85-852

сообщения
объединенного
института
ядерных
исследований
дубна

E17-85-852

Fam Le Kien*, A.S.Shumovsky

QUANTUM THEORY OF A MULTIPHOTON TWO-MODE LASER

* Moscow State University, USSR

1985

The laser action phenomenon involving one-photon emission per atomic transition has long been successfully explained [1-3]. The possibility of achieving laser action involving the emission of many photons per atomic transition was first suggested by Sorokin and Braslau [4] and Prokhorov [5]. It has received much attention in recent years, not only because this novel type of laser may be potential as a high power optical amplifier, but also because the selforganization features of a system with nonlinear interaction between the field and matter is interesting. Some theoretical analyses have been done [6-13] and an experimental realization has been reported [14].

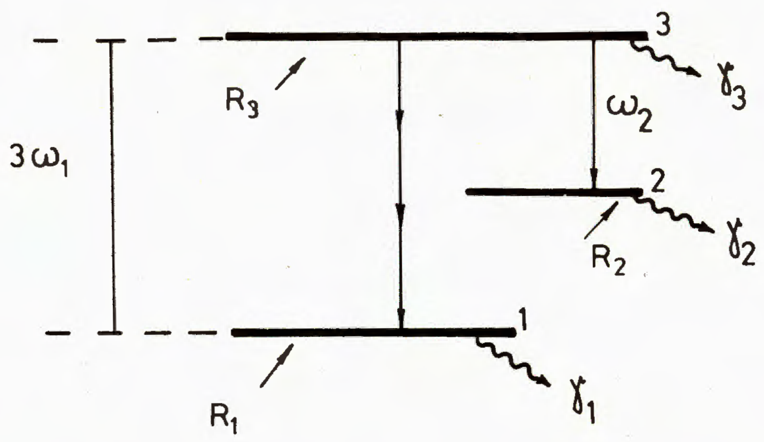
On the other hand, the theory of multimode lasers in multilevel atomic systems, which reflect the active medium more faithfully, has been developed by many authors. Some of these treatments have been semiclassical and deterministic, in which a set of coupled Maxwell-Bloch eguations is solved, see for example [15,16,3]. To discuss the statistical properties of the light field and include the effect of spontaneous emission noise, these theories have been generalized by replacing the Maxwell-Bloch equations with Langevin-type equations which may be converted into a Fokker-Planck equation, see e.g. [18,19]. In recent years, Scully and Lamb's guantum theory [20] of the one-photon single-mode laser has been extended to that of the multimode laser [21,22].

In this paper the quantum theory of a multiphoton two-mode laser with three-level atomic gain medium, in which all the levels are being pumped, is investigated, and the master equation of the laser operation is obtained. Our treatment is somewhat similar to that of Shi-yao Chu and Da-chun Su [22].

We consider a three-level atomic system as shown in the figure. We denote by R_1, R_2 and R_3 (γ_1, γ_2 and γ_3) the pumping rates (the decay constants) for level $|1>$, $|2>$ and $|3>$ respectively. An excited atom in state $|3>$ can make a transition to state $|1>$ (or $|2>$) by emitting m_1 (or m_2) photons into mode 1 (or 2) with frequency $\omega_1 = (\Omega_3 - \Omega_1)/m_1$ (or $\omega_2 = (\Omega_3 - \Omega_2)/m_2$). The transition between $|1>$ and $|2>$ is forbidden.

Let the function $p(n_1, n_2; t)$ represent the photon discribution in the system at the time t. As the three levels are all being pumped, and there exist cavity losses, we can write

$$\frac{d}{dt} p(n_1, n_2; t) = \dot{p}^{(1)}(t) + \dot{p}^{(2)}(t) + \dot{p}^{(3)}(t) + \dot{p}^{(\ell)}(t) , \qquad /1/$$



Three-level atomic system with multiphoton transitions in the particular case $m_1 = 3$, $m_2 = 1$.

where $\dot{p}^{(j)}(t)$ stands for the change caused by the pumping for the level $|j\rangle$ $(j = 1,2,3)$ and $\dot{p}^{(\ell)}(t)$ caused by the cavity losses. Following the procedure of [20,3,22]. They shall be determined in below.

