сообщения
объединенного
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ядерных
исследований
дубна

E17-85-784

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ON A JAYNES-CUMMINGS-TYPE MODEL

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1985

1. INTRODUCTION

The Jaynes-Cummings model [1] of a two-level atom interacting with a quantized single-mode radiation field is at the core of many problems in quantum optics, NMR and quantum electronics. The importance of this model lies in that it is perhaps the simplest solvable model that describes the essential physics of radiation-matter interaction. Recent studies of this model by Eberly et al. [2] and Knight and Radmore [3] have revealed quantum collapse and revival which are clearly a manifestation of the role of quantum mechanics in the coherence and fluctuation properties of radiation-matter systems. In a series of paper Buck and Sukumar [4-7] and Singh [8] have proposed three exactly solvably generations of the Jaynes-Cummings model one involving intensity dependent coupling, one involving multiphoton interaction between the field and atom, the other involving few-level structure of the atom. A generalized model describing a two-mode process in a three-level atom with one-photon transitions has been investigated by Li and Bei [9] and Bogolubov (Jr.) et al. [10-14] An excellent review of the dynamical theory of Jaynes-Cummings-type models has recently been given by Yoo and Eberly [40].

The possibility of a multiphoton transition, which proceeds via intermediate states, has first been pointed out by Mayer [15] Various multiphonon transition processes have been studied both theoretically and experimentally. Among them are two-photon and more general multiphoton lasers [16-22], two-photon decay [23,24] multiphoton absorption and emission in a two-level atomic system [25,26], Raman and hyper-Raman processes [27,28].

We wish to present in this paper a rigorous and fully quantum mechanical treatment of multiphoton two-mode processes in a three-level atom on the basis of an exactly solvable Jaynes-Cummings-type model [29].

In §2 we describe the model. Section 3 contains derivations of general explicit expressions for the time dependence of the level population and photon number operators. In §4 we study photon statistics. Section 5 gives a consideration of the quantum dressed states and transition probabilities. In §6 we summarize the results.

2. DESCRIPTION OF THE MODEL

We consider a three-level atom being at rest in a lossless cavity and interacting with a resonant quantized two-mode radiation field. The energy operator for the atom is

$$H_A = \sum_{j=1}^{3} \hbar \Omega_j R_{jj} . \tag{1}$$

Here, the operator $R_{jj} = |j><j|$ describes the population of level j and $\hbar\Omega_j$ is the corresponding level energy. The field Hamiltonian is

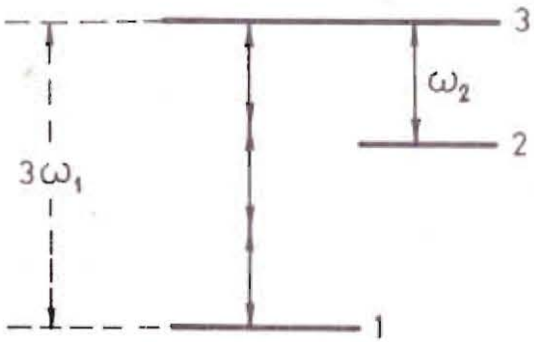
$$H_F = \sum_{\alpha=1}^{2} \hbar \omega_\alpha a_\alpha^+ a_\alpha . \tag{2}$$

The photon annihilation and creation operators a_α, a_α^+ $(\alpha=1,2)$ describe mode α of the quantized radiation field in the cavity. The ω_α's are the mode frequencies. Let the upper level 3 be coupled with the level 1 (level 2) due to the interaction with the field in mode 1 (mode 2) via a m_1-photon (m_2-photon) transition, see the figure in which the energy level structure and transition scheme are sketched for the case $m_1 = 3$, $m_2 = 1$. The corresponding multiphoton resonance conditions

$$\Omega_3 - \Omega_\alpha = m_\alpha \omega_\alpha , \quad (\alpha = 1,2) \tag{3}$$

are assumed to occur. As is well known, the atom-field interaction for a multiphoton process may be described by the effective Hamiltonian where a summation over intermediate states is implicit [25,30]. In the case of a three-level two-mode system considered here the effective Hamiltonian in the electric dipole and rotating wave approximation takes the form

$$H_{AF} = \hbar \sum_{\alpha=1}^{2} g_\alpha (R_{3\alpha} a_\alpha^{m_\alpha} + R_{\alpha 3} a_\alpha^{+m_\alpha}). \tag{4}$$



Energy level structure and transition scheme of the system considered in the particular case $m_1=3, m_2=1$.

