a]^{-1} \quad . \qquad (8)$$

We do not quote the boundary cyclic conditions as finally we are concerned with thermodynamic limit.

There, the nonstandard, planar models under consideration posses the Kramers-Wannier duality property.

For that to demonstrate let us introduce the operators

$$\prod_{r<k} X_r = \mathcal{Z}_k \quad \text{and} \quad Z_k^{-1} Z_{k+1} = \mathcal{X}_k, \quad k = 1, \dots, p. \qquad (9)$$

Note that these very operators do satisfy the same, generalized Glifford algebra defining relations as X_k and Z_k. Hence (9) defines an automorphism of the very algebra and this must be an inner automorphism because the generalized Glifford algebra with 2p generators is isomorphic to the algebra of all $n^p \times n^p$ matrices [6].

This in turn means that there exists an invertible matrix D such that

$$D \mathcal{Z}_k D^{-1} = Z_k \quad \text{and} \quad D \mathcal{X}_k D^{-1} = X_k. \qquad (10)$$

At the same time from (9) and (7) one gets

$$M = [g(a^*)]^p \exp\{b \sum_k (\mathcal{X}_k + \mathcal{X}_k^{-1})\} \exp\{a^* \sum_k (\mathcal{Z}_k^{-1} \mathcal{Z}_{k+1} + \mathcal{Z}_{k+1}^{-1} \mathcal{Z}_k)\}. \qquad (11)$$

Hence, (10) being also taken into account, one obtains [5] the following duality relation for the free energy

$$F(a, b) = -\frac{1}{n} \ln \frac{\det W[a] \det W[b]}{n^n} + F(b^*, a^*) . \qquad (12)$$

Using now arguments similar to those of Kramers and Wannier, under assumption of critical curves existence we conclude that their equation is of the form

$$\det W[a] \cdot \det W[b] = n^n . \qquad (13)$$

One readily verifies that for n = 2, (13) becomes the known equation from the Ising model case.

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Критические кривые в нестандартной модели Потса

Преобразование дуальности типа Крамерса-Ванье для планарной нестандартной модели Поттса используется для получения критических кривых в предположении об их существовании.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Critical Curves in Nonstandard Potts Models

Under assumption of their existence, critical curves are found for all nonstandard, planar Potts models using Kramers-Wannier duality.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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