18/VI-84

объединенный
институт
ядерных
исследований
дубна

E17-84-253

I.G.Gochev

QUANTUM DOMAIN WALL AND COHERENT STATES FOR THE HEISENBERG-ISING SPIN 1/2 CHAIN

Submitted to "Physics Letters A"

1984

The Hamiltonian of the exchange anisotropy spin chain is the familiar Heisenberg-Ising Hamiltonian:

$$\mathcal{H} = -\frac{J}{2} \sum_{m=1}^{\infty} [\frac{1}{g}(S^x_m S^x_{m+1} + S^y_m S^y_{m+1}) + S^z_m S^z_{m+1}]. \quad (1)$$

Here J is the exchange integral, $J > 0$; g is the anisotropy constant, $g > 1$.

Recently /1/, we have shown that the linear superposition $|\phi_{m_0}>$ of heavy spin complexes $|\psi_n>$

$$|\phi_{m_0}> = A \sum_{k=-\infty}^{\infty} \exp\{-\frac{\sigma}{2}[k + (\frac{1}{2} - a)]^2\} \cdot |\psi_{N_0+k}>, \quad (2)$$

where $m_0 = N_0 + a$, $|a| \le 1/2$, $N_0 \to \infty$, $A^{-2} = \sum_{k=-\infty}^{\infty} \exp\{-\sigma[k + (\frac{1}{2} - a)]^2\}$, is a complete quantum analog of the classical domain wall in the spin 1/2 chain (1). The expectation values of the spin components $S^{\beta}_m (\beta = x, y, z)$ and the energy of this state coincide with the corresponding classical expressions:

$$<\phi_{m_0}|S^x_m|\phi_{m_0}> = \frac{1}{2}[ch(m - m_0)\sigma]^{-1} = S^x_{m,class}; \; <\phi_{m_0}|S^y_m|\phi_{m_0}> = 0 = S^y_{m,class};$$

$$<\phi_{m_0}|S^z_m|\phi_{m_0}> = \frac{1}{2} th(m - m_0)\sigma = S^z_{m,class}; \quad (3)$$

$$\mathcal{H}|\phi_{m_0}> = \frac{J}{2} th\sigma \cdot |\phi_{m_0}> = W_{class}|\phi_{m_0}>; \quad \sigma = \ln(g + \sqrt{g^2 - 1}).$$

Since the position of the domain wall is arbitrary, m_0 is a free parameter. The expressions for $S^{\beta}_{m,class}$ and W_{class} in Eqs. (3) have been found independently by solving the domain wall problem for the classical counterpart of (1) (see /1/):

$$S^x_m = \frac{1}{2}\sin\theta_m, \quad S^y_m = 0; \quad S^z_m = \frac{1}{2}\cos\theta_m;$$
$$W\{\theta_m\} = \frac{J}{4}\sum (1 - \frac{1}{g}\sin\theta_m \sin\theta_{m+1} - \cos\theta_m \cos\theta_{m+1}); \; \partial W/\partial\theta_m = 0. \quad (4)$$

It is seen from Eqs. (3) that $|\phi_{m0}>$ realizes a complete correspondence between classical and quantum treatment of the domain wall in the exchange anisotropy spin chain (1). In the classical theory $\vec{S}_m$ is a vector of fixed length s. The condition

$$\sum_{\beta=x,y,z} (\langle\phi_{m_0}|S_m^{\beta}|\phi_{m_0}\rangle)^2 = \frac{1}{4}(=s^2), \quad (5)$$

which follows from (3), is its quantum analog.

On the other hand, Bishop, Domany, and Krumhansl[2] have proposed a variational procedure for establishing such a correspondence between both pictures. The trial wave function is chosen by these authors in a form (here we restrict ourselves to the case of $s = 1/2$ only):

$$|\tilde{\phi}\rangle = \prod_m \begin{pmatrix} \cos\theta_m/2 \\ \sin\theta_m/2 \end{pmatrix}. \quad (6)$$

If one minimizes $\langle\tilde{\phi}|\mathcal{H}|\tilde{\phi}\rangle$ with respect to $\{\theta_m\}$, the classical equations for θ_m are obtained[2]. Moreover, as Pokrovskii and Khokhlachev[3] have shown, if, for the spin chain (1), θ_m satisfies the equations equivalent to the classical domain wall equation (4), there exists a stationary state of the form (5). Now the exact solution (3) of Eq.(4) is available. This means that we have two explicit quantum states $|\phi_{m_0}\rangle$ and $|\tilde{\phi}_{m_0}\rangle$ fulfilling formulae (3). Hence, one can ask how they are related. If $|\tilde{\phi}_{m_0}\rangle = \sum \tilde{C}_n|\psi_n\rangle$, then $\tilde{C}_n = \langle\psi_n|\tilde{\phi}_{m_0}\rangle$, and, using the exact wave function $|\psi_n\rangle$ (see [4]), one gets:

$$\tilde{C}_k = \frac{\exp\{-\frac{\sigma}{2}[k^2 + (1-2a)k]\}}{\prod_{\nu=1}^{\infty}\sqrt{(1-e^{-2\sigma\nu})(1+e^{-2\sigma(\nu-1+a)})(1+e^{-2\sigma(\nu-a)})}}, \quad (7)$$

where $n = N_0+k$, $\tilde{C}_{N_0+k} \to \tilde{C}$, $m_0 = N_0 + a$, $N_0 \to \infty$. Next, we shall use the Gauss-Jacobi identity[5]:

$$\prod_{k=1}^{\infty}(1-q^{2k})(1-q^{2k-1}\cdot z^2)(1-q^{2k-1}\cdot z^{-2}) = \sum_{k=-\infty}^{\infty}(-1)^k\cdot z^{2k}\cdot q^{k^2}, \quad (8)$$
$$z \neq 0,\ |q| < 1.$$