If the average change of field due to one atom in the state $|j\rangle$ is $\delta p^{(j)}(t)$, and the pumping rate at which atoms are introduced into the cavity in the state $|j\rangle$ is R_j, we have

$$\dot{p}^{(j)}(t) = R_j\, \delta p^{(j)}(t), \quad j = 1, 2, 3, \qquad /2/$$

assuming that the change of field is very slow compared with that of atoms[22]. We consider the homogeneous broadened atomic system with the level widenesses γ_j. Taking into account the atom decay which is described by the probability distributions $P_j(\tau) = \gamma_j \exp(-\gamma_j \tau)$ of atomic level lifetimes, the average change of field per atom is given by

$$\delta p^{(j)}(t) = \int_0^\infty d\tau P_j(\tau)\, \delta^\tau p^{(j)}(t) = \int_0^\infty d\tau \gamma_j\, e^{-\gamma_j \tau}\, \delta^\tau p^{(j)}(t), \qquad /3/$$

where $\delta^\tau p^{(j)}(t)$ is the change in $p(n_1, n_2; t)$ due to one unbroadened three-level atom in the state $|j\rangle$ interacting for a time τ with the two-mode resonant radiation field in a lossless cavity throughout the multiphoton transition mechanism (the ideal model). This model is exactly soluble and has been investigated in [2,3]. By using the results of this paper we have

$$\delta^\tau p^{(j)}(t) = p^{(j)}_{id}(n_1, n_2; t+\tau) - p(n_1, n_2; t), \qquad /4/$$

where

$$p^{(j)}_{id}(n_1, n_2; t+\tau) =$$
$$= p(n_1 + m_1\delta_{1j} - m_1, n_2 + m_2\delta_{2j}; t) R_1(j, n_1 + m_1\delta_{1j} - m_1, n_2 + m_2\delta_{2j}; \tau) +$$
$$+ p(n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j} - m_2; t) R_2(j, n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j} - m_2; \tau) + \qquad /5/$$
$$+ p(n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j}; t) R_3(j, n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j}; \tau).$$

The functions $R_i(j, n_1, n_2; \tau)$ being in eq.(5) are[23]

$$R_1(j, n_1, n_2; \tau) = -2\mu(j, n_1, n_2) \sin^2 \frac{\lambda(j, n_1, n_2)\tau}{2} -$$
$$- 2\lambda_1^2(j, n_1, n_2)\, u(j, n_1, n_1) \sin^2 \lambda(j, n_1, n_2)\tau + \delta_{1j},$$
$$R_2(j, n_1, n_2; \tau) = 2\mu(j, n_1, n_2) \sin^2 \frac{\lambda(j, n_1, n_2)\tau}{2} - \qquad /6/$$
$$- 2\lambda_2^2(j, n_1, n_2)\, u(j, n_1, n_2) \sin^2 \lambda(j, n_1, n_2)\tau + \delta_{2j},$$
$$R_3(j, n_1, n_2; \tau) = 2\lambda^2(j, n_1, n_2)\, u(j, n_1, n_2) \sin^2 \lambda(j, n_1, n_2)\tau + \delta_{3j}.$$

