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TWO-PHOTON PROCESS IN A THREE-LEVEL SYSTEM OF THE CASCADE TYPE

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An adequate description of dynamics for a three-level system interacting with an electromagnetic field is an important problem of the modern quantum radiophysics. First of all, this fact is due to a close connection between the theory of two-mode laser and the above-mentioned problem /1/. It is also connected with the theory of the so-called simulton /2/. Still another reason consists in the examination of a pumping influence on dynamics of a radiating system /3,4/. It is obvious that the exact results are of the greatest interest for the investigators.

In our previous paper /5/ the dynamics of a special two-photon process in the three-level system with a common upper level of transitions (fig.1) was rigorously examined. The temporal behaviour of the photon-mode occupation numbers and of the level fillings was found. Now we shall investigate the dynamics of a cascade process in the three-level system. In this case there is a common middle level of transitions in the system (fig.2). For generality we shall consider the photon modes detuning with respect to this middle level.

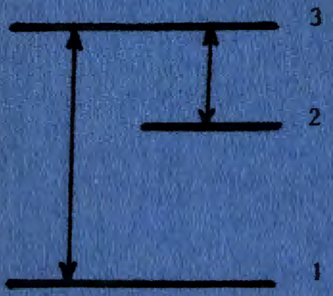

Fig. 1. Energy-level structure and transition scheme of the model considered in /5/.

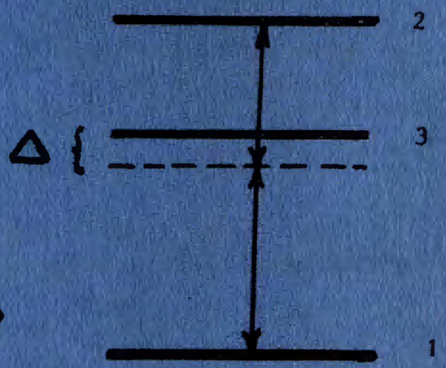

Fig. 2. Energy-level structure and transition scheme of the model considered in this work.

The Hamiltonian of the system under consideration has the form

$$H = H_A + F_F + H_{AF}. \tag{1}$$

Here H_A describes the energy of a free three-level emitter

$H_A = \sum_{j=1}^{3} \hbar\Omega_j R_{jj}$. The operator $R_{jj} = |j\rangle\langle j|$ describes a filling of a j-th level /2-4/, while the operator H_F describes the energy of two cascade modes of an electromagnetic field

$$H_F = \sum_{\alpha=1}^{2} \hbar\omega_\alpha a_\alpha^+ a_\alpha ,$$

where $\omega_1 + \omega_2 = \Omega_2 - \Omega_1$. For the energy of an emitter-field interaction in the dipole and the rotating wave approximations we have

$$H_{AF} = -i\hbar g_1 (a_1 R_{31} - a_1^+ R_{13}) - i\hbar g_2 (a_2 R_{23} - a_2^+ R_{32}),$$

where $g_\alpha = \text{const}$ and $R_{ij} = |i\rangle\langle j|$ /2-4/. The operator R_{ij} describes a transition from the state $|j\rangle$ to the state $|i\rangle$ $(i \neq j)$. Such operators R_{ij} are generators of SU(3) group and obey the following commutation rules

$$[R_{ij}, R_{k\ell}] = R_{i\ell}\delta_{kj} - R_{kj}\delta_{i\ell}, \quad R_{ij} R_{k\ell} = R_{i\ell}\delta_{kj}. \tag{2}$$

The basis states $|j\rangle$ $(j = 1,2,3)$ obey the conditions

$$H_A |j\rangle = \hbar\Omega_j |j\rangle, \quad \langle i|j\rangle = \delta_{ij}, \quad \sum_{i=1}^{3} |i\rangle\langle i| = 1.$$

It follows that

$$\sum_{j=1}^{3} R_{jj} = 1. \tag{3}$$

Let us now consider the equations of motion for the operators $R_{jj}(t)$ in the Heisenberg representation

$$\dot R_{11}(t) = g_1 \{a_1(t) R_{31}(t) + a_1^+(t) R_{13}(t)\} \equiv g_1 A_1(t),$$
$$\dot R_{22}(t) = -g_2 \{a_2(t) R_{23}(t) + a_2^+(t) R_{32}(t)\} \equiv -g_2 A_2(t). \tag{4}$$

