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KINETICS OF n-LEVEL SYSTEM INTERACTING WITH ELECTROMAGNETIC FIELD

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A considerable growth of interest in the theoretical examination of macroscopic systems interacting with an electromagnetic field observed at present is connected with numerous applied problems of quantum and nonlinear optics. Although the most problems of interest from a physical point of view have to deal with the multy-level systems, the Dicke-type models describing only two-level transitions are usually studied[1]. But even for such a simple case an exact hierarchy of the kinetic equations has been derived quite recently [2,3].

It should be emphasised that the method of papers [2,3] is connected with the linear character of interaction between the field and emitters with respect to the boson variables. So it can be applied to the dynamical description of any system of the mentioned "linear" type.

In the present paper some example of such a "linear" system, the so-called Gilmore's model[4,5] describing the n-level emitters interacting with $\binom{n}{2}$ modes of E.M. field, will be examined on the basis of the above-mentioned method.

The system under consideration is described by the following Hamiltonian [5].

$$H = \sum_{1\le\alpha<\beta\le n} h\omega_{\beta\alpha} a^+_{\beta\alpha} a_{\beta\alpha} + \sum_{f=1}^{N} \{ \sum_{\alpha=1}^{n} \epsilon_\alpha E_{\alpha\alpha}(f) + \sum_{1\le\alpha<\beta\le n} \frac{\lambda_{\beta\alpha}}{\sqrt{N}} (a^+_{\beta\alpha} E_{\alpha\beta}(f) + a_{\beta\alpha} E_{\beta\alpha}(f))\}. \qquad (1)$$

Here N is the number of emitters, the photon operators obey the commutation rules

$$[a_{\beta'\alpha'} ; a^+_{\beta\alpha}] = \delta_{\beta'\beta}\delta_{\alpha'\alpha}, \quad a_{\beta\alpha} = a^+_{\alpha\beta} . \qquad (2)$$

The operators $E_{\alpha\beta}(f)$ describe the f-th emitter and belong to the Lie algebra of $SU(n)$ group. The operators $E_{\alpha\alpha}(f)$ are diagonal generators of $SU(n)$ group and $E_{\alpha\beta}(f)$ are the operators of Okubo [6] describing the transition from state $|\beta\rangle$ to $|\alpha\rangle$. Parameters $\lambda_{\beta\alpha} = \lambda^*_{\beta\alpha}$ are coupling constants and the parameters ϵ_α describe the free emitters. For the case of $n = 2$

$$E_{11} = -E_{22} = \sigma^z, \quad E_{12} = \sigma^-, \quad E_{21} = \sigma^+, \quad \sigma^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y),$$

where $\sigma^{\cdots}$ are the Pauli operators and the Hamiltonian (1) describes the Dicke model.

Let B be the operator of the dynamical variable in the Schrödinger representation. Then its Heisenberg representation is

$$B(t) = U^{-1}(t, t_0)\, B U(t, t_0),$$

where the operator $U(t, t_0)$ obeys the following equation $ihU(t, t_0) = HU(t, t_0)$, $U(t_0, t_0) = 1$, $U^{-1}\cdot(t, t_0) = U^+(t, t_0)$ and t_0 is an initial time.

Now let us consider the Heisenberg equations of motion for the photon operators. We have

$$\dot a_{\beta\alpha}(t) = -i\omega_{\beta\alpha} a_{\beta\alpha}(t) - \frac{i}{h}\frac{\lambda_{\beta\alpha}}{\sqrt{N}} E_{\beta\alpha}(t), \quad \dot a^+_{\beta\alpha}(t) = i\omega_{\beta\alpha} a^+_{\beta\alpha}(t) + \frac{i}{h}\frac{\lambda_{\beta\alpha}}{\sqrt{N}} E_{\beta\alpha}(t). \qquad (3)$$