Here the notation

$$\lambda_\alpha(j, n_1, n_2) = g_\alpha \left[\frac{(n_\alpha - m_\alpha\delta_{\alpha j} + m_\alpha)!}{(n_\alpha - m_\alpha\delta_{\alpha j})!} \right]^{1/2},$$

$$\lambda(j, n_1, n_2) = [\lambda_1^2(j, n_1, n_2) + \lambda_2^2(j, n_1, n_2)]^{1/2}, \qquad /7/$$
$$\mu(j, n_1, n_2) = 2\lambda_1^2(j, n_1, n_2)\lambda_2^2(j, n_1, n_2)\{\delta_{1j} - \delta_{2j}\}/\lambda^4(j, n_1, n_2),$$
$$u(j, n_1, n_2) = \{\lambda_1^2(j, n_1, n_2)\delta_{1j} + \lambda_2^2(j, n_1, n_2)\delta_{2j} -$$
$$- \lambda^2(j, n_1, n_2)\delta_{3j}\}/[2\lambda^4(j, n_1, n_2)]$$

has been introduced; g_α's are mode-atom coupling constants.

Now we can easily find from Eqs. (2)-(7) the expressions for $\dot{p}^{(j)}(t)$. For $\dot{p}^{(3)}(t)$ we get

$$\dot{p}^{(3)}(t) = -p(n_1, n_2; t) \frac{A_{13}\frac{(n_1+m_1)!}{n_1!} + A_{23}\frac{(n_2+m_2)!}{n_2!}}{1 + \frac{B_{13}}{A_{13}}\frac{(n_1+m_1)!}{n_1!} + \frac{B_{23}}{A_{23}}\frac{(n_2+m_2)!}{n_2!}} +$$

$$+\,p(n_1-m_1,\,n_2;t)\,\frac{A_{13}\dfrac{n_1!}{(n_1-m_1)!}}{1+\dfrac{B_{13}}{A_{13}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{23}}{A_{23}}\dfrac{(n_2+m_2)!}{n_2!}}\,+ \qquad /8/$$

$$+\,p(n_1,\,n_2-m_2;t)\,\frac{A_{23}\dfrac{n_2!}{(n_2-m_2)!}}{1+\dfrac{B_{13}}{A_{13}}\dfrac{(n_1+m_1)!}{n\,!}+\dfrac{B_{23}}{A_{23}}\dfrac{n_2!}{(n_2-m_2)!}}\,,$$

where $A_{\alpha 3}\equiv 2g_\alpha^2R_3/\gamma_3^2$ and $B_{\alpha 3}/A_{\alpha 3}\equiv 4g_\alpha^2/\gamma_3^2,\ \alpha=1,2$. The expression of $\dot p^{(1)}(t)$ is found to be

$$\dot p^{(1)}(t) = -p(n_1,n_2;t)\,\frac{A_{11}\dfrac{n_1!}{(n_1-m_1)!}}{\dfrac{B_{11}}{A_{11}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{21}}{A_{21}}\dfrac{(n_2+m_2)!}{n_2!}}\times$$

$$\times\left\{\frac{\dfrac{B_{11}}{A_{11}}\dfrac{n_1!}{(n_1-m_1)!}}{1+\dfrac{B_{11}}{A_{11}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{21}}{A_{21}}\dfrac{(n_2+m_2)!}{n_2!}}+\frac{4\dfrac{B_{21}}{A_{21}}\dfrac{(n_2+m_2)!}{n_2!}}{4+\dfrac{B_{11}}{A_{11}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{21}}{A_{21}}\dfrac{(n_2+m_2)!}{n_2!}}\right\}+$$

$$+\,p(n_1+m_1,\,n_2-m_2;t)\times \qquad /9/$$

$$\times\frac{3A_{11}\dfrac{B_{21}}{A_{21}}\dfrac{(n_1+m_1)!}{n_1!}\dfrac{n_2!}{(n_2-m_2)!}}{\left[1+\dfrac{B_{11}}{A_{11}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{21}}{A_{21}}\dfrac{n_2!}{(n_2-m_2)!}\right]\left[4+\dfrac{B_{11}}{A_{11}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{21}}{A_{21}}\dfrac{n_2!}{(n_2-m_2)!}\right]}+$$

$$+\,p(n_1+m_1,\,n_2;t)\,\frac{A_{11}\dfrac{(n_1+m_1)!}{n_1!}}{1+\dfrac{B_{11}}{A_{11}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{21}}{A_{21}}\dfrac{(n_2+m_2)!}{n_2!}}\,,$$

