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INTERACTING WALKERS ON THE CAYLEY TREE AND POLYMER STATISTICS

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1. INTRODUCTION

The properties of one-dimensional interacting strings which are embedded in three dimensions are of great importance both in polymer physics and biology. A model that reproduces the configurational properties of hydrocarbon chains inside a lipid membrane has been proposed by Izuyama and Akutsu [1] (to be referred to as IA). This model is a generalization of the two-dimensional dimer model [2] used by Nagle [3] to describe a phase transition in the system of noncontact flexible polymer chains. Polymers or "dislocation lines" in the IA model appear above the critical temperature T_c and may be regarded as directed strings which run vertically through the lattice and do not intersect one another.

IA attempted to prove that the model exhibits a phase transition with a jump in the specific heat C(T) at the critical point, i.e., the specific heat is finite as $T \to T_c + 0$ and zero for $T < T_c$. However, Bhattacharjee, Nagle, Huse, and Fisher (see, also [5]) have reconsidered the IA model with a random walk analogy and found that C(T) diverges as $\ln(T - T_c)$ when $T \to T_c + 0$ for $d = 3$ and as finite for higher dimensions.

The random walk analogy can be elucidated by identification of the vertical z-coordinate with discrete time. An actual question to be solved is a random walk problem of n walkers on an $x - y$ plane lattice with the restriction that after all walkers have taken the same number of steps, any two of them are not at the same site. When $n = 2$ the problem can be solved exactly [4]. The logarithmic law for C(T) follows then from a finite-size scaling ansatz [4], namely, the supposition that the asymptotic behaviour of two walkers remains true for large n.

Another approach to this problem has recently been proposed [6] which deals directly with an arbitrary number of walkers. Unfortunately, the sign of contribution to the partition function in this method depends on periodicity of polymers in the vertical direction (assuming periodic boundary conditions). Neglect of the sign difference called "generalized Bethe approximation" leads to the finite jump in C(T). It was noted in Ref. [6] that the method becomes exact if the $x - y$ plane lattice has the Bethe structure. The purpose of the present paper is to obtain explicitly the generating function of the above formulated random walk problem on the Cayley tree.

In Section 2 we use the general method [6] to reduce the original problem to statistics of a single Polya walk. A Polya walk on a Bethe lattice was investigated by Hughes and Sahimi [7] who extended the Montroll generating function formalism [8] to this case and showed that random walk on a Bethe lattice do have some qualitative similarities to random walks on a hypercubic lattice of dimension $d > 4$. It is natural to expect that the finiteness of the specific heat at T_c follows from this result according to the analysis of Ref. [4]. However, the true answer is quite different. In Section 3 we show that the model exhibits "3/2-order" transition in which the specific heat diverges as $(T - T_c)^{-1/2}$. Thus the IA model on the Bethe lattice demonstrates the two-dimensional behaviour [2,3] in spite of apparent multi-dimensional properties of related randon walks.

If we consider the random walk problem on the complete Cayley tree, then the generating function contains contributions from both sites deep within the lattice and sites close to the boundary. The results obtained below show that the contribution from the latter gives in thermodynamic limit the same singularity as the bulk term.

2. GENERAL CONSIDERATIONS

Consider the complete Cayley tree with a coordination number z and a central site O. Any other site of the lattice is connected with O by a unique sequence of bonds. If this sequence consists of ℓ bonds, we assign to the site the coordinate ℓ. There are $z(z-1)^{\ell-1}$ sites with the coordinate ℓ and the total number of sites in the graph is

$$N = z[(z-1)^L - 1]/(z-2), \qquad (1)$$

where L is the coordinate of boundary sites.

We define an M-stepped walk as a connected path along M bonds (perhaps with repetitions) starting and ending at the same point. Two walks do not intersect if they are not at the same point after equal numbers of steps. The statistical weight of a single M-stepped walk P is defined as

$$W(P) = x^M. \qquad (2)$$

Let g_n be an arbitrary configuration of n nonintersecting M-stepped walks on the Cayley tree. The weight of configuration g_n is given by the product

$$\chi(g_n) = \prod_{i=1}^{n} W(P_i) = x^{nM}. \qquad (3)$$

The problem consists in determining the generating function

$$\Lambda(x) = \sum_g \chi(g), \qquad (4)$$

where summation runs over all possible configurations of M-stepped walks and the weight of the void lattice is unity.

