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N.N.Bogolubov Jr., Fam Le Kien*,
A.S.Shumovsky

PHOTON STATISTICS AND ATOMIC DYNAMICS IN A THREE-LEVEL PLUS TWO-MODE MODEL

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* Moscow State University

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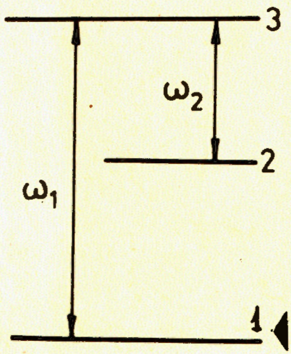
As is well known, the three-level system is the most useful model in the study of resonance phenomena in atoms (or molecules) and has been extensively discussed in papers [1-7] and references therein. In the last time the efforts have been directed towards the investigation of the behaviour of a three-level system interacting with a two-mode quantized radiation field [3-7]. The dynamics of this model is examined rigorously in ref. [6]. The novel properties exhibited in the case of the initially coherent radiation field are obtained in [5-7]. In the present paper we discuss how to obtain information about the photon statistics when the dynamical behaviour of the system is known.

The three-level atomic model considered here is shown in the figure. The upper level 3 is connected with the levels 1 and 2 by dipole transitions whereas the transition 1-2 is forbidden. The Hamiltonian describing the interaction of the atom with the two-mode resonant radiation field in RWA is [5,6]

$$H = \sum_{j=1}^{3} \hbar\Omega_j \hat{R}_{jj} + \sum_{a=1}^{2} \hbar\omega_a \hat{a}_a^+ \hat{a}_a + \hbar \sum_{a=1}^{2} g_a (\hat{a}_a \hat{R}_{3a} + \hat{a}_a^+ \hat{R}_{a3}) . \tag{1}$$

Here, the operator $\hat{R}_{jj} \equiv |j\rangle\langle j|$ describes the population of level j with the corresponding energy $\hbar\Omega_j$ and the state vector $|j\rangle$. $\hat{R}_{ij} \equiv |i\rangle\langle j|$ $(i \neq j)$ is the transition operator from level j to level i. The operators $\hat{R}_{ij}$, $i,j = 1,2,3$ are generators of SU(3) group and obey [6]

$$\hat{R}_{ij}\hat{R}_{k\ell} = \hat{R}_{i\ell}\,\delta_{jk} , \quad \sum_{i=1}^{3} \hat{R}_{ii} = 1 . \tag{2}$$



The photon operators $\hat{a}_a, \hat{a}_a^+$ describe two modes of the radiation field with the resonance frequencies $\omega_a = \Omega_3 - \Omega_a$, and g_a are the corresponding atom-mode coupling.

Let us denote the photon-number operator of mode a by $\hat{N}_a$, $\hat{N}_a \equiv \hat{a}_a^+\hat{a}_a$. Then, the photon-number statistical moments and the correlations between the modes are defined by $\hat{N}_a^m$ and $\hat{N}_1^k \hat{N}_2^\ell$ $(m, k, \ell = 1,2...)$, respectively.

Energy level structure of the atom considered.

It can be shown that the operators $\hat{N}_a(t) - \hat{R}_{aa}(t)$, $a = 1,2$, commute with the Hamiltonian (1) and with each other. They are therefore constants of motion

$$\hat{N}_a(t) - \hat{R}_{aa}(t) = \hat{M}_a = \text{const}, \quad a = 1,2. \tag{3}$$

For the operators of the m-th order statistical moments $\hat{N}_a^m(t)$ of the photon numbers we obtain from (3) the equation

$$\hat{N}_a^m(t) = \sum_{q=0}^{m} \frac{m!}{q!(m-q)!} \hat{M}_a^{m-q} \hat{R}_{aa}^q(t). \tag{4}$$

With the help of the relation (2): $\hat{R}_{aa}\hat{R}_{aa} = \hat{R}_{aa}$ we can replace in (4) all $\hat{R}_{aa}^q$ with $q = 1,2\ldots, m$ by $\hat{R}_{aa}$. Then

$$\hat{N}_a^m(t) = \{(\hat{M}_a + 1)^m - \hat{M}_a^m\}\hat{R}_{aa}(t) + \hat{M}_a^m. \tag{5}$$

Analogously, the equation for the operator of mode correlation $\hat{N}_1^k(t)\hat{N}_2^\ell(t)$ is found to be

$$\hat{N}_1^k(t)\hat{N}_2^\ell(t) = \hat{M}_1^k\hat{M}_2^\ell + \hat{M}_1^k\{(\hat{M}_2+1)^\ell - \hat{M}_2^\ell\}\hat{R}_{22}(t) + \hat{M}_2^\ell\{(\hat{M}_1+1)^k - \hat{M}_1^k\}\hat{R}_{11}(t), \tag{6}$$

where the relation (2): $\hat{R}_{11}\hat{R}_{22} = 0$ and eq. (5) have been used.