Here, the operator $R_{ij} = |i><j|$ describes the atomic transition from level j to level $i (i \neq j)$. The mode α-atom coupling constant g_α is proportional to $\chi^{(m_\alpha)}$, the dipole matrix element for a m_α-photon transition between levels 3 and α. The operators $R_{ij} = |i><j|$ $(i, j = 1,2,3)$ obey the relations:

$$R_{ij} R_{k\ell} = R_{i\ell} \delta_{kj} , \tag{5a}$$

$$[R_{ij}, R_{k\ell}] = R_{i\ell}\delta_{kj} - R_{kj}\delta_{i\ell} , \tag{5b}$$

$$\sum_{i=1}^{3} R_{ii} = 1 . \tag{5c}$$

Thus, the full model Hamiltonian of the "atom-field" system is

$$H = H_A + H_F + H_{AF} = \sum_{j=1}^{3} \hbar\Omega_j R_{jj} + \sum_{\alpha=1}^{2} \hbar\omega_\alpha a_\alpha^+ a_\alpha + \hbar \sum_{\alpha=1}^{2} g_\alpha (R_{3\alpha} a_\alpha^{m_\alpha} + R_{\alpha 3} a_\alpha^{+m_\alpha}). \tag{6}$$

Note, the case $m_1 = m_2 = 1$ has been considered by Bogolubov (Jr.) et al. [10-14]. In the special case when the second mode is excluded from consideration, i.e., when $g_2 = 0$ we can obtain from the Hamiltonian (6) that examined by Buck and Sukumar [5,7] and Singh [8].

3. TIME-DEPENDENT LEVEL POPULATION AND PHOTON NUMBER OPERATORS

3.1. Equations of Motion

Starting from the Hamiltonian (6) we write down the Heisenberg equations for various operators in the usual way, i.e., $\dot{\mathcal{O}} = (i\hbar)[H, \mathcal{O}]$. First of all we define for convenience the subsidiary operators

$$A_\alpha = i(R_{3\alpha} a_\alpha^{m_\alpha} - R_{\alpha 3} a_\alpha^{+m_\alpha}). \tag{7}$$

Then, the Heisenberg equations for the level-population operators $R_{\alpha\alpha}$ and the photon-number operators $N_\alpha = a_\alpha^+ a_\alpha$ $(\alpha = 1,2)$ are quickly established

$$\dot{R}_{\alpha\alpha}(t) = g_\alpha A_\alpha(t), \tag{8a}$$

$$\dot{N}_\alpha(t) = m_\alpha g_\alpha A_\alpha(t). \tag{8b}$$

From these equations it follows that

$$N_\alpha(t) - m_\alpha R_{\alpha\alpha}(t) = \mathrm{const} \equiv M_\alpha , \qquad (9)$$

where M_α's are constants of motion.

By using relations (5) the Heisenberg equations for A_α are found to be

$$g_\alpha \dot{A}_\alpha(t) = 2 g_\alpha^2 \frac{(M_\alpha+m_\alpha)!}{M_\alpha !}[1 - R_{11}(t) - R_{22}(t) - R_{\alpha\alpha}(t)] - g_1 g_2 B(t), \qquad (10)$$

where

$$B = R_{21} a_1^{m_1} a_2^{+m_2} + R_{12} a_1^{+m_1} a_2^{m_2} . \qquad (11)$$

The operator B obeys the equation of motion

$$\dot{B}(t) = g_1 \frac{(M_1+m_1)!}{M_1 !} A_2(t) + g_2 \frac{(M_2+m_2)!}{M_2 !} A_1(t). \qquad (12)$$

Equations (8a), (10) and (12) form a closed system of linear equations that has the following integral of motion:

$$g_1 g_2 B(t) - \lambda_1^2 R_{22}(t) - \lambda_2^2 R_{11}(t) = \mathrm{const} = K. \qquad (13)$$

Here the notation

$$\lambda_\alpha^2 = g_\alpha^2 \frac{(M_\alpha+m_\alpha)!}{M_\alpha !} \qquad (14)$$

has been introduced.