The polymer model arises from these definitions if one associates time (or the number of steps after the start) with the space z-coordinate. Indeed, the trajectories of walkers moving in a Cayley tree may be regarded as noncontact polymer chains or "dislocation lines" of the IA model. The condition for an M-stepped walk to start and to end at the same point means the periodic boundary conditions in z direction for the obtained three-dimensional lattice.

The partition function of the polymer model results from generating function (4) if we attach to the variable x a statistical meaning by setting $x = \exp(-\beta\mu)$, where β is the inverse temperature and μ is a chemical potential of a polymer link.

Instead of the original problem we consider first a modification of it. Let P be a K-stepped aperiodic walk returning to the starting point after $K = kM$ steps, where $k \geq 1$ is an integer. The absence of periodicity means that neither the walk itself nor any part of it can be represented in the form $(P')^m$, where P' is a closed path and $m \geq 2$ is an integer.

We introduce the auxiliary functions

$$\bar{W}(P) = (-1)x^K \qquad (5)$$

and

$$\bar{\chi}(g_n) = (-1)^n x^{nM}. \qquad (6)$$

There holds the following proposition.

Theorem. The product

$$\prod_P [1 + \bar{W}(P)] \qquad (7)$$

over all possible K-stepped aperiodic walks on the Cayley tree equals to the sum $\Sigma_g \bar{\chi}(g)$ over all configurations of M-stepped nonintersecting walks including the void lattice:

$$\prod_P [1 + \bar{W}(P)] = \sum_g \bar{\chi}(g). \qquad (8)$$

Sketch of Proof. It should be remarked that Eq.(8) is very similar to the identity which is known as Feynman's conjecture and is proved by Sherman [9] to get the combinatorical solution of the planar Ising model.

We will say that an K-stepped walk ($K = kM$, $k > 1$) is self-intersecting if the walker being at some point at the moment ℓ ($0 \le \ell \le (k-1)M$) visits the same point after rM steps, $r < k$ is an integer. An essential property of the Cayley tree is that any K-stepped walk with $K = kM$, $k > 1$ is self-intersecting.

Let us consider the formal infinite product on the left-hand side of Eq.(8), decomposing it into a sum of products of the form $\bar{W}(P_1)\bar{W}(P_2)\ldots\bar{W}(P_n)$. If among the set g_n of walks $P_1, \ldots, P_n$ there are not intersecting and self-intersecting ones, the product equals $\bar{\chi}(g_n)$ and contributes to the right-hand side.

Consider now a configuration g_n containing two walks P_i and P_j intersecting at some point. Then there exists a configuration g_n' which contains in place of two walks P_i and P_j a single walk P' with self-intersection at the same point. The first case is described by the term $(-1)x^{K_i}(-1)x^{K_j}$ in the expansion of (7), while the second by the term $(-1)x^{K_i+K_j}$. Therefore, the contributions from intersecting and self-intersecting walks cancel. Similar arguments can be used in the case of several intersection points. Thus, only terms of the sum $\Sigma_g \bar{\chi}(g)$ survive, where all configurations g consist of solely M-stepped walks.

Equation (8) makes it possible to reformulate the random walk problem of many walkers into a more simple one of a single particle. For this goal it is necessary to establish a relation between the configuration weight $\chi(g)$ and the auxiliary function $\bar{\chi}(g)$. Let us put $\tilde{x} = xe^{i\pi/M}$ and note that the change of variables $x \to \tilde{x}$ alters the sign of each M-stepped walk in (6). Then $\Sigma_g \bar{\chi}(g) \to \Sigma_g \chi(g)$ and since we assume $M \to \infty$, $\Lambda(x)$ and $\Lambda(\tilde{x})$ coincide in the thermodynamic limit. As a result, we may write the generating function in the form

$$\Lambda(x) = \sum_g \bar{\chi}(g) = \prod_P [1 + \bar{W}(P)] . \qquad (9)$$

On the basis of this equation we have

$$\ln \Lambda(x) = \sum_P \ln[1 + \bar{W}(P)] = -\sum_P \sum_{j=1}^{\infty} \frac{(-\bar{W}(P))^j}{j} . \qquad (10)$$