Here $E_{\beta\alpha} \equiv \sum_{f=1}^{N} E_{\beta\alpha}(f)$. The formal solution of equations (3) is

$$a_{\beta\alpha}(t) = e^{-i\omega_{\beta\alpha}(t-t_0)} a_{\beta\alpha}(t_0) - \frac{i}{h}\frac{\lambda_{\beta\alpha}}{\sqrt{N}} \int_{t_0}^{t} e^{-i\omega_{\beta\alpha}(t-\tau)} E_{\alpha\beta}(\tau)\, d\tau,$$

$$a^+_{\beta\alpha}(t) = e^{i\omega_{\beta\alpha}(t-t_0)} a^+_{\beta\alpha}(t_0) + \frac{i}{h}\frac{\lambda_{\beta\alpha}}{\sqrt{N}} \int_{t_0}^{t} e^{i\omega_{\beta\alpha}(t-\tau)} E_{\beta\alpha}(\tau)\, d\tau. \qquad (4)$$

We define now the total radiation intensity in the usual way

$$I(t) = \sum_{1\le\alpha<\beta\le n} I_{\beta\alpha}(t) = \sum_{1\le\alpha<\beta\le n} h\omega_{\beta\alpha} \frac{d}{dt} \langle a^+_{\beta\alpha}(t)\, a_{\beta\alpha}(t)\rangle, \qquad (5)$$

where $\langle\ldots\rangle$ denotes the mean-value of the following type

$$\langle B(t)\rangle = \mathrm{Tr}\, BD(t) = \mathrm{Tr}\, BU(t, t_0)\, D(t_0)\, U^{-1}(t, t_0) = \mathrm{Tr}\, B(t)\, D(t_0). \qquad (6)$$

Here $D(t)$ is the statistical operator of system (1) with the initial condition $D(t) = D(E) \otimes D(F)$, where $D(\ldots)$ are the statistical operators for subsystems of emitters (E) and of field (F) at the initial time when the interaction is absent in the system.

Below we shall use the following lemma of Bogolubov and Bogolubov Jr. [7]:

$$\langle B(t)\, e^{-i\omega_{\beta\alpha}(t-t_o)}\, a_{\beta\alpha}(t_0)\rangle = N_{\beta\alpha}\langle [e^{-i\omega_{\beta\alpha}(t-t_o)}\, a_{\beta\alpha}(t_0)\,;\, B(t)]\rangle ,$$
$$\langle e^{-i\omega_{\beta\alpha}(t-t_o)}\, a_{\beta\alpha}(t_0)\, B(t)\rangle = (1+N_{\beta\alpha})\langle [e^{-i\omega_{\beta\alpha}(t-t_o)}\, a_{\beta\alpha}(t_0); B(t)]\rangle , \tag{7}$$

where $N_{\beta\alpha} = e^{-\hbar\omega_{\beta\alpha}/\theta}/(1-e^{-\hbar\omega_{\beta\alpha}/\theta})$, $N_{\alpha\beta} \equiv N_{\alpha\beta}$ and θ is an initial temperature of the E.M. field.

Now from the expressions (4), (5), and (7) one can obtain for the intensity

$$I(t) = \sum_{1\le\alpha<\beta\le n} \frac{\lambda^2_{\beta\alpha}\omega_{\beta\alpha}}{N\hbar}\int_{t_o}^{t} d\tau\, \{\langle e^{-i\omega_{\beta\alpha}(t-\tau)}\, ((1+N_{\beta\alpha}) \times$$
$$\times E_{\beta\alpha}(t)\, E_{\alpha\beta}(\tau) - N_{\beta\alpha}E_{\alpha\beta}(\tau)\, E_{\beta\alpha}(t))\rangle + \langle e^{i\omega_{\beta\alpha}(t-\tau)}\, ((1+N_{\beta\alpha}) \times \tag{8}$$
$$\times E_{\beta\alpha}(\tau)\, E_{\alpha\beta}(t) - N_{\beta\alpha}E_{\alpha\beta}(t)\, E_{\beta\alpha}(\tau))\rangle \}.$$

This is an exact expression of the total radiation intensity in terms of the variables of the E-subsystem. To obtain its time dependence, it is necessary to calculate the two-time averages in the right-hand side of (8). For this aim let us now construct an exact hierarchy of the kinetic equations for system (1).