Let us introduce the following characteristic function operators [8]

$$\hat{\chi}(\xi_1,\xi_2) = \exp[i\xi_1\hat{N}_1(t) + i\xi_2\hat{N}_2(t)], \tag{7}$$

$$\hat{\chi}_a(\xi) = \exp[i\xi\hat{N}_a(t)]. \tag{8}$$

From (5) and the definition (8) of the characteristic function operators $\hat{\chi}_a(\xi)$ of the a-th mode photon distribution we find

$$\hat{\chi}_a(\xi) = \exp(i\xi\hat{M}_a)\{[\exp(i\xi) - 1]\hat{R}_{aa}(t) + 1\}. \tag{9}$$

Using this equation together with the relation (2) we can obtain for the characteristic function operator $\hat{\chi}(\xi_1,\xi_2)$ of the joint photon distribution the expression

$$\hat{\chi}(\xi_1,\xi_2) = \exp(i\xi_1\hat{M}_1 + i\xi_2\hat{M}_2) \times \{[\exp(i\xi_1) - 1]\hat{R}_{11}(t) + [\exp(i\xi_2) - 1]\hat{R}_{22}(t) + 1\}. \tag{10}$$

Note, that the time dependence of $\hat{N}_a^m(t)$, $\hat{N}_1^k(t)\hat{N}_2^\ell(t)$, $\hat{\chi}_a(\xi)$ and $\hat{\chi}(\xi_1,\xi_2)$ described by eqs. (5), (6), (9) and (10), respectively, is included only in the population operators $\hat{R}_{aa}(t)$ ($a = 1,2$). All the operators $\hat{M}_a$ are constant, diagonal and defined by, according to (3), the expression $\hat{M}_a = \hat{N}_a(0) - \hat{R}_{aa}(0)$. The explicit expressions of the time-dependent population operators $\hat{R}_{aa}(t)$ have been found in ref. [6]. They are

$$\hat{R}_{11}(t) = \hat{\mu}_1(\cos\hat{\lambda}t - 1) + \hat{\beta}_1\sin\hat{\lambda}t + \hat{\lambda}_1^2\{\hat{\mu}_2(\cos 2\hat{\lambda}t - 1) + \hat{\beta}_2\sin 2\hat{\lambda}t\} + \hat{R}_{11}(0),$$

$$\hat{R}_{22}(t) = -\hat{\mu}_1(\cos\hat{\lambda}t - 1) - \hat{\beta}_1\sin\hat{\lambda}t + \hat{\lambda}_2^2\{\hat{\mu}_2(\cos 2\hat{\lambda}t - 1) + \hat{\beta}_2\sin 2\hat{\lambda}t\} + \hat{R}_{22}(0). \tag{11}$$

Here, the frequency operators $\hat{\lambda}, \hat{\lambda}_1$ and $\hat{\lambda}_2$ are [5,6]: $\hat{\lambda}_a = g_a\sqrt{\hat{M}_a + 1}$, $\hat{\lambda} = \sqrt{\hat{\lambda}_1^2 + \hat{\lambda}_2^2}$. The "amplitude operators" $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are defined by the initial conditions [6]

$$\hat{\mu}_1 = \{\hat{\lambda}^2[\hat{\lambda}_2^2\hat{R}_{11}(0) - \hat{\lambda}_1^2\hat{R}_{22}(0)] + [\hat{\lambda}_2^2 - \hat{\lambda}_1^2]\hat{K}\}\hat{\lambda}^{-4},$$

$$\hat{\mu}_2 = \{\hat{\lambda}^2[1 - 2\hat{R}_{33}(0)] + \hat{K}\}(2\hat{\lambda}^4)^{-1},$$

$$\hat{\beta}_1 = \{\hat{\lambda}_2^2\dot{\hat{R}}_{11}(0) - \hat{\lambda}_1^2\dot{\hat{R}}_{22}(0)\}\hat{\lambda}^{-3}, \quad \hat{\beta}_2 = \{\dot{\hat{R}}_{11}(0) + \dot{\hat{R}}_{22}(0)\}(2\hat{\lambda}^3)^{-1}.$$

