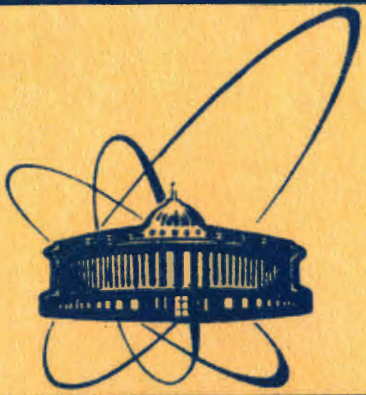

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ON THE MODEL OF HETEROPHASE SUPERCONDUCTOR

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1. INTRODUCTION

A microscopic model of heterophase superconductor has been recently [1] advanced. This model, formulated in the frames of the statistical theory of heterophase fluctuations [2,3], treats superconductor as a mixture of superconducting and normal states. It is worth to stress that such a mixture is uniform and should not be confused with the well-known mixed state consisting of spatially separated fully normal and superconducting regions (some new data concerning the mixed state can be found in ref. [4]).

In the present communication we consider the model of heterophase superconductor concentrating on the following aspects which have not yet been discussed.

1) We generalize the investigation on the arbitrary spectrum of pairing particles. This does not lead to difficulties of principle and is reasonable, because electron spectra in real metals can be quite various. The latter is the more so for metastable superconducting alloys [5-7] and metallic glasses [8]. Nonparabolic spectra can also appear when dealing with superconducting transitions in heated rotating nuclei [9-11] or in the quark-gluon matter [12-14].

2) We analyze stability conditions showing which connection between the Coulomb interaction, pairing interaction and the kinetic energy should be valid in order that the heterophase system be preferable comparing with the purely superconducting phase.

3) We discuss the transition temperature obtained and confirm its excellent agreement with experimentally measured superconducting temperatures for the majority of superconductors known.

Throughout the paper $h = 1$, $k_B = 1$.

2. HETEROPHASE MIXTURES

Let us first of all remember the main points of heterophase theory. Consider the system of N particles in the volume V. If we suppose that the system is spatially separated by two parts containing in the subvolume V_1 one thermodynamic phase of N_1 particles and in the subvolume V_2 another phase of N_2 particles, then the Hamiltonian of the system H should be

expressed through the integrals $\int_{V_i} \dots d\vec{r}$ $(i = 1,2)$. In this case we come to the nonuniform and nonequilibrium problem with given initial conditions, when we are forced to deal with kinetic equations describing phase nucleation (or spinodal decomposition), domain growth and movement, dynamics and structure of random interfaces [15] and all that. However, if the system in average is in equilibrium, and the characteristic life time of phase nuclei is much less than measuring time, then we have to average over all possible phase configurations [2,3], which is equivalent to the replacement

$$\int_{V_i} \dots d\vec{r} \to \frac{V_i}{V} \int_{V} \dots d\vec{r}. \tag{1}$$

It is convenient to introduce the geometrical probability

$$w_i \equiv \frac{V_i}{V} \quad (V_1 + V_2 = V), \tag{2}$$

by definition having the property

$$0 \le w_i \le 1 \quad (i = 1, 2) \tag{3}$$

and satisfying the normalization condition

$$w_1 + w_2 = 1. \tag{4}$$

The Hamiltonian H, being the functional of the volumes V_i, becomes dependent on the probabilities: $H = H(w_1, w_2)$. For the same reason thermodynamic potentials, for example free energy or grand potential, are the functions of probabilities:

$$f(w_1, w_2) = -\frac{\Theta}{N} \ln \mathrm{Sp} \exp\left[-\frac{1}{\Theta} H(w_1, w_2)\right]. \tag{5}$$

Using the normalization (4), one can simplify the notations:

$$w_1 \equiv w, \quad w_2 \equiv 1 - w, \quad f(w) \equiv f(w, 1 - w).$$

The probabilities (2) are such that they make the thermodynamic state advantageous, that is, they minimize the thermodynamic potential:

$$\frac{\partial f(w)}{\partial w} = 0, \quad \frac{\partial^2 f(w)}{\partial w^2} > 0. \tag{6}$$