Let us now differentiate each of equations (8a) with respect to time. Taking into account eqs.(10) and the constant of motion (13) we get then

$$\ddot{R}_{11}(t) + (4\lambda_1^2 + \lambda_2^2) R_{11}(t) + 3\lambda_1^2 R_{22}(t) = 2\lambda_1^2 - K,$$

$$\ddot{R}_{22}(t) + (4\lambda_2^2 + \lambda_1^2) R_{22}(t) + 3\lambda_2^2 R_{11}(t) = 2\lambda_2^2 - K. \qquad (15)$$

Note that equations (15) are the same as the equations obtained previously in the paper of Bogolubov (Jr.) et al.[10] for the case $m_1 = m_2 = 1$. One can consider these second-order differential equations as a system of equations for bounded quantum oscillators[31] generating Rabi nonlinear oscillations of level populations and photon numbers[32] in our model. The dependence of (15) upon the numbers of multiple photons per atomic transition m_1 and m_2 is included in the expressions of λ_1, λ_2 and K only.

The solutions of the system (15) can easily be found and represented in the form

$$R_{11}(t) = \mu(\cos\lambda t - 1) + \beta \sin\lambda t + \lambda_1^2 \{u(\cos 2\lambda t - 1) + v \sin 2\lambda t\} + R_{11}(0),$$

$$R_{22}(t) = -\mu(\cos\lambda t - 1) - \beta \sin\lambda t + \lambda_2^2 \{u(\cos 2\lambda t - 1) + v \sin 2\lambda t\} + R_{22}(0), \qquad (16)$$

where the operator

$$\lambda = (\lambda_1^2 + \lambda_2^2)^{1/2} = [g_1^2 \frac{(M_1+m_1)!}{M_1 !} + g_2^2 \frac{(M_2+m_2)!}{M_2 !}]^{1/2} \qquad (17)$$

describes the Rabi oscillation frequencies. The "amplitude operators" μ , β, u , v are defined by the initial conditions as follows:

$$\mu = \{\lambda^2 [\lambda_2^2 R_{11}(0) - \lambda_1^2 R_{22}(0)] + (\lambda_2^2 - \lambda_1^2) K\}/\lambda^4,$$

$$u = \{\lambda^2 [1 - 2R_{33}(0)] + K\}/(2\lambda^4),$$

$$\beta = \{\lambda_2^2 g_1 A_1(0) - \lambda_1^2 g_2 A_2(0)\}/\lambda^3 , \qquad (18)$$

$$v = \{g_1 A_1(0) + g_2 A_2(0)\}/(2\lambda^3).$$

By using the conservation laws (5c) and (9) together with eqs. (16) we can obtain

$$R_{33}(t) = -\lambda^2 [u(\cos 2\lambda t - 1) + v \sin 2\lambda t] + R_{33}(0),$$

$$N_1(t) = m_1 \{\mu(\cos\lambda t - 1) + \beta \sin\lambda t + \lambda_1^2 [u(\cos 2\lambda t - 1) + v \sin 2\lambda t]\} + N_1(0), \qquad (19)$$

$$N_2(t) = m_2 \{-\mu(\cos\lambda t - 1) - \beta \sin\lambda t + \lambda_2^2 [u(\cos 2\lambda - 1) + v \sin 2\lambda t]\} + N_2(0).$$

Thus, we have found the solutions of the equations of motion for the level-population and photon-number operators in the Heisenberg picture. Since the operators M_α, therefore, and the operators λ_α and λ are diagonal in the space of the basis states, we can use the solutions (16), (19) as conventional means to find the time dependences of the level populations and photon numbers. By using these solutions we can find also the statistical characteristics of the photons in the system (see Bogolubov (Jr.) et al.[12] and §4).

3.2. Time Evolution Operator

We denote the free Hamiltonian of the atom and field by H_0

$$H_0 = H_A + H_F \quad . \tag{20}$$

Then, the full Hamiltonian (6) can be written as

$$H = H_0 + H_{AF} \ . \tag{21}$$

It is easily shown that both H_0 and H_{AF} are constants of motion, i.e.,

$$[H, H_0] = [H, H_{AF}] = [H_0, H_{AF}] = 0. \tag{22}$$

This allows the time evolution operator U(t) to be written as

$$U(t) = \exp[-iHt/\hbar] = \exp[-iH_0 t/\hbar] \exp[-iH_{AF} t/\hbar] = = \exp[-iH_0 t/\hbar] U_{int}(t), \tag{23}$$

where

$$U_{int}(t) = \exp[-iH_{AF} t/\hbar] \tag{24}$$

is the time evolution operator in the interaction picture.