where $A_{\alpha 1}\equiv 2g_\alpha^2R_1/\gamma_1^2$ and $B_{\alpha 1}/A_{\alpha 1}\equiv 4g_\alpha^2/\gamma_1^2,\ \alpha=1,2$. Finally, for $\dot p^{(2)}(t)$ we find

$$\dot p^{(2)}(t) = -p(n_1,n_2;t)\,\frac{A_{22}\dfrac{n_2!}{(n_2-m_2)!}}{\dfrac{B_{12}}{A_{12}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{22}}{A_{22}}\dfrac{n_2!}{(n_2-m_2)!}}\times$$

$$\times\left\{\frac{4\dfrac{B_{12}}{A_{12}}\dfrac{(n_1+m_1)!}{n_1!}}{4+\dfrac{B_{12}}{A_{12}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{22}}{A_{22}}\dfrac{n_2!}{(n_2-m_2)!}}+\frac{\dfrac{B_{22}}{A_{22}}\dfrac{n_2!}{(n_2-m_2)!}}{1+\dfrac{B_{12}}{A_{12}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{22}}{A_{22}}\dfrac{n_2!}{(n_2-m_2)!}}\right\}+$$

$$+\,p(n_1-m_1,\,n_2+m_2;t)\times$$

$$\times\frac{3A_{22}\dfrac{B_{12}}{A_{12}}\dfrac{n_1}{(n_1-m_1)!}\dfrac{(n_2+m_2)!}{n_2!}}{\left[1+\dfrac{B_{12}}{A_{12}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{22}}{A_{22}}\dfrac{(n_2+m_2)!}{n_2!}\right]\left[4+\dfrac{B_{12}}{A_{12}}\dfrac{n_1!}{(n_1-m_1)!}+\dfrac{B_{22}}{A_{22}}\dfrac{(n_2+m_2)!}{n_2!}\right]}+ \qquad /10/$$

$$+\,p(n_1,\,n_2+m_2;t)\,\frac{A_{22}\dfrac{(n_2+m_2)!}{n_2!}}{1+\dfrac{B_{12}}{A_{12}}\dfrac{(n_1+m_1)!}{n_1!}+\dfrac{B_{22}}{A_{22}}\dfrac{(n_2+m_2)!}{n_2!}}\,,$$

where $A_{\alpha 2}\equiv 2g_\alpha^2R_2/\gamma_2^2$ and $B_{\alpha 2}/A_{\alpha 2}\equiv 4g_\alpha^2/\gamma_2^2,\ \alpha=1,2$. Note that all the coefficients $A_{\alpha j}$'s and $B_{\alpha j}$'s ($\alpha=1,2,\ j=1,2,3$) can be written in the following general form

$$A_{\alpha j}\equiv 2R_j(g_\alpha/\gamma_j)^2,\quad B_{\alpha j}\equiv 8R_j(g_\alpha/\gamma_j)^4. \qquad /11/$$

The coefficients $A_{\alpha j}$'s are gain parameters, while $B_{\alpha j}$'s are the so-called nonlinear parameters.

In practice, losses are due to the escape of photons from the cavity. To simulate losses we can introduce fictitious sets of atoms that absorb the laser radiation. We assume that losses are linear in photon numbers and independent for the two modes. Then the change of $p(n_1,n_2;t)$ caused by cavity losses is found to be [21]:

$$\dot p^{(\ell)}(t) = C_1(n_1+1)p(n_1+1,\,n_2;t)+C_2(n_2+1)p(n_1,\,n_2+1;t)\,-$$
$$-\,C_1n_1p(n_1,n_2;t)-C_2n_2p(n_1,n_2;t)\,. \qquad /12/$$