The first of eqs.(6) has the sense of an equilibrium equation and can be rewritten in the form

$$\left\langle \frac{\partial H}{\partial w} \right\rangle = 0, \tag{7}$$

while the second of eqs.(6) is the stability condition and can be written as

$$\left[\Theta \left\langle \frac{\partial^2 H}{\partial w^2} \right\rangle - \left\langle \left(\frac{\partial H}{\partial w}\right)^2 \right\rangle\right] > 0. \tag{8}$$

This inequality will be called later on the heterophase stability condition. An absolutely stable state corresponds to $f_{abs} = \inf\{f(w), f(1), f(0)\}$.

When taking the thermodynamic limit for a heterophase system we shall abbreviate notations and write $\lim_{N\to\infty}$, meaning that

$$N \to \infty, \quad V \to \infty, \quad N_i \to \infty, \quad V_i \to \infty,$$

$$\frac{N}{V} \to \mathrm{const}, \quad \frac{N_i}{V_i} \to \mathrm{const}.$$

If the limiting constants for N_1/V_1 and N_2/V_2 are the same, that is

$$\lim_{N\to\infty} \frac{N_i}{V_i} = \rho \quad (i = 1, 2), \tag{9}$$

then it is easy to check that

$$\lim_{N\to\infty} \frac{V_i}{V} = \lim_{N\to\infty} \frac{N_i}{N}.$$

Therefore the probability (2) can be defined as an asymptotic value for the concentration

$$w_i \simeq \frac{N_i}{N} \quad (N_1 + N_2 = N). \tag{10}$$

3. FORMULATION OF THE MODEL

We shall investigate here the model of heterophase superconductor [1] being described by the Hamiltonian of the BCS (Bogolubov, Bardeen, Cooper, Schrieffer) type

$$H = H_1 \oplus H_2,$$

$$H_i = w_i \sum_k \epsilon_i(k) a_i^+(k) a_i(k) - \frac{w_i^2}{2V} \sum_{kk'} J(k,k') a_i^+(k) a_i(-k) a_i^+(-k') a_i(k'), \tag{11}$$

where $k=\{\vec{k}, \sigma\}$; $\sigma = +, -$ is the spin index,

$$\epsilon_i(k) = \epsilon_{eff}(k) + Qw_i - \mu, \tag{12}$$

$\epsilon_{eff}(\vec{k})$ is the electron spectrum of an arbitrary form, Q is the effective Coulomb interaction between electrons, μ is the chemical potential, $J(k,k') = J(k',k)$ is the pairing interaction. The Hamiltonian H_1 is defined on $\mathcal{F}_1$ which is U(1)-non-invariant space, and H_2 is defined on U(1)-invariant space $\mathcal{F}_2$. The direct sum $\oplus_i H_i$ acts on the space $\mathcal{F} = \mathcal{F}_1 \otimes \mathcal{F}_2$.

The heterophase-equilibrium equation (7) with the notations

$$\Lambda_i = \frac{1}{N}\sum_k \epsilon_{eff}(\vec{k}) \langle a_i^+(k)\, a_i(k)\rangle,$$

$$J_i = \frac{1}{2NV}\sum_{kk'} J(k,k') \langle a_i^+(k)\, a_i^+(-k)\, a_i(-k')\, a_i(k')\rangle, \tag{13}$$

$$Q_i = \frac{Q}{N}\sum_k \langle a_i^+(k)\, a_i(k)\rangle$$

gives

$$w = \frac{Q_2 - J_2 + (\Lambda_2 - \Lambda_1)/2}{Q_1 + Q_2 - J_1 - J_2}. \tag{14}$$

Using the expression for the grand potential

$$\Omega = \Omega_1 + \Omega_2, \quad \Omega_i = -\Theta \ln \mathrm{Sp}\, e^{-H_i/\Theta},$$

we can find the numbers of particles in each phase

$$N_i = -\frac{\partial \Omega_i}{\partial \mu} = w_i \sum_k \langle a_i^+(k)\, a_i(k)\rangle. \tag{15}$$