By using the identities

$$a_\alpha^{+m} a_\alpha^m = \frac{N_\alpha!}{(N_\alpha - m)!}, \quad a_\alpha^m a_\alpha^{+m} = \frac{(N_\alpha + m)!}{N_\alpha!} \tag{25}$$

and the relations (5a) we can easily show that

$$(H_{AF}/\hbar)^2 = K + \lambda^2, \quad H_{AF} K = 0, \tag{26}$$

where the constant operators K and λ have been defined in the previous subsection by eqs.(13) and (17), respectively. From eqs.(26) it follows that for an integer number $n \geq 1$

$$(H_{AF}/\hbar)^{2n} = \frac{K + \lambda^2}{\lambda^2} \lambda^{2n}, \quad (H_{AF}/\hbar)^{2n+1} = (H_{AF}/\hbar) \lambda^{2n}. \tag{27}$$

Hence, it is easy to express the time evolution operators $U_{int}(t)$ and U(t) in the form

$$U_{int}(t) = \frac{K + \lambda^2}{\lambda^2} \cos\lambda t - i(H_{AF}/\hbar) \frac{1}{\lambda} \sin\lambda t - \frac{K}{\lambda^2}, \tag{28a}$$

$$U(t) = \exp(-iH_0 t/\hbar) U_{int}(t). \tag{28b}$$

The time evolution of any operator is now determined by applying the transformation (28) to its value at the initial time t=0. In particular, the density operator $\rho(t)$ of the system "atom-field" in the Schrödinger picture will be given by

$$\rho(t) = U(t) \rho(0) U^+(t) \tag{29}$$

in terms of its value at time t=0. The density matrix $\rho_F(t)$ of the radiation field and the probability $P(n_1, n_2; t)$ of finding n_1 photons in mode 1 and n_2 photons in mode 2 are found from eq. (29) to be

$$\rho_F(t) = \mathrm{Tr}_A [U(t) \rho(0) U^+(t)], \quad P(n_1, n_2; t) = \langle n_2, n_1 | \rho_F(t) | n_1, n_2 \rangle. \tag{30}$$

Using eqs.(28)-(30) we can examine photon statistics for a given initial state of the system in the manner of Singh /8/.

On the other hand, the time evolution of the operator $\mathcal{O}$ in the Heisenberg picture is given by

$$\mathcal{O}(t) = U^+(t) \mathcal{O} U(t). \tag{31}$$

Using eqs.(28) and (31) we can quickly come to the same equations (16)-(19) and examine the time behaviour of the level populations and photon numbers for any initial state of the system.

4. PHOTON STATISTICS

Let us introduce the following operators of the characteristic function of photon distribution

$$\chi(\xi_1, \xi_2) = \exp[i\xi_1 N_1(t) + i\xi_2 N_2(t)]. \tag{32}$$

Using the conservation laws (9) we find

$$\chi(\xi_1, \xi_2) = \exp(i\xi_1 M_1 + i\xi_2 M_2) \{[\exp(i\xi_1 m_1) - 1] R_{11}(t) + + [\exp(i\xi_2 m_2) - 1] R_{22}(t) + 1\}. \tag{33}$$

Denote by $\rho(0)$ the density operator describing an initial state of the "atom-field" system. Then, the characteristic function $\langle\chi(\xi_1, \xi_2)\rangle$ is defined as

$$\langle\chi(\xi_1, \xi_2)\rangle = \mathrm{Tr}\, \chi(\xi_1, \xi_2) \rho(0). \tag{34}$$

It is connected with the photon distribution function $P(n_1,n_2;t)$ by the relation

$$<\chi(\xi_1,\xi_2)> = \sum_{n_1,n_2} \exp(i\xi_1 n_1 + i\xi_2 n_2)\, P(n_1,n_2;t) \tag{35}$$

which allows us to get the latter if the former is known.