Here, $\hat{K}$ is the constant of motion [6]

$$\hat{K} = g_1g_2\{\hat{a}_1\hat{a}_2^+\hat{R}_{21} + \hat{a}_1^+\hat{a}_2\hat{R}_{12}\} - \hat{\lambda}_1^2\hat{R}_{22} - \hat{\lambda}_2^2\hat{R}_{11} = \text{const},$$

and the first derivatives $\dot{\hat{R}}_{aa}(0)$ obey the equations

$$\dot{\hat{R}}_{aa}(0) = ig_a[\hat{a}_a(0)\hat{R}_{3a}(0) - \hat{a}_a^+(0)\hat{R}_{a3}(0)], \quad a = 1,2.$$

Thus, formulas (5), (6), (9) and (10) allow us to obtain the information about the photon statistical properties when the dynamic behaviour of the level populations (11) is known.

Let $\rho(0)$ be a density matrix corresponding to some initial state of the atom-field system (1). Then, the characteristic functions $\chi(\xi_1\xi_2)$ and $\chi_a(\xi)$ can be defined as [8]

$$\chi(\xi_1,\xi_2) \equiv \langle\hat{\chi}(\xi_1,\xi_2)\rangle = \mathrm{Tr}\,\hat{\chi}(\xi_1,\xi_2)\hat{\rho}(0),$$

$$\chi_1(\xi) \equiv \langle\hat{\chi}_1(\xi)\rangle = \chi(\xi,0), \quad \chi_2(\xi) \equiv \langle\hat{\chi}_2(\xi)\rangle = \chi(0,\xi). \tag{12}$$

They are connected with the photon distributions $P_t(n_1,n_2)$ and $P_{ta}(n)$ by the relations

$$\chi(\xi_1,\xi_2) = \sum_{n_1,n_2}\exp(i\xi_1n_1 + i\xi_2n_2)P_t(n_1,n_2), \quad \chi_a(\xi) = \sum_n \exp(i\xi n)P_{ta}(n), \tag{13}$$

which allow us to get the latter if the former is known.

Below we consider the special case when at $t=0$ the atom is on level 1, and therefore

$$\hat{\rho}(0) = |1\rangle\langle 1| \otimes \hat{\rho}_F . \tag{14}$$

Here, $\hat{\rho}_F$ describes the initial state of the field.

Then, by using eqs. (10) and (11) the characteristic functions (12) are found to be:

$$\chi(\xi_1,\xi_2) = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2) \exp[i\xi_1(n_1-1) + i\xi_2 n_2] \times$$
$$\times \{[\exp(i\xi_1)-1] R_{1n_1n_2}(t) + [\exp(i\xi_2)-1] R_{2n_1n_2}(t) + 1\}, \tag{15}$$
$$\chi_1(\xi) = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2) \exp[i\xi(n_1-1)]\{[\exp(i\xi)-1] R_{1n_1n_2}(t) + 1\},$$
$$\chi_2(\xi) = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2) \exp(i\xi n_2)\{[\exp(i\xi)-1] R_{2n_1n_2}(t) + 1\},$$

where

$$R_{1n_1n_2}(t) = 1 - \frac{4g_1^2 g_2^2 n_1(n_2+1)}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^2\left[\frac{t}{2}\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}\right] -$$
$$- \frac{g_1^4 n_1^2}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^2\left[t\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}\right], \tag{16}$$
$$R_{2n_1n_2}(t) = \frac{4g_1^2 g_2^2 n_1(n_2+1)}{[g_1^2 n_1 + g_2^2(n_2+1)]^2} \sin^4\left[\frac{t}{2}\sqrt{g_1^2 n_1 + g_2^2(n_2+1)}\right],$$

and the weight factor $P(n_1,n_2)$ is an initial distribution function of the photon numbers

$$P(n_1,n_2) = \langle n_1, n_2 | \hat{\rho}_F | n_1, n_2 \rangle . \tag{17}$$

Corresponding to (15) the photon distribution functions $P_t(n_1,n_2)$, $P_{t1}(n_1)$ and $P_{t2}(n_2)$ are

$$P_t(n_1,n_2) = P(n_1,n_2) R_{1n_1n_2}(t) + P(n_1+1, n_2-1) R_{2n_1+1n_2-1}(t) +$$
$$+ P(n_1+1, n_2)[1 - R_{1n_1+1n_2}(t) - R_{2n_1+1n_2}(t)],$$

$$P_{t1}(n_1) = \sum_{n_2=0}^{\infty} P_t(n_1,n_2) = \sum_{n_2=0}^{\infty} \{P(n_1,n_2) R_{1n_1n_2}(t) +$$
$$+ P(n_1+1, n_2)[1 - R_{1n_1+1n_2}(t)]\},$$
$$P_{t2}(n_2) = \sum_{n_1=0}^{\infty} P_t(n_1,n_2) = \sum_{n_1=0}^{\infty} \{P(n_1, n_2-1) R_{2n_1n_2-1}(t) +$$
$$+ P(n_1,n_2)[1 - R_{2n_1n_2}(t)]\}. \tag{18}$$