If we decide to accept (10), then from (15) it follows

$$\sum_k \langle a_i^+(k) a_i(k)\rangle \simeq N. \tag{16}$$

After this we see that $Q_1 \simeq Q_2 \simeq Q$, and (14) becomes

$$w \simeq \frac{Q - J_2 + \Lambda}{2Q - J_1 - J_2}, \quad \Lambda \equiv \frac{1}{2}(\Lambda_2 - \Lambda_1). \tag{17}$$

In order that (3) is valid, i.e., $0 \le w \le 1$, the effective Coulomb interaction has to satisfy one of the inequalities

$$Q \ge \sup\{\frac{1}{2}(J_1 + J_2),\ J_1 + \Lambda,\ J_2 - \Lambda\},$$

$$Q \le \inf\{\frac{1}{2}(J_1 + J_2),\ J_1 + \Lambda,\ J_2 - \Lambda\}.$$

Using the known methods [16], one is able to check that for the model considered

$$\langle (\frac{\partial H}{\partial w})^2 \rangle \simeq \langle \frac{\partial H}{\partial w} \rangle^2 \quad (N \to \infty),$$

and the latter expression is zero according to (7). Taking this into account and substituting

$$\langle \frac{\partial^2 H}{\partial w^2} \rangle = 2N(2Q - J_1 - J_2)$$

into (8), we find the heterophase-stability condition

$$Q > \frac{1}{2}(J_1 + J_2). \tag{18}$$

Thus, from the two limitations, corresponding to $0 < w < 1$, we much choose only one

$$Q > \sup\{J_i + \Lambda,\ J_2 - \Lambda\}, \tag{19}$$

which is not less strong than (18).

At the end of this section let us give the quasi-spin representation [17] of the Hamiltonian (11). Invoking the notations

$$\sigma_i^+(k) = a_i(-k)\, a_i(k), \quad \sigma_i^z(k) = \hat{1} - 2\hat{n}_i(k), \quad \hat{n}_i(k) = a_i^+(k)\, a_i(k),$$

we get

$$H_i = \frac{w_i}{2}\sum_k \epsilon_i(k)[\hat{1} - \sigma_i^z(k)] - \frac{w_i^2}{2V}\sum_{kk'} J(k,k')\, \sigma_i^-(k)\, \sigma_i^+(k'). \tag{20}$$

Remember, that everywhere above $k = \{\vec{k}, \sigma\}$ and $\sum_k = \sum_{\vec{k}} \sum_\sigma$.

4. ASYMPTOTIC SOLUTION

In the thermodynamic limit the Hamiltonian (20) is asymptotically equivalent [18] to

$$H \simeq \frac{w_i}{2}\sum_k \epsilon_i(k)[\hat{1} - \sigma_i^z(k)] - \frac{w_i^2}{2}\sum_k \Delta_i(k)[\sigma_i^+(k) + \sigma_i^-(k) - \langle \sigma_i^+(k)\rangle], \tag{21}$$

where

$$\Delta_i(k) = \frac{1}{V}\sum_{k'} J(k,k') < \sigma_i^+(k') > \qquad (22)$$

is the gap, and the identity $<a_i^+(k)> = <\sigma_i^-(k)>$ is taken into consideration.

The expressions (13) are now

$$\Lambda_i = \frac{1}{2N}\sum_k \epsilon_{eff}(\vec{k})[1 - <\sigma_i^z(k)>], \quad J_i = \frac{1}{2N}\sum_k \Delta_i(k) <\sigma_i^+(k)>.$$