Once the characteristic and photon distribution functions are known, it is easy to find the statistical moments of photon number $<N_\alpha^m(t)>$ and the correlations of modes $<N_1^k(t)\, N_2^\ell(t)>$ using the relations

$$<N_\alpha^m(t)> = \sum_{n_1 n_2} n_\alpha^m P(n_1,n_2;t) = \frac{\partial^m}{\partial(i\xi_\alpha)^m} <\chi(\xi_1=0,\ \xi_2=0)>$$

$$<N_1^k(t)\, N_2^\ell(t)> = \sum_{n_1 n_2} n_1^k n_2^\ell P(n_1,n_2;t) = \frac{\partial^{k+\ell}}{\partial(i\xi_1)^k\, \partial(i\xi_2)^\ell} <\chi(\xi_1=0,\xi_2=0)>. \tag{36}$$

Equations (33)-(36) together with eqs.(16) allow us to discuss photon statistics for a given initial state of the system. A detailed consideration of this problem will be given below.

We first assume that the atom is initially on a definite level i, i.e.,

$$\rho(0) = |i><i| \otimes \rho_F\ , \tag{37}$$

where the density matrix ρ_F describes the initial state of the field. Then, by using eqs.(33), (16) and (37) the characteristic function (34) is found to be

$$<\chi(\xi_1,\xi_2)> = \sum_{n_1 n_2} P(n_1,n_2) \exp[\, i\xi_1(n_1 - m_1\delta_{1i}) + i\xi_2(n_2 - m_2\delta_{2i})]$$

$$\{[\exp(i\xi_1 m_1) - 1]\, R_1(i,n_1,n_2;t) + [\exp(i\xi_2 m_2) - 1]\, R_2(i,n_1,n_2;t) + 1\}. \tag{38}$$

Here $P(n_1,n_2)$ is the initial distribution of photon numbers

$$P(n_1,n_2) = <n_2,n_1|\rho_F|n_1,n_2>. \tag{39}$$

The functions $R_\alpha(i,n_1,n_2;t)$ in eq.(38) are determined as

$$R_1(i,n_1,n_2;t) = -2\mu(i,n_1,n_2)\sin^2\frac{\lambda(i,n_1,n_2)t}{2} -$$

$$-2\lambda_1^2(i,n_1,n_2)\, u(i,n_1,n_2)\sin^2\lambda(i,n_1,n_2)t + \delta_{1i}\ , \tag{40}$$

$$R_2(i,n_1,n_2;t) = 2\mu(i,n_1,n_2)\sin^2\frac{\lambda(i,n_1,n_2)t}{2} -$$

$$-2\lambda_2^2(i,n_1,n_2)\, u(i,n_1,n_2)\sin^2\lambda(i,n_1,n_2)t + \delta_{2i}\ ,$$

where

$$\lambda_\alpha(i,n_1,n_2) = g_\alpha\sqrt{\frac{(n_\alpha - m_\alpha\delta_{\alpha i} + m_\alpha)!}{(n_\alpha - m_\alpha\delta_{\alpha i})!}}\ ,$$

$$\lambda(i,n_1,n_2) = \sqrt{\lambda_1^2(i,n_1,n_2) + \lambda_2^2(i,n_1,n_2)}\ , \tag{41}$$

$$\mu(i,n_1,n_2) = 2\lambda_1^2(i,n_1,n_2)\,\lambda_2^2(i,n_1,n_2)\{\delta_{1i} - \delta_{2i}\}/\lambda^4(i,n_1,n_2)\ ,$$

$$u(i,n_1,n_2) = \{\lambda_1^2(i,n_1,n_2)\delta_{1i} + \lambda_2^2(i,n_1,n_2)\delta_{2i} - \lambda^2(i,n_1,n_2)\delta_{3i}\}/[2\lambda^4(i,n_1,n_2)].$$

Comparing eq.(38) with eq.(35) we obtain

$$P(n_1,n_2;t) =$$

$$= P(n_1 + m_1\delta_{1i} - m_1, n_2 + m_2\delta_{2i})R_1(i;n_1 + m_1\delta_{1i} - m_1, n_2 + m_2\delta_{2i};t) +$$

$$+ P(n_1 + m_1\delta_{1i}, n_2 + m_2\delta_{2i} - m_2)R_2(i;n_1 + m_1\delta_{1i}, n_2 + m_2\delta_{2i} - m_2;t) + \tag{42}$$

$$+ P(n_1 + m_1\delta_{1i}, n_2 + m_2\delta_{2i})R_3(i;n_1 + m_1\delta_{1i}, n_2 + m_2\delta_{2i};t),$$

where

$$R_3(i,n_1,n_2;t) = 2\lambda^2(i,n_1,n_2)\, u(i,n_1,n_2)\sin^2\lambda(i,n_1,n_2)t + \delta_{3i}\ . \tag{43}$$