One can obtain a suitable equation for $R_{33}(t)$ from exp. (3). Let $N_\alpha \equiv a_\alpha^+ a_\alpha$. Then

$$\dot N_\alpha(t) = g_\alpha A_\alpha(t), \quad \alpha = 1,2. \tag{5}$$

From the equations (4) and (5) it follows that

$$N_1(t) - R_{11}(t) = M_1, \qquad N_2(t) + R_{22}(t) = M_2 + 1, \tag{6}$$

where M_a are some time-independent operators.
The equations of motion for the operators $A_a(t)$ are

$$\dot A_1(t) = i\Delta C_1(t) + 2g_1(M_1 + 1)[1 - 2R_{11}(t) - R_{22}(t)] + g_2 B(t),$$
$$\dot A_2(t) = -i\Delta C_2(t) - 2g_2(M_2 + 1)[1 - 2R_{22}(t) - R_{11}(t)] - g_1 B(t). \tag{7}$$

Here Δ is the detuning parameter

$$\Delta \equiv (\Omega_3 - \Omega_1) - \omega_1 = \omega_2 - (\Omega_2 - \Omega_3)$$

and

$$B \equiv a_1 a_2 R_{21} + a_1^+ a_2^+ R_{12}, \quad C_1 \equiv a_1 R_{31} - a_1^+ R_{13}, \quad C_2 \equiv a_2 R_{23} - a_2^+ R_{32}. \tag{8}$$

The operators $B(t)$ and $C_a(t)$ obey the following equations of motion

$$\dot B(t) = g_1(M_1 + 1) A_2(t) - g_2(M_2 + 1) A_1(t),$$
$$\dot C_1(t) = i\Delta A_1(t) + g_2 D(t), \quad \dot C_2(t) = -i\Delta A_2(t) - g_1 D(t), \tag{9}$$

where $D \equiv a_1 a_2 R_{21} - a_1^+ a_2^+ R_{12}$ and

$$\dot D(t) = g_1(M_1 + 1) C_2(t) - g_2(M_2 + 1) C_1(t). \tag{10}$$

The equations (4), (5), (7), (9), and (10) form a closed system and have two following integrals of motion

$$g_1 g_2 B(t) + g_1^2(M_1 + 1) R_{22}(t) + g_2^2(M_2 + 1) R_{11}(t) = K,$$
$$g_1 C_1(t) + g_2 C_2(t) - i\Delta\{R_{11}(t) + R_{22}(t)\} = Q. \tag{11}$$

Here K and Q are time-independent operators. The operators M_a, K, and Q are commutating with each other. Under the condition (11) one can obtain from the equations (4), (5), (7), (9), and (10)

$$\ddot R_{11}(t) = -(3\lambda_1^2 + \lambda_0^2) R_{11}(t) - 3\lambda_1^2 R_{22}(t) + i\Delta g_1 C_1(t) + 2\lambda_1^2 + K,$$
$$\ddot R_{22}(t) = -(3\lambda_2^2 + \Delta^2) R_{11}(t) - (3\lambda_2^2 + \lambda_0^2 + \Delta^2) R_{22}(t) - i\Delta g_1 C_1(t) + 2\lambda_2^2 + K + i\Delta Q, \tag{12}$$
$$g_1 \ddot C_1(t) = i\Delta \ddot R_{11}(t) + i\Delta\lambda_1^2\{R_{11}(t) + R_{22}(t)\} - \lambda_0^2 g_1 C_1(t) + \lambda_1^2 Q.$$

Here λ_a ($a = 1,2$) are operators of the Rabi frequency for the transition $a \to 3$ at the resonance

$$\lambda_a \equiv g_a\sqrt{M_a + 1}; \qquad \lambda_0 \equiv \sqrt{\sum_{a=1}^{2} \lambda_a^2}.$$