For the averages of the Pauli matrices $\sigma_i^z(k)$ and $\sigma_i^{\pm}(k) = \sigma_i^x(k) \pm i\sigma_i^y(k)$ one obtains

$$<\sigma_i^z(k)> = \frac{w_i \epsilon_i(k)}{E_i(k)} \operatorname{th}\frac{E_i(k)}{2\Theta}, \quad <\sigma_i^{\pm}(k)> = \frac{w_i^2 \Delta_i(k)}{2E_i(k)} \operatorname{th}\frac{E_i(k)}{2\Theta},$$

here the spectrum of pairing particles is

$$E_i(k) = w_i[\epsilon_i^2(k) + w_i^2 \Lambda_i^2(k)]^{1/2}. \qquad (23)$$

In this way (22) leads to the equation for the gap

$$\Delta_i(k) = \frac{w_i^2}{2V}\sum_{k'} \frac{J(k,k')\Lambda_i(k')}{E(k')} \operatorname{th}\frac{E_i(k')}{2\Theta}. \qquad (24)$$

As far as the spectrum $\epsilon_{eff}(k)$ does not depend on the spin index, then, according to (12), $\epsilon_i(k) = \epsilon_i(k)$. Suppose that $J(k,k') = \sum_{\sigma'} J(k,k')$ is also a function of wave vectors only. Therefore

$$\Lambda_i = \frac{1}{N}\sum_{\vec{k}} \epsilon_{eff}(\vec{k})\left[1 - \frac{w_i \epsilon_i(\vec{k})}{E_i(\vec{k})} \operatorname{th}\frac{E_i(\vec{k})}{2\Theta}\right],$$

$$J_i = \frac{w_i^2}{2N}\sum_{\vec{k}} \frac{\Lambda_i^2(\vec{k})}{E_i(\vec{k})} \operatorname{th}\frac{E_i(\vec{k})}{2\Theta},$$

$$\Delta_i(\vec{k}) = \frac{w_i^2}{2V}\sum_{k'} \frac{J(\vec{k},\vec{k}')\Delta_i(\vec{k}')}{E_i(\vec{k}')} \operatorname{th}\frac{E_i(\vec{k}')}{2\Theta}.$$

Owing to U(1)-noninvariance of the space $\mathcal{F}_1$ and U(1)-invariance of the space $\mathcal{F}_2$ the following conditions are true:

$$<a_1(-k)a_1(k)> = <\sigma_1^+(k)> \neq 0, \quad <a_2(-k)a_2(k)> = <\sigma_2^+(k)> = 0, \qquad (25)$$

Consequently,

$$\Delta_1(\vec{k}) \equiv \Delta(\vec{k}) \neq 0, \quad \Delta_2(\vec{k}) \equiv 0. \qquad (26)$$

Fulfilling the standard substitution $\frac{1}{V}\sum_{\vec{k}} \to \int \frac{d\vec{k}}{(2\pi)^3}$ and using eq. (23), from which

$$E_1(\vec{k}) = wE(\vec{k}), \quad E(\vec{k}) \equiv \sqrt{\epsilon_1^2(\vec{k}) + w^2\Delta^2(\vec{k})}, \quad E_2(\vec{k}) = (1-w)\epsilon_2(\vec{k}),$$

we obtain

$$\Lambda_1 = \frac{1}{\rho}\int \epsilon_{eff}(\vec{k})\left[1 - \frac{\epsilon_1(\vec{k})}{E(\vec{k})} \operatorname{th}\frac{wE(\vec{k})}{2\Theta}\right]\frac{d\vec{k}}{(2\pi)^3},$$

$$\Lambda_2 = \frac{1}{\rho}\int \epsilon_{eff}(\vec{k})\left[1 - \operatorname{th}\frac{(1-w)\epsilon_2(\vec{k})}{2\Theta}\right]\frac{d\vec{k}}{(2\pi)^3}, \qquad (27)$$

$$J_1 = \frac{w}{2\rho}\int \frac{\Delta^2(\vec{k})}{E(\vec{k})} \operatorname{th}\frac{wE(\vec{k})}{2\Theta}\frac{d\vec{k}}{(2\pi)^3}, \quad J_2 = 0.$$

The equation (24) for the gap becomes of the form

$$\Delta(\vec{k}) = \frac{w}{2}\int \frac{J(\vec{k},\vec{k}')\Delta(\vec{k}')}{E(\vec{k}')} \operatorname{th}\frac{wE(\vec{k}')}{2\Theta}\frac{d\vec{k}'}{(2\pi)^3}. \qquad (28)$$