Let R be an arbitrary K-stepped walk not restricted by the aperiodicity condition, returning to the initial point after $K = kM$ steps, $k \ge 1$. We denote by $R_m(i)$ a set of walks which occur in the site i after m steps, $0 \le m \le M$. The total number of such walks $|R_m(i)|$ obeys for each i the following translation relations:

$$|R_0(i)| = |R_1(i)| = \ldots = |R_{M-1}(i)|, \qquad (11)$$

A walk containing kM steps with $k > 1$ being at site i at the moment m and passing some sequence of sites coincides with walks being at the site i at the moments $m + M$, $M + 2M$, ... and passing the same sequence of sites. Hence, each K-stepped walk enters into the sum

$$\sum_{m=0}^{M-1} \sum_i |R_m(i)| \qquad (12)$$

K times if it is aperiodic, and K/j times if it has a periodicity j. Then we can continue Eqs.(10):

$$-\sum_P \sum_{j=1}^{\infty} \frac{(-\bar{W}(P))^j}{j} = -\sum_{m=0}^{M-1} \sum_i \sum_K \frac{S_K^m(i)\, x^K}{K} = -M \sum_i \sum_K \frac{S_K^0(i)\, x^K}{K} , \qquad (13)$$

where $S_k^m(i)$ is the number of K-stepped walks in the set $R_m(i)$. The sum over lattice sites can be rearranged due to the symmetry of the Cayley tree. As a result, we obtain for the lattice with a coordination number z:

$$\ln \Lambda(x) = -M \sum_K \frac{S_K(0)x^K}{K} - M \sum_{\ell=1}^{L} z(z-1)^{\ell-1} \sum_K \frac{S_K(\ell)\, x^K}{K} , \qquad (14)$$

where

$$S_K(\ell) = S_K^0(i) \qquad (15)$$

if the site i has the coordinate ℓ.

3. SINGLE WALK GENERATING FUNCTION

The considerations in the preceding section lead to the expression (14) which we shall now make explicit by calculating the sums

$$\sum_K \frac{S_K(\ell)x^K}{K}, \quad \ell = 0, 1, \ldots, L \qquad (16)$$

including $S_k(\ell)$ - the number of arbitrary closed K-stepped walks.

Let $W_n(\ell|m)$ be a number of walks starting with coordinate m and terminating after n steps with coordinate ℓ. Following the treatment of Hughes and Sahimi /7/ we begin with the evolution equation

$$W_{n+1}(\ell|m) = \sum_{\ell'} \gamma(\ell, \ell')\, W_n(\ell'|m), \qquad (17)$$

where

$$\gamma(\ell, \ell') = \begin{cases} (z-1)\,\delta_{\ell,\ell'+1} + \delta_{\ell,\ell'-1} & 0 < \ell' < L \\ z\,\delta_{\ell,\ell'+1} & \ell' = 0 \\ \delta_{\ell,\ell'-1} & \ell' = L \end{cases} . \tag{18}$$

The origin $\ell = 0$ and the last shell $\ell = L$ act as reflecting barriers. Thus the random walks on the Cayley tree can be represented as effective biased walks on the one-dimensional lattice with two "defects". The initial condition is

$$W_0(\ell|m) = \delta_{\ell,m} . \tag{19}$$

To separate translation invariant and "defect" parts of $\gamma(\ell,\ell')$ we write

$$\gamma(\ell,\ell') = p(\ell-\ell') + q(\ell,\ell'), \tag{20}$$

where

$$P(\ell) = (z-1)\,\delta_{\ell,1} + \delta_{\ell,-1} \tag{21}$$

and

$$q(\ell,\ell') = \begin{cases} 0 & \ell' \neq 0,\ \ell' \neq L \\ \delta_{\ell,1} - \delta_{\ell,-1} & \ell' = 0 \\ -(z-1)\,\delta_{\ell,L+1} & \ell' = L \end{cases} . \tag{22}$$

Inserting this notation into Eq.(17) we obtain

$$W_{n+1}(\ell|m) - \sum_{\ell'} p(\ell-\ell')\,W_n(\ell'|m) = \sum_{\ell'} q(\ell,\ell')\,W_n(\ell'|m). \tag{23}$$