To solve the system of second-order differential equations (12), we ought to determine the eigenvalues of the linear coefficients matrix. This leads to the following equation

$$\det\begin{pmatrix} X^2 - (3\lambda_1^2 + \lambda_0^2) & -3\lambda_1^2 & i\Delta \\ -(3\lambda_2^2 + \Delta^2) & X^2 - (3\lambda_2^2 + \lambda_0^2 + \Delta^2) & -i\Delta \\ i\Delta(\lambda_1^2 - X^2) & i\Delta\lambda_1^2 & X^2 - \lambda_0^2 \end{pmatrix} =$$
$$= X^6 - 2(3\lambda_0^2 + \Delta^2) X^4 + (3\lambda_0^2 + \Delta^2) X^2 - \lambda_0^4(4\lambda_0^2 + \Delta^2) = 0.$$

These eigenvalues are 2λ, $\lambda_\pm$, where

$$\lambda \equiv \sqrt{\lambda_0^2 + \Delta^2/4}, \qquad \lambda_\pm \equiv \lambda \pm \Delta/2.$$

They are the operators of the frequencies of a nonlinear optical oscillation in the three-level system. Now the solution of the system (12) can be presented in the form

$$R_{11}(t) = P_+(t) + P_-(t) + \lambda_1^2 P(t) + R_{11}(0),$$
$$R_{22}(t) = -P_+(t) - P_-(t) + \lambda_2^2 P(t) + R_{22}(0), \tag{13}$$

where

$$P_\pm(t) = \mu_\pm(\cos\lambda_\pm t - 1) + \beta_\pm \sin\lambda_\pm t,$$
$$P(t) = \mu(\cos 2\lambda t - 1) + \beta\sin 2\lambda t.$$

The operator amplitudes $\mu_\pm$, μ, $\beta_\pm$, β are

$$\mu = -\{\ddot R_{11}(0) + \ddot R_{22}(0)\}(4\lambda_0^2\lambda^2)^{-1},$$
$$\mu_\pm = \{\lambda_1^2 \ddot R_{22}(0) - \lambda_2^2 \ddot R_{11}(0)\}(2\lambda_0^2\lambda\lambda_\pm)^{-1} \pm i g_1 g_2 \dot D(0)(2\lambda\lambda_\pm^2)^{-1},$$
$$\beta = \{\dot R_{11}(0) + \dot R_{22}(0)\}(2\lambda_0^2\lambda)^{-1},$$
$$\beta_\pm = \{\lambda_2^2 \dot R_{11}(0) - \lambda_1^2 \dot R_{22}(0)\}(2\lambda_0^2\lambda)^{-1} \mp i g_1 g_2 D(0)(2\lambda\lambda_\pm)^{-1}.$$

In other words, these operator amplitudes are determined by the initial conditions only.

With the aid of the conservation laws (3) and (6) one can obtain from (13)

$$R_{33}(t) = -\lambda_0^2 P(t) + R_{33}(0), \quad N_1(t) = P_+(t) + P_-(t) + \lambda_1^2 P(t) + N_1(0), \tag{14}$$
$$N_2(t) = P_+(t) + P_-(t) - \lambda_2^2 P(t) + N_2(0).$$

The expressions (13), (14) present an exact solution both for the operators of the level fillings and for the operators of the photon-mode occupation numbers in the cascade-type three-levels system with the Hamiltonian (1).

Now the observable values of the level fillings and of the photon-mode occupation numbers and their time dependence can be found by averaging over the initial statistical operator $\rho(0)$

$$\langle \mathcal{O}(t) \rangle = \mathrm{Sp}\, \mathcal{O}(t)\, \rho(0). \tag{15}$$

This problem will be examined elsewhere.

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Боголюбов Н.Н. /мл./, Фам Ле Киен, Шумовский А.С. E17-84-39

Двухфотонный процесс в трехуровневой системе типа каскада

Точно исследовано динамическое поведение населенностей уровней и чисел заполнения фотонных мод для трехуровневой двухфотонной системы типа каскада. Получены квантовые выражения для частот нелинейной оптической нутации.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Two-Photon Process in a Three-Level System of the Cascade Type

A dynamical behaviour of the level fillings and of the photon-mode occupation numbers is rigorously examined for a three-level two-photon cascade type system. Quantum expressions are obtained for the frequencies of a nonlinear optical oscillation in the system.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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