Let $\mathcal{O}(t)$ be an arbitrary operator of E-sybsystem in the Heisenberg representation (2). The equation of motion for $\mathcal{O}$ has the form $i\hbar\dot{\mathcal{O}}(t) = [\mathcal{O}(t); H(t)]$. Using now averaging (6) together with lemma (7) and expressions (4) we obtain

$$\frac{d}{dt}\langle \mathcal{O}(t)\rangle - \frac{i}{\hbar}\sum_{\alpha=1}^{n}\epsilon_\alpha\langle [E_{\alpha\alpha}(t);\ \mathcal{O}(t)]\rangle =$$
$$= \sum_{1\le\alpha<\beta\le n}\frac{\lambda^2_{\beta\alpha}}{N\hbar^2}\int_{t_o}^{t} d\tau\, \langle\{N_{\beta\alpha}\, E_{\alpha\beta}(\tau)[\,\mathcal{O}(t)\, ;\, E_{\beta\alpha}(t)] +$$
$$+ (1+N_{\beta\alpha})[E_{\beta\alpha}(t)\, ;\ \mathcal{O}(t)]\, E_{\alpha\beta}(\tau)\}\, e^{-i\omega_{\beta\alpha}(t-\tau)}\rangle + \tag{9}$$
$$+ \sum_{1\le\alpha<\beta\le n}\frac{\lambda^2_{\beta\alpha}}{N\hbar^2}\int_{t_o}^{t} d\tau\, \langle\{(1+N_{\beta\alpha})E_{\beta\alpha}(\tau)[\,\mathcal{O}(t); E_{\alpha\beta}(t)] +$$
$$+ N_{\beta\alpha}[E_{\alpha\beta}(t)\, ;\ \mathcal{O}(t)]\, E_{\beta\alpha}(\tau)\}\, e^{i\omega_{\beta\alpha}(t-\tau)}\rangle .$$

In the limit $t_0 \to -\infty$ expression (9) is the exact hierarchy to be found. In the case of the spontaneous radiation when $D(F) = |0\rangle\langle 0|$, where $|0\rangle$ is the vacuum of E.M. field, instead of (9) we have

$$\frac{d}{dt}\langle \mathcal{O}(t)\rangle - \frac{i}{\hbar}\sum_{\alpha=1}^{n}\epsilon_\alpha\langle [E_{\alpha\alpha}(t);\ \mathcal{O}(t)]\rangle =$$
$$= \sum_{1\le\alpha<\beta\le n}\frac{\lambda^2_{\beta\alpha}}{N\hbar^2}\int_{-\infty}^{t} d\tau \langle e^{-i\omega_{\beta\alpha}(t-\tau)}\, [E_{\beta\alpha}(t);\ \mathcal{O}(t)]\, E_{\alpha\beta}(\tau) + \tag{10}$$
$$+ e^{i\omega_{\beta\alpha}(t-\tau)}\, E_{\beta\alpha}(\tau)[\,\mathcal{O}(t); E_{\alpha\beta}(t)]\rangle .$$

The corresponding intensity of the spontaneous radiation is

$$I(t) = \sum_{1\le\alpha<\beta\le n}\frac{\lambda^2_{\beta\alpha}\omega_{\beta\alpha}}{N\hbar}\int_{-\infty}^{t} d\tau \langle e^{-i\omega_{\beta\alpha}(t-\tau)}\, E_{\beta\alpha}(t)\, E_{\alpha\beta}(\tau) + e^{i\omega_{\beta\alpha}(t-\tau)}\, E_{\beta\alpha}(\tau)\, E_{\alpha\beta}(t)\rangle . \tag{11}$$

If $I(t) > 0$, then there is the radiation in the system.

It should be emphasised that the exact hierarchies of papers /2,3/ for the two-level Dicke-type system follow from equations (9) and (10) when $n = 2$.

Now the time dependence of intensity (8) or (11) can be obtained from the solution of equations (9) or (10) with the aid of some physical decoupling.

On the basis of hierarchies (9) and (10) various physical processes can be described. The dynamics of the two-modes laser /8/ can be mentioned as an example. The consideration of concrete physical realisations of model (1) will be performed in subsequent more detailed paper.

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Кинетика n-уровневой системы, взаимодействующей с электромагнитным полем

Получена точная иерархия кинетических уравнений для n-уровневой системы, описываемой моделью Джилмора. Дано точное выражение для интенсивности излучения как функции от переменных подсистемы излучателей.

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Kinetics of n-Level System Interacting with Electromagnetic Field

An exact hierarchy of the kinetic equations for an n-level system described by Gilmore's model is obtained. An expression for the radiation intensity as a function of the emitter-subsystem variables is found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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