The statistical moments of photon number and the correlations of modes are found from eqs.(36) and (38) to be

$$<N_\alpha^m(t)> = \sum_{n_1 n_2} P(n_1,n_2)\{(n_\alpha - m_\alpha\delta_{\alpha i})^m +$$

$$+ [(n_\alpha - m_\alpha\delta_{\alpha i} + m_\alpha)^m - (n_\alpha - m_\alpha\delta_{\alpha i})^m]\, R_\alpha(i,n_1,n_2;t)\}, \tag{44}$$

$$<N_1^k(t)\, N_2^\ell(t)> = \sum_{n_1 n_2} P(n_1,n_2)\{(n_1 - m_1\delta_{1i})^k (n_2 - m_2\delta_{2i})^\ell +$$

$$+ (n_1 - m_1\delta_{1i})^k [(n_2 - m_2\delta_{2i} + m_2)^\ell - (n_2 - m_2\delta_{2i})^\ell]\, R_2(i,n_1,n_2;t) +$$

$$+ (n_2 - m_2\delta_{2i})^\ell [(n_1 - m_1\delta_{1i} + m_1)^k - (n_1 - m_1\delta_{1i})^k]\, R_1(i,n_1,n_2;t).$$

In particular, we find

$$\langle N_\alpha(t)\rangle = \sum_{n_1 n_2} P(n_1,n_2)\{n_\alpha - m_\alpha\delta_{\alpha i} + m_\alpha R_\alpha(i,n_1,n_2;t)\},$$

$$\langle N_\alpha^2(t)\rangle = \sum_{n_1 n_2} P(n_1,n_2)\{(n_\alpha - m_\alpha\delta_{\alpha i})^2 + [2m_\alpha(n_\alpha - m_\alpha\delta_{\alpha i}) + m_\alpha^2]R_\alpha(i,n_1,n_2;t)\},$$

$$\langle N_1(t)N_2(t)\rangle = \sum_{n_1 n_2} P(n_1,n_2)\{(n_1-m_1\delta_{1i})(n_2-m_2\delta_{2i}) + (n_1-m_1\delta_{1i})m_2R_2(i,n_1,n_2;t) + (n_2-m_2\delta_{2i})m_1R_1(i,n_1,n_2;t)\}. \tag{45}$$

Note, in the case $i = 1$, $m_1 = m_2 = 1$ equations (45) reduce to the results obtained by Bogolubov (Jr.) et al.[12] Equations (42) for the distribution function of photon numbers can easily be found by other ways using either the time evolution operators (28) and eqs.(30) in the Schrödinger picture or the dressed state formalism (see section 5). With the aid of the above-obtained eqs.(42), (44) and (45) we can examine the time behaviour of various photon statistical characteristics, mean photon numbers and mean atomic level populations[13,14]. In particular, the interesting effects such as quantum collapse and revival[2,3,13], quantum chaos[33,34], photon antibunching[14] in exactly soluble models can be investigated.

5. QUANTUM DRESSED STATES AND TRANSITION PROBABILITIES

We represent an eigenstate vector of the free Hamiltonian H_0 by $|i;n_1,n_2\rangle$, where $|i\rangle$ is an atomic eigenstate vector corresponding to level i, and $|n_1,n_2\rangle$ denotes a Fock state with n_1 photons in mode 1 and n_2 photons in mode 2. This vector describes the so-called undressed state of the system[35-37]. The eigenstates of the full Hamiltonian are easily found by solving the stationary Schrödinger equation

$$H\psi = E\psi. \tag{46}$$

Their expressions in terms of the undressed states $|i;n_1,n_2\rangle$ are given by

$$\psi_{+;n_1,n_2} = \frac{\lambda_1(n_1)}{\sqrt{2}\lambda(n_1,n_2)}|1;n_1+m_1,n_2\rangle + \frac{\lambda_2(n_2)}{\sqrt{2}\lambda(n_1,n_2)}|2;n_1,n_2+m_2\rangle + \frac{1}{\sqrt{2}}|3;n_1,n_2\rangle,$$

$$\psi_{-;n_1,n_2} = \frac{\lambda_1(n_1)}{\sqrt{2}\lambda(n_1,n_2)}|1;n_1+m_1,n_2\rangle + \frac{\lambda_2(n_2)}{\sqrt{2}\lambda(n_1,n_2)}|2;n_1,n_2+m_2\rangle - \frac{1}{\sqrt{2}}|3;n_1,n_2\rangle, \tag{47}$$