It is convenient to introduce a generating function by

$$W(\ell|m;\xi) = \sum_{n=0}^{\infty} W_n(\ell|m)\,\xi^n. \tag{24}$$

From Eq. (23) using Eq.(14) we have

$$W(\ell|m;\xi) - \xi\sum_{\ell'} p(\ell-\ell')\,W(\ell'|m;\xi) = \delta_{\ell m} + \xi\sum_{\ell'} q(\ell,\ell')\,W(\ell'|m;\xi). \tag{25}$$

A discrete Fourier transform

$$\tilde{W}(\phi|m;\xi) = \sum_{\ell=-\infty}^{\infty} e^{i\ell\phi}\,W(\ell|m;\xi) \tag{26}$$

yields

$$\tilde{W}(\phi|m;\xi) = \frac{e^{im\phi}}{1-\xi\lambda(\phi)} + \frac{\xi(e^{i\phi}-e^{-i\phi})}{1-\xi\lambda(\phi)}\,W(0|m;\xi) - \frac{\xi(z-1)e^{i(L+1)\phi}}{1-\xi\lambda(\phi)}\,W(L|m;\xi), \tag{27}$$

where $\lambda(\phi)$ is the "structure function"

$$\lambda(\phi) = \sum_{\ell=-\infty}^{\infty} e^{i\ell\phi}\,p(\ell) = (z-1)e^{i\phi} + e^{-i\phi}. \tag{28}$$

Inverting the Fourier transform we find that

$$W(\ell|m;\xi) = G(\ell|m;\xi) + \xi W(0|m;\xi)\,H(\ell;\xi) - \xi W(L|m;\xi)\,F(\ell;\xi) \tag{29}$$

with $G(\ell|m;\xi)$, $H(\ell;\xi)$, $F(\ell;\xi)$ defined by

$$G(\ell|m;\xi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i\phi(m-\ell)}}{1-\xi\lambda(\phi)}\,d\phi, \tag{30}$$

$$H(\ell;\xi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{-i\ell\phi}(e^{i\phi}-e^{-i\phi})}{1-\xi\lambda(\phi)}\,d\phi \tag{31}$$

and

$$F(\ell;\xi) = \frac{(z-1)}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i(L-\ell+1)\phi}}{1-\lambda\lambda(\phi)}\,d\phi. \tag{32}$$

Put $\ell = 0$ and $\ell = L$ in Eq.(29). We get the system of linear equations

$$\begin{aligned} W(0|m;\xi) &= G(0|m;\xi) + \xi W(0|m;\xi)\,H(0;\xi) - \xi W(L|m;\xi)\,F(0;\xi) \\ W(L|m;\xi) &= G(L|m;\xi) + \xi W(0|m;\xi)\,H(L;\xi) - \xi W(L|m;\xi)\,F(L;\xi) \end{aligned} \tag{33}$$

which has the solutions

$$W(0|m;\xi) = D^{-1}\det\begin{pmatrix} G(0|m;\xi) & \xi F(0;\xi) \\ G(L|m;\xi) & 1+\xi F(L;\xi) \end{pmatrix}, \tag{34}$$

$$W(L|m;\xi) = D^{-1}\det\begin{pmatrix} 1-\xi H(0;\xi) & G(0|m;\xi) \\ -\xi H(L;\xi) & G(L|m;\xi) \end{pmatrix} . \tag{35}$$

where

$$D \equiv \det \begin{pmatrix} 1-\xi H(0;\xi) & \xi F(0;\xi) \\ -\xi H(L;\xi) & 1+\xi F(L;\xi) \end{pmatrix}. \qquad (36)$$

The integrals (30), (31), (32) are simply evaluated. Denoting

$$d(\xi) = [1 - 4\xi^2 (z-1)]^{1/2} \qquad (37)$$

and

$$t_{\pm}(\xi) = \frac{1 \pm d(\xi)}{2\xi(z-1)} \qquad (38)$$

one can show that

$$G(\ell\,|\,m;\xi) = \begin{cases} [-t_+^{m-\ell}\,\theta(1-|t_+|) + t_-^{m-\ell}]/d(\xi) & m \ge \ell; \\ t_+^{m-\ell}\,\theta(|t_+|-1)/d(\xi) & m < \ell; \end{cases} \qquad (39)$$

$$H(\ell;\xi) = \begin{cases} [-t_+\theta(1-|t_+|) + t_- - t_+^{-1}\theta(|t_+|-1)]/d(\xi) & \ell = 0; \\ t_+^{-\ell}(t_+ - t_+^{-1})\,\theta(|t_+|-1)/d(\xi) & \ell \ge 1; \end{cases} \qquad (40)$$

and

$$F(\ell;\xi) = (z-1)[-t_+^{L-\ell+1}\theta(1-|t_+|) + t_-^{L-\ell+1}]/d(\xi), \qquad (41)$$

where $\theta(x) = 1,\ x>0$ and $\theta(x) = 0,\ x \le 0$.