The loss parameters C_1 and C_2 are related to the cavity Q by

$$C_i=\omega_i/Q_i,\quad i=1,2. \qquad /13/$$

It follows from Eqs. (1), (8)-(10) and (12) that $p(n_1,n_2;t)$ satisfies the following equation

$$\frac{d}{dt}p(n_1,n_2;t) = -p(n_1,n_2;t)\ \left\{\frac{A_{13}G_1(n_1+m_1)+A_{23}G_2(n_2+m_2)}{1+B'_{13}G_1(n_1+m_1)+B'_{23}G_2(n_2+m_2)}+\right.$$

$$+\frac{A_{11}G_1(n_1)}{1+B'_{11}G_1(n_1)+B'_{21}G_2(n_2+m_2)}+\frac{A_{22}G_2(n_2)}{1+B'_{12}G_1(n_1+m_1)+B'_{22}G_2(n_2)}+$$

$$+\frac{3A_{11}B'_{21}G_1(n_1)G_2(n_2+m_2)}{[1+B'_{11}G_1(n_1)+B'_{21}G_2(n_2+m_2)][4+B'_{11}G_1(n_1)+B'_{21}G_2(n_2+m_2)]}+$$

$$+\frac{3A_{22}B'_{12}G_1(n_1+m_1)G_2(n_2)}{[1+B'_{12}G_1(n_1+m_1)+B'_{22}G_2(n_2)][4+B'_{12}G_1(n_1+m_1)+B'_{22}G_2(n_2)]}+$$

$$+C_1n_1+C_2n_2\}+p(n_1-m_1,n_2;t)\,\frac{A_{13}G_1(n_1)}{1+B'_{13}G_1(n_1)+B'_{23}G_2(n_2+m_2)}+$$

$$+p(n_1,n_2-m_2;t)\,\frac{A_{23}G_2(n_2)}{1+B'_{13}G_1(n_1+m_1)+B'_{23}G_2(n_2)}+$$

$$+p(n_1+m_1,n_2;t)\,\frac{A_{11}G_1(n_1+m_1)}{1+B'_{11}G_1(n_1+m_1)+B'_{21}G_2(n_2+m_2)}+$$

$$+p(n_1,n_2+m_2;t)\,\frac{A_{22}G_2(n_2+m_2)}{1+B'_{12}G_1(n_1+m_1)+B'_{22}G_2(n_2+m_2)}+$$

$$+p(n_1+m_1,n_2-m_2;t)\times$$

$$\times\frac{3A_{11}B'_{21}G_1(n_1+m_1)G_2(n_2)}{[1+B'_{11}G_1(n_1+m_1)+B'_{21}G_2(n_2)][4+B'_{11}G_1(n_1+m_1)+B'_{21}G_2(n_2)]}+$$

$$+p(n_1-m_1,n_2+m_2;t)\times$$

$$\times\frac{3A_{22}B'_{12}G_1(n_1)G_2(n_2+m_2)}{[1+B'_{12}G_1(n_1)+B'_{22}G_2(n_2+m_2)][4+B'_{12}G_1(n_1)+B'_{22}G_2(n_2+m_2)]}+$$

$$+C_1(n_1+1)p(n_1+1,n_2;t)+C_2(n_2+1)p(n_1,n_2+1;t)\,, \qquad /14/$$

where the notation

$$G_a(n_a)\equiv\frac{n_a!}{(n_a-m_a)!}\,, \qquad /15.a/$$

$$B'_{aj}\equiv\frac{B_{aj}}{A_{aj}}=4g_a^2/\gamma_j^2 \qquad /15.b/$$

has been introduced. This is the desired master equation that we set out to derive. Various terms on the right-hand side of this equation can be interpreted as out-flow and in-flow of probabilities in two dimensions /21,22/.

In Eq. (14) there are four terms with the coefficients of the type AB. They represent two-photon processes between the two modes /22/ and are due to pumping to the lower levels. The atom pumped into one lower state can make a transition to the upper state by absorbing photons of the corresponding mode, then make another transition to the other lower state by emitting photons of the other mode. Each process plays a role of a loss to one mode, while a gain to the other mode.