$$\psi_{0;n_1,n_2} = \frac{\lambda_2(n_2)}{\lambda(n_1,n_2)}|1;n_1+m_1,n_2\rangle - \frac{\lambda_1(n_1)}{\lambda(n_1,n_2)}|2;n_1,n_2+m_2\rangle,$$

and also by $\psi_{1;\tilde{n}_1,n_2} = |1;\tilde{n}_1,n_2\rangle$ with $\tilde{n}_1 \le m_1-1$ and $\psi_{2;n_1,\tilde{n}_2} = |2;n_1,\tilde{n}_2\rangle$ with $\tilde{n}_2 \le m_2-1$. Here for convenience we have denoted

$$\lambda_1(n_1) = g_1\sqrt{\frac{(n_1+m_1)!}{n_1!}}, \qquad \lambda_2(n_2) = g_2\sqrt{\frac{(n_2+m_2)!}{n_2!}},$$

$$\lambda(n_1,n_2) = \sqrt{\lambda_1^2(n_1)+\lambda_2^2(n_2)} = \sqrt{g_1^2\frac{(n_1+m_1)!}{n_1!} + g_2^2\frac{(n_2+m_2)!}{n_2!}} \tag{48}$$

The eigenenergies $E_{\nu;n_1,n_2}$ ($\nu = 0, \pm, 1, 2$) of the full Hamiltonian H that correspond to the eigenstates $|\psi_{\nu;n_1,n_2}\rangle$ are found to be

$$E_{0;n_1,n_2} = \hbar(\Omega_3 + n_1\omega_1 + n_2\omega_2),$$

$$E_{\pm;n_1,n_2} = E_{0;n_1,n_2} \pm \hbar\lambda(n_1,n_2), \tag{49}$$

and

$$E_{1;\tilde{n}_1,n_2} = \hbar(\Omega_1 + \tilde{n}_1\omega_1 + n_2\omega_2) \qquad \tilde{n}_1 \le m_1 - 1$$

$$E_{2;n_1,\tilde{n}_2} = \hbar(\Omega_2 + n_1\omega_1 + \tilde{n}_2\omega_2) \qquad \tilde{n}_2 \le m_2 - 1. \tag{50}$$

Thus, the spectrum of the Hamiltonian H consists of a lattice of triplets of closely spaced eigenstates $\psi_{s;n_1,n_2}$ ($s=0,\pm$) and two sets of equally spaced undressed states $|1;\tilde{n}_1,n_2\rangle$ with $\tilde{n}_1 \le m_1-1$ and $|2;n_1,\tilde{n}_2\rangle$ with $\tilde{n}_2 \le m_2-1$. Each triplet is characterized by a pair of indices (n_1,n_2) that indicates that those triplet states are linear combinations of the three degenerate states $|1;n_1+m_1,n_2\rangle$, $|2;n_1,n_2+m_2\rangle$ and $|3;n_1,n_2\rangle$, see (33). The energy splittings $\pm\hbar\lambda(n_1,n_2)$ within the triplet (n_1,n_2) are of course due to the coupling of the atom to the field and are referred to as the resonant Stark effect. The triplet eigenstates $\psi_{s;n_1,n_2}$ ($s = 0, \pm$) are called quantum dressed states of the system[35-37,40]. It is interesting to note that the dressed states $\psi_{0;n_1,n_2}$, see the last eq. in (47), are the coherent superpositions of only the undressed states $|1;n_1+m_1,n_2\rangle$ and $|2;n_1,n_2+m_2\rangle$ but not $|3;n_1,n_2\rangle$. The existence of such dressed states uncoupled with the upper level 3 plays the important role in the mechanism of the population trapping effect[37-40] due to which the decay channels in multiphoton excitation can be turned off.