Up to now we were dealing with the number of arbitrary walks on the Cayley tree. To calculate the sums (16), it is necessary to adapt the general generating function (24) for M-stepped walks starting and ending at the same point. For this goal we put in (24): $\xi = xt\exp(2\pi i \frac{j}{M})$.

The summation over j gives

$$\sum_K S_K(\ell)x^K t^K = \sum_{n=0}^{\infty} \frac{1}{M}\sum_{j=1}^{M} W_n(\ell\,|\,\ell)x^n t^n \exp(2\pi i \frac{jn}{M}) - 1 \qquad (42)$$

because only terms with $n = 0$ (mod M) will survive in the right-hand side of Eq.(42). Performing the integration over t and changing the summation by integration, i.e., setting $\beta = 2\pi j/M$, $d\beta = 2\pi/M$ we obtain for large M

$$\sum_K \frac{S_K(\ell)x^K}{K} = \frac{1}{2\pi}\int_0^{2\pi} d\beta \int_0^1 \frac{dt}{t}[W(\ell\,|\,\ell;\, xte^{i\beta}) - 1]. \qquad (43)$$

Now, the solution (29) together with Eqs.(34), (35) and (39)-(41) can be used for the derivation of thermodynamic properties of the system from the partition function (14).

4. THERMODYNAMIC PROPERTIES

In this section we concentrate on the analysis of the partition function near $x_c = 1/z$ that is the critical point according to simple arguments from Ref./4/. Indeed, energy-entropy estimates show that lowest-lying excited states consist of one M-stepped walk and have the free energy $RT(-\ln x - \ln z)$. Such states will be thermodynamically preferred to the ground state only when $x > z^{-1}$ which locates the critical point of the model. The free energy is zero below x_c. We have now to determine the singularity of $\Lambda(x)$ when $x \to x_c$ from the disordered phase.

We put

$$x = \frac{1}{z} + \omega \qquad (44)$$

and substitute the first terms from Eq.(29) into (43). From Eq.(39) we find that

$$I_1 \equiv \frac{1}{2\pi}\int_0^{2\pi} d\beta \int_0^1 \frac{dt}{t}[G(\ell\,|\,\ell;\, xte^{i\beta}) - 1] = -\frac{1}{2\pi}\int_0^1 \frac{dt}{t}\int_{-h(\omega,t)}^{h(\omega,t)} d\beta\, d^{-1}(xte^{i\beta}), \qquad (45)$$

where the function $h(\omega,t)$ is defined by

$$|t_+(xte^{i\beta})| \le 1 \qquad -h \le \beta \le h. \qquad (46)$$

The condition (46) gives the integration range in (45):

$$h(\omega,t) = \left[\frac{z(z-2)(z\omega + t - 1)}{2}\right]^{1/2} \qquad (47)$$

so that

$$I_1 = -\frac{2^{1/2} z^3}{3\pi(z-2)^{1/2}}\,\omega^{3/2} + O(\omega^2). \qquad (48)$$

The second term in Eq.(29) appears due to reflection of effective one-dimensional walks from the origin $\ell = 0$. Note, that at $\xi = 1/z$

$$F(0;\xi) = \frac{z}{(z-2)(z-1)^L}. \qquad (49)$$

Then the simple estimates in Eqs.(34), (36) show that the non-vanishing contribution from the second term has the form

$$\xi H(\ell;\xi)\frac{G(0|\ell;\xi)}{1-\xi H(0;\xi)} \tag{50}$$

in the thermodynamic limit $L\to\infty$. Hence,

$$I_2=\frac{1}{2\pi}\int_0^{2\pi}d\beta\int_0^1\frac{dt}{t}\,\xi W(0|m;\xi)H(\ell;\xi)=\frac{1}{2\pi}\int_0^{2\pi}d\beta\int_0^t\frac{dt}{t}\,\xi H(\ell;\xi)\frac{G(0|\ell;\xi)}{1-\xi H(0;\xi)}. \tag{51}$$