The probability of superconducting phase (17) is now

$$w = \frac{Q + \Lambda}{2Q - J_1}, \qquad (29)$$

where

$$\Lambda = \frac{1}{2\rho}\int \epsilon_{eff}(\vec{k})\left\{\frac{\epsilon_1(\vec{k})}{E(\vec{k})} \operatorname{th}\frac{wE(\vec{k})}{2\Theta} - \operatorname{th}\frac{(1-w)\epsilon_2(\vec{k})}{2\Theta}\right\}\frac{d\vec{k}}{(2\pi)^3}.$$

Eq. (16), defining the chemical potential μ as a function of ρ and Θ, may be written down as

$$\rho = \int\left[1 - \frac{w_i \epsilon_i(\vec{k})}{E_i(\vec{k})} \operatorname{th}\frac{E_i(\vec{k})}{2\Theta}\right]\frac{d\vec{k}}{(2\pi)^3}.$$

For the stability condition (19) we have

$$Q > \sup\{J_1 + \Lambda, -\Lambda\}. \qquad (30)$$

As can be seen from (27), $\Lambda_1 \to \infty$ if $w \to 0$ at $\Theta > 0$; correspondingly, $\Lambda_2 \to \infty$ if $w_2 \to 0$ at $\Theta > 0$. But $\Lambda \to \pm\infty$ has no sense in eq. (29). Thus, there is no continuous nucleation at any finite temperature $\Theta > 0$. Nuclei of one phase in the other could appear either just at $\Theta = 0$, or, if not, by a jump at $\Theta > 0$.

5. CRITICAL TEMPERATURE

In order to proceed in further calculations let us concretize the pairing potential choosing the Bardeen form

$$J(\vec{k},\vec{k}') = \begin{cases} J_0, & |\epsilon_1(k)| < \omega_0, \ |\epsilon_1(k')| < \omega_0 \\ 0, & |\epsilon_1(.)| > \omega_0 \end{cases}, \qquad (31)$$

in which ω_0 is an effective frequency of crystalline lattice. Then (22) yields the known result for the gap

$$\Delta(k) = \begin{cases} \Delta, & |\epsilon_1(\vec{k})| < \omega_0 \\ 0, & |\epsilon_1(\vec{k})| > \omega_0 \end{cases}. \qquad (32)$$

Consider the isotropic medium, when $\epsilon_{eff}(\vec{k}) = \epsilon_{eff}(|\vec{k}|)$. Define the function $k_1(\epsilon) = |\vec{k}|$, given by the equation

$$\epsilon_{eff}(k_1(\epsilon)) + Qw - \mu = \epsilon. \qquad (33)$$

After this eq.(28) transforms to

$$\frac{wJ_0}{2} \int_{-\omega_0}^{+\omega_0} \frac{n_1(\epsilon)}{\sqrt{\epsilon^2 + w^2\Delta^2}} \, th \, \frac{w\sqrt{\epsilon^2 + w^2\Delta^2}}{2\Theta} \, d\epsilon = 1,$$

where the level density is

$$n_1(\epsilon) \equiv \frac{k_1^2(\epsilon)}{2\pi^2} \frac{dk_1(\epsilon)}{d\epsilon}.$$

The integrals (27) can be also rewritten using $n_1(\epsilon)$; for example $J_1 = \Delta^2/\rho J_0$. The law of the mean helps to simplify more such integrals. Thus, for the gap equation (28) one has

$$w\lambda \int_0^{\omega_0} th \, \frac{w\sqrt{\epsilon^2 + w^2\Delta^2}}{2\Theta} \frac{d\epsilon}{\sqrt{\epsilon^2 + w^2\Delta^2}} = 1, \quad \lambda \equiv n_1(0)J_0. \qquad (34)$$

The constant λ plays the role of an effective interaction of electrons due to the phonon exchange. That is why this constant may be called the electron-phonon-electron coupling, or the effective electron-phonon coupling. The value of λ, being defined through the electronic density of states $n_1(\epsilon)$, depends on different model parameters [19] as well as on thermodynamic parameters, e.g., it can depend on pressure-induced electron changes [20]. In the case of an excitonic superconductor λ is the effective electron-exciton coupling [21].