Choosing $q = e^{-\sigma}$ and $z = ie^{-\frac{\sigma}{2}(1-2a)}$ from (2), (7), and (8) we conclude that $\tilde{C}_k = C_k = A\exp\{-\frac{\sigma}{2}[k+(\frac{1}{2}-a)]^2\}$, thus $|\phi_{m_0}\rangle = |\tilde{\phi}_{m_0}\rangle$.

In this way, the state $|\phi_{m_0}\rangle$ is rewritten as a direct product of one-site Bloch's coherent states $|\chi\rangle = \begin{pmatrix} e^{i\varphi}\cos\theta/2 \\ e^{-i\varphi}\sin\theta/2 \end{pmatrix}$ (in the domain wall case $\varphi = 0$). The properties of the $|\chi\rangle$ state are well known[6]. In particular, this is a minimum uncertainty

state. Thus, $|\phi_{m_0}\rangle$ is a minimum uncertainty state too, and, as it is seen from (6), the correlation functions factorize:

$$\langle\phi_{m_0}|S_{m_1}^{\beta_1}S_{m_2}^{\beta_2}|\phi_{m_0}\rangle = \langle\phi_{m_0}|S_{m_1}^{\beta_1}|\phi_{m_0}\rangle\langle\phi_{m_0}|S_{m_2}^{\beta_2}|\phi_{m_0}\rangle,\ m_1 \neq m_2.$$

The properties of $|\phi_{m_0}\rangle$ indicate that this is a coherent state for the considered interacting spin system (1). To best author's knowledge, up to now the coherent states have been found only for noninteracting magnons[7]. For such systems the direct product of Glauber's coherent states is a complete quantum analog of the classical spin wave. It is clear from our results that the complete quantum analog (6) of classical domain wall (i.e., a nonlinear excitation) is a direct product of Bloch's coherent states. On the other hand, this quantum analog is a Gaussian superposition (1) of heavy spin complexes. Usually, the unique spin complex is considered as a quantum analog of the classical soliton[8]. However, condition (5) cannot be fulfilled in the case of unique spin complex as has been pointed out in[1].

We thank Dr.V.Priezzhev for bringing the Gauss-Jacobi identity (8) to our attention.

REFERENCES

1. Gochev I.G. Zh.Eksp.Teor.Fiz., 1983, 85, p.199.
2. Bishop A.R., Domany E., Krumhansl J.A. Ferroelectrics, 1977, 16, p.183.
3. Pokrovskii V.L., Khokhlachev S.B. Zh.Eksp.Teor.Fiz. (Pis'ma Red.), 1975, 22, p.371.
4. Gochev I.G. Zh.Eksp.Teor.Fiz. (Pis'ma Red.), 1977, 26, p.136.
5. Hall M., Jr. Combinatorial Theory. Blaisdell Publ.Comp., Waltham-Toronto-London, 1967, ch.4.
6. Arrecchi F. et al. Phys.Rev., 1972, A6, p.2211.
7. Rezende S., Zaguru N. Phys.Rev., 1971, B4, p.201.
8. Kosevich A.M., Ivanov B.A. Zh.Eksp.Teor.Fiz., 1977, 72, p.2000; Tjon J., Wright J. Phys.Rev., 1977, B15, p.391; Bishop A.R. J.Phys., 1980, C13, p.L67; Fogedby H. J.Phys., 1980, C13, p.L195; Schneider T. Phys.Rev., 1981, B24, p.5327; Gochev I.G. Phys.Lett., 1982, A89, p.31; Kosevich A.M., Ivanov B.A., Kovalev A.S. Nonlinear Magnetization Waves: Dynamical and Topological Solitons. "Naukova dumka", Kiev, 1983 (in Russian).

Received by Publishing Department
on April 17, 1984.

Гочев И.Г. E17-84-253

Квантовая домéнная стенка и когерентные состояния для спиновой цепочки Гейзенберга-Изинга (спин 1/2)

Показано, что состояние типа квантовой домéнной стенки $|\phi>$, ранее найденное для анизотропной спиновой цепочки, может быть представлено как прямое произведение одноузельных блоховских когерентных состояний. Из этого следует, что для указанной взаимодействующей спиновой системы $|\phi>$ является пакетом с минимальным произведением неопределенностей. В этом состоянии корреляционные функции факторизуются.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Gochev I.G. E17-84-253

Quantum Domain Wall and Coherent States for the Heisenberg-Ising Spin 1/2 Chain

It is shown that the quantum domain wall state $|\phi>$, recently found for the exchange anisotropy spin 1/2 chain, can be written as a direct product of one site Bloch´s coherent states. From this representation it follows that for this particular system of interacting spins $|\phi>$ is a minimum uncertainty packet and the correlation functions in this state factorize.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984