Thus, in this paper we have obtained the master equation for the multiphoton two-mode laser operating in three-level atomic gain medium. It should be noted that in the particular case of single photon transitions $m_1=m_2=1$, and of the equal decay constants $\gamma_1=\gamma_2=\gamma_3$ equation (14) reduces to that obtained by Shi-yao Chu and Da-chun Su for single-photon-transition two-mode lasers /22/. In the other case, when one mode, for example, mode 2, is excluded from consideration by putting $g_2=0$ and, therefore, $A_{2j}=0$, and when there is no pumping to the lower level $|1\rangle$, i.e. $R_1=0$, from (14) we can easily get the master equation for the multiphoton one-mode laser operating in two-level gain medium /6,21/. Investigations of the general master equation (14) will be the subject of a publication in future.

REFERENCES

1. Lax.M. Statistical Physics. Vol.2. Brandeis University Summer Institute in Theoretical Physics, 1968.
2. Haken H. Handbuch der Physik. Vol.XXV/2C. Berlin, Heidelberg, New York, Springer 1970.
3. Sargent M., Scully M., Lemb W.E. Laser Physics, Addison-Wesley, Reading, Mass., 1974.
4. Sorokin P., Braslau N. IBM J.Res.Dev., 1964, 8, p.177.
5. Prokhorov A. Science, 1965. 149, p.828.
6. McNeil K., Walls D.J.Phys. 1975, A8, p.104; 1975, 8, p.111.
7. Ito H., Nakagomi T. Prog.Theor.Phys., 1977, 57,p.54.
8. Nayak N., Mohanty B. Phys.Rev., 1979, A19, p.1204.
9. Bulsara A., Schieve W. Phys.Rev., 1979, A19, p.2046.
10. Sczaniecki L. Optica Acta. 1980,27, p.125; 1982, 29, p.69.
11. Reid M., McNeil K., Walls D. Phys.Rev., 1981, A24, p.2029.
12. Singh S. Phys.Rev., 1982, A25, p.3206.

13. Wang Z.C., Haken H. Z.Phys., 1984,B 55, p.361.; 1984, 56, p.77.
14. Nikolaus B., Zlang D.Z., Toschek P. Phys.Rev.Lett.,1981, 47, p.171.
15. Menegozzi L., Lamb W.E. Phys.Rev., 1973, A8, p.2103.
16. O'Bryan C.L., Sargent M. Phys.Rev., 1973, A8, p.3071.
17. Arecchi F.T., Ricca A.M. Phys.Rev., 1977, A15, p.308.
18. M-Tehrani M., Mandel L. Phys.Rev., 1978, A17,p.679.
19. Hioe F.T. J.Math.Phys., 1978, 19, p.1307.
20. Scully M.O., Lamb W.E. Phys.Rev. 1967, A159, p.208.
21. Singh S., Zubairy M.S. Phys.Rev, 1980, A21, p.281.
22. Shi-yao Chu, Da-chun Su. Phys.Rev.,1982, A25, p.3169.
23. Aliskenderov E.I., Fam Le Kien, Shumovsky A.S. JINR, P17-85-574, Dubna, 1985; J.Phys. A: to be published.

Received by Publishing Department
on November 26, 1985.

Фам Ле Киен, Шумовский А.С. E17-85-852

Квантовая теория двухмодового лазера с многофотонными переходами

Изучена модель двухмодового лазера, реализуемого на трехуровневых излучателях с многофотонными переходами при наличии накачки всех трех уровней. Для такой системы построено кинетическое уравнение типа "Master Equation" для функции распределения фотонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Fam Le Kien, Shumovsky A.S. E17-85-852

Quantum Theory of a Multiphoton Two-Mode Laser

The quantum theory of a multiphoton two-mode laser operating in three-level atomic gain medium, in which all levels are being pumped is investigated. The master equation for the photon distribution function which describes the laser action is obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985