The critical temperature Θ_c is to be defined by the condition $\Delta = 0$. This gives together with (34) the equation

$$w_c\lambda \int_0^{\omega_0} \frac{1}{\epsilon} \, th \, \frac{w_c\epsilon}{2\Theta_c} \, d\epsilon = 1, \qquad (35)$$

in which $w_c \equiv w(\Theta_c)$. Eq.(35) can be analytically solved in two opposite cases: this one of weak coupling, when

$$\Theta_c \simeq 1{,}134 w_c \omega_0 \exp\left(-\frac{1}{w_c\lambda}\right), \quad \left(\Theta_c < \frac{\omega_0}{2\pi}\right), \qquad (36)$$

and of strong coupling, when

$$\Theta_c \simeq 0{,}5 w_c^2 \lambda \omega_0, \quad \left(\Theta_c >> \frac{\omega_0}{2\pi}\right). \qquad (37)$$

The first case corresponds to $\lambda < 1.574$, and the second to $\lambda >> 1.273$. As is evident, (36) transforms to the usual BCS formula [22], if we put $w_c \to 1$. The dependence (37), where $\Theta_c \sim \lambda$, is analogous to the Rowell [23] phenomenological formula, which is in good agreement with experimental transition temperatures for superconductors with strong coupling.

The critical probability of superconducting phase has to be found from eq.(29). At the critical temperature, where $\Delta = 0$, one gets from (27)

$$J_1 = J_2 = 0, \quad \Lambda_1 = \Lambda_2, \quad \Lambda = \frac{1}{2}(\Lambda_2 - \Lambda_1) = 0.$$

Because of this

$$w_c = w(\Theta_c) = \frac{1}{2}. \qquad (38)$$

The stability condition (30) shows, that in the left vicinity of the critical point Θ_c there is a mixture of superconducting and normal phases only if the effective Coulomb interaction is positive: $Q > 0$.

With $w_c = 1/2$ the expressions (36) and (37) lead to

$$\Theta_c = \begin{cases} 0{.}567 \, \omega_0 e^{-2/\lambda} & (\lambda < 1.6) \\ 0{.}125 \, \omega_0 \lambda & (\lambda >> 1.3) \end{cases}$$

The constant λ can be found either from electron-tunneling experiments or from calculations based on empirical-pseudo-potential methods (see, e.g., ref. [24]). The frequency ω_0 is defined in literature by various ways: as proportional to the Debye temperature Θ_D (McMillan [25], Rowell [23]), to $\omega_{ln} \equiv \exp\langle \ln \omega \rangle$

(Allen and Dynes [26]), to $\langle\omega\rangle$ (McMillan [25]), to $\langle\omega^2\rangle^{1/2}$ (Allen and Dynes [26], Kresin, Gutfreund and Little [27]). In all cases mentioned, except the Rowell [23] one, the transition temperature contains an additional parameter - an effective electron-electron repulsion μ^*, which gives some limitations for the applicability of the formulae derived. For example, the McMillan formula is wrong for $\mu^* > 0.245$ (see the discussion in ref. [28]). The value of ω_{ln} can be connected with the zero-temperature gap Δ_0 within Eliashberg theory [29]. There exists also a phenomenological connection [30] between Δ_0, Θ_c, λ and μ^*. In our case Q is equivalent to μ^*.

We would like to emphasize, that all theoretical formulae known for Θ_c are valid either for strongly coupled, or weakly coupled superconductors only. This is because of the fact that in the process of derivation of Θ_c one uses, roughly speaking, either expansions over $e^{-1/\lambda}$, as for weak coupling, or in powers of $1/\lambda$, as for strong coupling. That is why, for instance, in the BCS case $\Theta_c \sim e^{-1/\lambda}$. The inequality $e^{-1/\lambda} \ll 1$ requires $\lambda < 1$. In our case the validity of the weak-coupling approximation has a bit larger region, because our $\Theta_c \sim e^{-2/\lambda}$; and the inequality $e^{-2/\lambda} \ll 1$ needs only $\lambda < 2$. The strong coupling limit always calls for $\lambda \gg 1$. As far as we know, overwhelming majority of superconductors (if not all) have $\lambda \lesssim 2.59$ (the latter is for $Pb_{0.45}Bi_{0.55}$). Therefore in search of a common formula for all superconductors we would choose