In the vicinity of $1/z$ take

$$H(\ell;\xi)=-\frac{2z^3(z-1)}{(z-2)^2}\,\omega+O(\omega^2). \tag{52}$$

From Eqs.(39), (40), (46), (52) we find that the expression (51) for I_2 becomes

$$I_2=\frac{z^2\omega}{\pi(z-2)^3(z-1)^{\ell-2}}\int_0^1\frac{dt}{t}\int_{-h(\omega,t)}^{h(\omega,t)}d\beta+O(\omega^3) \tag{53}$$

and because of (47)

$$I_2=\frac{2^{3/2}z^4\omega^{5/2}}{3\pi(z-2)^{5/2}(z-1)^{\ell-2}}+O(\omega^3). \tag{54}$$

The third term in Eq.(29) is associated with the boundary shell $\ell=L$. In the thermodynamic limit $L\to\infty$ it reduces to

$$\xi F(\ell;\xi)\frac{G(L|m;\xi)}{1+\xi F(L;\xi)}. \tag{55}$$

Proceeding as above we obtain from (30), (41), (46), (55)

$$I_3=\frac{1}{2\pi}\int_0^{2\pi}d\beta\int_0^1\frac{dt}{t}\,\xi F(\ell;\xi)\,W(L|m;\xi)=\frac{-z(z-1)^{m-L-1}}{2\pi(z-2)}\int_0\frac{dt}{t}\int_{-h(\omega,t)}^{h(\omega,t)}d\beta+O(\omega^2) \tag{56}$$

and, using (47)

$$I_3=-\frac{2^{1/2}z^3\omega^{3/2}}{3\pi(z-2)^{1/2}(z-1)^{L-\ell+1}}+O(\omega^2). \tag{57}$$

Substitution of the leading terms I_1 and I_3 into (14) finally gives

$$\frac{1}{MN}\ln\Lambda(x)=\frac{2^{1/2}z^2(z-1)}{3\pi(z-2)^{1/2}}\,\omega^{3/2}+O(\omega^2). \tag{58}$$

Thus, we obtain the free energy per lattice site above $x_c=1/z$. In the same way, the integral (43) can be calculated below x_c.

This gives the obvious result

$$\Lambda(x)=0,\ x<x_c. \tag{59}$$

The obtained "3/2-order" transition calls for some comments. It was noted in Introduction that the phase transition of this type occurs in two-dimensional polymer models [2,3]. In our notation it is the case $z=2$. There is a drastic difference between random walk behaviour for $z=2$ and $z>2$. For any pair of walkers in the former case its coordinates ℓ_1 and ℓ_2 are strongly ordered, say $\ell_1>\ell_2$, at any moment of time, whereas in the latter one permutations of ℓ_1 and ℓ_2 are permitted. The sole restriction on a walk configuration for $z>2$ is the absence of K-stepped walks with the period $k=K/M>1$. Nevertheless, our results show that this reduced constraint is still too strong to give the logarithmic singularity of the partition function.

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Приезжев В.Б. E17-85-318

Взаимодействующие случайные блуждания на решетке Кэли и статистика полимеров

Получена производящая функция для ансамбля случайных блужданий на решетке Кэли с координационным числом z. Учтено парное взаимодействие между блужданиями, которое препятствует попаданию двух блуждающих частиц в одну точку решетки после равного числа шагов. Статистика взаимодействующих полимеров следует из этой модели, если отождествить время /или число шагов/ с дополнительной пространственной координатой. Из предельного выражения для свободной энергии следует, что в системе происходит фазовый переход "3/2 рода".

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Priezzhev V.B. E17-85-318

Interacting Walkers on the Cayley Tree and Polymer Statistics

We obtain the generating function for an ensemble of random walkers on the Cayley tree of coordination number z. The pair interaction between walkers is taken into account, which forbids two walkers to occur at the same lattice point after equal numbers of steps. Interacting polymer statistics results from this model if one associates time (or the number of steps) with an additional space coordinate. The limiting free energy appears in a form which corresponds to the phase transition of "3/2 order".

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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