Here, the relations (13) have been used. It is clear from (18) that $R_{1n_1n_2}(t)$, $R_{2n_1n_2}(t)$ and $R_{3n_1n_2}(t) \equiv 1 - R_{1n_1n_2}(t) - R_{2n_1n_2}(t)$ are the probabilities of the processes $|1, n_1, n_2\rangle \to |1, n_1, n_2\rangle$, $|1, n_1, n_2\rangle \to |2, n_1-1, n_2+1\rangle$ and $|1, n_1, n_2\rangle \to |3, n_1-1, n_2\rangle$, respectively. Their expressions (16) are in agreement with the results of /4,7/. Now, we consider some consequences of the results obtained above.

Once the characteristic and photon distribution functions are known, it is easy to compute the statistical moments $\langle \hat{N}_a^m(t)\rangle$ of the photon numbers of the modes and their correlations $\langle \hat{N}_1^k(t)\hat{N}_2^\ell(t)\rangle$ by using the relations

$$\langle \hat{N}_a^m(t)\rangle = \sum_{n=0}^{\infty} n^m P_{ta}(n) = \frac{\partial^m}{\partial(i\xi)^m}\chi_a(\xi=0),$$

$$\langle \hat{N}_1^k(t)\hat{N}_2^\ell(t)\rangle = \sum_{n_1,n_2=0}^{\infty} n_1^k n_2^\ell P_t(n_1,n_2) = \frac{\partial^{(k+\ell)}}{\partial(i\xi_1)^k\,\partial(i\xi_2)^\ell}\chi(\xi_1=0,\xi_2=0).$$

In particular, we find

$$\langle \hat{N}_1(t)\rangle = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2)\{n_1 - 1 + R_{1n_1n_2}(t)\},$$
$$\langle \hat{N}_2(t)\rangle = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2)\{n_2 + R_{2n_1n_2}(t)\}, \tag{19}$$

and

$$\langle \hat{N}_1^2(t)\rangle = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2)\{(n_1-1)^2 + (2n_1-1)R_{1n_1n_2}(t)\},$$
$$\langle \hat{N}_2^2(t)\rangle = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2)\{n_2^2 + (2n_2+1)R_{2n_1n_2}(t)\},$$
$$\langle \hat{N}_1(t)\hat{N}_2(t)\rangle = \sum_{n_1,n_2=0}^{\infty} P(n_1,n_2)\{(n_1-1)n_2 + n_2 R_{1n_1n_2}(t) + (n_1-1)R_{2n_1n_2}(t)\}. \tag{20}$$

Note, that the formulas (19) and (20) can be also derived directly by using eqs. (5), (6) and (11).

Let two modes of the radiation field be initially uncorrelated. Then, we have

$$\hat{\rho}_F = \hat{\rho}_{F_1} \otimes \hat{\rho}_{F_2}, \qquad (21)$$

and, according to (17)

$$P(n_1,n_2) = \langle n_1|\hat{\rho}_{F_1}|n_1\rangle\langle n_2|\hat{\rho}_{F_2}|n_2\rangle \equiv P_1(n_1)\,P_2(n_2), \qquad (22)$$

where $\hat{\rho}_{F_1}$ and $\hat{\rho}_{F_2}$ describe the initial state of mode 1 and mode 2, respectively. It is easy to see that for $t>0$

$$P_t(n_1,n_2) \neq P_{t1}(n_1)\,P_{t2}(n_2), \quad \langle \hat{N}_1(t)\,\hat{N}_2(t)\rangle \neq \langle \hat{N}_1(t)\rangle\langle \hat{N}_2(t)\rangle$$

in spite of (22). These differences imply the appearance of correlation between the modes because of the interaction with the atom. Finally, we note that the mean photon numbers calculated in refs. /5,6/ for some particular cases of initial field state are in agreement with our more general result (19).

Thus, in this paper the relations (5), (6), (9) and (10) between the photon statistical characteristics and atomic level populations are established and examined rigorously. The characteristic and photon distribution functions have been found for the case of the initially unexcited atom. Further discussion will be made in a future publication.

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Photon Statistics and Atomic Dynamics in a Three-Level Plus Two-Mode Model

The relations between photon statistical characteristics and atomic level populations are examined rigorously. The characteristics and photon distribution functions are found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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