$$\Theta_c = 0.567\,\omega_0 e^{-2/\lambda}. \qquad (39)$$

Let us mention, that in the interval $1.3 < \lambda < 1.6$ both the expressions (36) as well as (37) give the values which are close to each other. Thus, for $\lambda = 1.4$ they yield $\Theta_c/\omega_0 \approx 0.1$. Making the trivial substitution $\tilde{\omega} = 1.134\,\omega_0$, one may rewrite (36) and (39) in the equivalent form

$$\Theta_c = \frac{\tilde{\omega}}{2} \exp\left(-\frac{2}{\lambda}\right). \qquad (40)$$

This formula in the representations (36) and (39) was published in 1981 and 1982 (Shumovsky and Yukalov [1,31]). Later, in 1983 and 1984 its applicability for about 100 metals and alloys - strong and weak superconductors, was checked by Surma [32,33]. He considered three variants for $\tilde{\omega}$: $\tilde{\omega} = \omega_{ln}$, $\tilde{\omega} = 0.9\langle\omega\rangle$, and $\tilde{\omega} = 0.7\Theta_D$, where 0.9 and 0.7 are fitting parameters. The values for $\tilde{\omega}$ were taken from calorimetric and neutron-scattering experiments, and for λ from tunneling experiments. In the cases available experimental data for $\tilde{\omega}$ and λ were compared with theoretically calculated. For the majority of known superconductors the deviation of Θ_c, given by (40), from experimental transition temperatures is of order of 10%. This has to be recognized

as a very good agreement, because the typical experimental uncertainties for λ and $\tilde{\omega}$ are of about 10% too.

It is quite interesting, that the equation $\tilde{\omega} = 0.7\,\Theta_D$, used by Surma [32] as a fitting relation, can be obtained theoretically when applying to the theory of heterophase crystals [34-36]. According to this theory $\omega_0 \approx \sqrt{w}\,\Theta_D$. As far as $w = 1/2$ at $\Theta = \Theta_c$, we get just $\tilde{\omega} = 0.7\,\Theta_D$.

In our model of heterophase superconductor above the critical temperature $w \equiv 1/2$, although $\Delta \equiv 0$. It is possible to interpret this as the existence of some atavism of superconducting state (may be a sort of gapless superconductivity, taking place in alloys with admixtures [37] and in rotating nuclei [10,11]). Such an atavism is not surprising from the experimental point of view, because a number of different materials is known, in which a high-temperature disordered phase remembers its low-temperature ordered origin. For instance, an antiferromagnetic-resonance mode persists [38] in the paramagnetic state of the layer compound $(C_2H_5NH_3)_2\,CuCl_4$ up to almost $2\Theta_N$. At the same time this compound is spatially uniform and is not separated into stable clusters as in superantiferromagnetic [39] and superparamagnetic [40] matters. However, an investigation of the heterophase-superconductor properties above the critical temperature is not the aim of the present paper.

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Юкалов В.И. E17-85-114

К модели гетерофазного сверхпроводника

Изучается двухжидкостная модель сверхпроводника. Исследование основывается на микроскопических методах статистической физики. Спектр электронов предполагается произвольным. Условия устойчивости показывают, что система представляет собой смесь сверхпроводящей и нормальной фаз только при достаточно сильном кулоновском взаимодействии. Найденная температура сверхпроводящего перехода прекрасно согласуется с экспериментом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Yukalov V.I. E17-85-114

On the Model of Heterophase Superconductor

The microscopic model of a two-fluid superconductor with arbitrary electron spectrum is considered. Stability conditions show that the system is a mixture of superconducting and normal phases only when the Coulomb interaction is strong enough. Superconducting transition temperature is obtained, which is in excellent agreement with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985