We now proceed to calculate the probabilities for the multiphoton transitions of the atom. Let us denote by $\phi(t)$ the wave function of the total system "atom+field" in the Schrödinger picture. Then, the probability of finding the atom on its j-th level at time t as a result of the transition $i \to j$ initiated by n_1 photons in mode 1 and n_2 photons in mode 2 of the field is defined by the formula

$$P(t;i\to j) = \sum_{n_1' n_2'} |<\phi_{in_1n_2}(t)|j;n_1',n_2'>|^2. \tag{51}$$

Here, the initial condition

$$\phi_{in_1n_2}(0) = |i;n_1,n_2> \tag{52}$$

has been assumed. By expanding $\phi_{in_1n_2}(0)$ in terms of the dressed eigenstates (47) we can easily find the time dependent wave functions $\phi_{in_1n_2}(t)$. They read

$$\phi_{1n_1+m_1n_2}(t)\exp[i(\Omega_3+n_1\omega_1+n_2\omega_2)t] =$$

$$=|1;n_1+m_1,n_2>[\lambda_1^2(n_1)\cos[\lambda(n_1,n_2)t]+\lambda_2^2(n_2)]/\lambda^2(n_1,n_2)+$$

$$+|2;n_1,n_2+m_2>[\cos[\lambda(n_1,n_2)t]-1]\lambda_1(n_1)\lambda_2(n_2)/\lambda^2(n_1,n_2)-$$

$$-i|3;n_1,n_2>\lambda_1(n_1)\sin[\lambda(n_1,n_2)t]/\lambda(n_1,n_2).$$

$$\phi_{2n_1n_2+m_2}(t)\exp[i(\Omega_3+n_1\omega_1+n_2\omega_2)t]=$$

$$=|1;n_1+m_1,n_2>[\cos[\lambda(n_1,n_2)t]-1]\lambda_1(n_1)\lambda_2(n_2)/\lambda^2(n_1,n_2)+ \tag{53}$$

$$+|2;n_1,n_2+m_2>[\lambda_2^2(n_2)\cos[\lambda(n_1,n_2)t]+\lambda_1^2(n_1)]/\lambda^2(n_1,n_2)-$$

$$-i|3;n_1,n_2>\lambda_2(n_2)\sin[\lambda(n_1,n_2)t]/\lambda(n_1,n_2).$$

$$\phi_{3n_1n_2}(t)\exp[i(\Omega_3+n_1\omega_1+n_2\omega_2)t]=$$

$$=-i|1;n_1+m_1,n_2>\lambda_1(n_1)\sin[\lambda(n_1,n_2)t]/\lambda(n_1,n_2)-$$

$$-i|2;n_1,n_2+m_2>\lambda_2(n_2)\sin[\lambda(n_1,n_2)t]/\lambda(n_1,n_2)+|3;n_1,n_2>\cos[\lambda(n_1,n_2)t].$$

Hence, the expressions of the probabilities (51) for the multiphoton transitions are found to be

$$P(t;i\to j)=R_j(i,n_1,n_2;t), \tag{54}$$

where the functions $R_j(i,n_1,n_2;t)$ have been defined by eqs.(40) and (43). Equation (54) implies that the transition probability $P(t;i\to j)$ is equal to the population of level j under the initial state (52). Using eq.(54) and the detailed balance principle, and under the initial condition (37) we can easily obtain the same eq.(42) for the photon distribution function $P(n_1,n_2,t)$.

6. SUMMARY

In this paper we have presented and studied a soluble Jaynes-Cummings-type model. The model considered consists of a lambda configuration three-level atom interacting with a two-mode resonant radiation field through the multiphoton transition mechanism. The general explicit expressions for the time-dependent level population and photon number operators have been derived by various ways using either equations of motion or time evolution operators. The quantum electrodynamic expression of Rabi oscillation frequencies has been obtained. Photon statistics in the model has been studied. Expressions for the photon distribution, characteristic function, mean photon numbers, statistical moments and correlations of photon numbers in the modes are presented for various initial conditions. The quantum dressed eigenstates and the energy spectrum have been found. The probabilities for multiphoton transitions from a level to a level of the atom have been calculated. Application of the model to the study of multiphoton two-mode laser will be discussed in a future work.

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Received by Publishing Department
on November 1, 1985.

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Представлена точно решаемая модель трехуровневого атома, взаимодействующего с двухмодовым резонансным полем излучения через механизм многофотонных переходов. Исследованы динамика населенностей уровней и статистика фотонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Aliskenderov E.I. et al. E17-85-784
On a Jaynes-Cummings-Type Model

We present a soluble quantum electrodynamic model of a three-level atom interacting with a two-mode resonant radiation field through the multiphoton transition mechanism. Population dynamics and photon statistics in this Jaynes-Cummings-type model are examined.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985