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SELF-CONSISTENT THEORY OF ELEMENTARY EXCITATIONS WITH DAMPING IN THE SYSTEMS WITH MANY-BRANCH SPECTRUM.

Ferromagnetic Semiconductors

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1. INTRODUCTION

In this paper we present a unified and complete self-consistent consideration of the mutual influence of the electron and spin subsystems in the s-f model of the ferromagnetic semiconductors by taking explicitly into account damping effects and finite lifetimes. A great deal of effort is being made today to understand the physical properties of magnetic semiconductors. An important problem is the relationship between their magnetic and electrical properties. The interaction between conduction electrons and localized moments in degenerate ferromagnetic semiconductors is usually described by the s-f model [1,2]. The many-body problem formulated by the s-f model Hamiltonian has been discussed in many previous calculations for ferromagnetic semiconductors. Nolting [3,4] and Nolting and Oles [5,6] calculated the electronic excitation spectrum of a ferromagnetic semiconductor in the strong-coupling limit by an improved moment method. Sinkkonen [7] developed an intermediate-coupling theory for the s-f model using the functional-derivative method. In papers [1-10] were calculated the magnon spectrum and Curie temperature, quasi-particle density of states, and edge shifts of doped ferromagnetic semiconductors. Allan and Edwards [11] in a very elegant way investigated the exact nature of the conduction band states (retaining a predominant d-character of the conduction electrons) and find the most important effect of the electron-magnon interaction. In connection with the discussion about the true quasi-particle spectrum of a ferromagnetic semiconductor a significant contribution has been made by Edwards [12], who has clarified the theoretical situation regarding the structure of the conduction band of a ferromagnetic semiconductor such as EuS and a correct explanation of the observed spin-polarization of electrons. A very detailed investigation of the spectrum of magnetic excitations for s-f model has been given in the random-phase approximation [13]. These authors give a much more detailed description of the magnetic excitation spectrum than Woolsey and White [14], including an optical branch to the magnon spectrum and a Stoner-like continuum of excitations as well as the usual acoustic magnons. Unfortunately, the damping effects and finite lifetimes have not been taken into account. The only mechanism for the dampling of the magnon modes which has been considered is the decay of a magnon into an electron-hole pair with a spin-flip.

The purpose of our paper is to develop the complete self-consistent theory of electronic and magnetic elementary excitations in ferromagnetic semiconductors by taking explicitly into account magnon-magnon, electron-magnon, and electron-electron inelastic scattering processes. For this aim we use the novel irreducible Green function method (IGF) developed by Plakida [15,16] for the self-consistent phonon theory and Heisenberg ferromagnet and by Kuzemsky [17] for the Hubbard model. The IGF method completely describes the quasiparticle inelastic scattering processes in a many-body system and finds quasiparticle spectra with damping in a very general way. From a technical point of view the IGF methos is a special kind of the projection-operator approach in the theory of two-time Green functions [18]. Introducing "irreducible" parts of the GF (or the "irreducible" parts of the operators, from which the GF is constructed) the equation of motion for the GF can be exactly transformed into a Dyson equation with an exact representation of the self-energy operator which is represented by higher order Green functions. In order to calculate the self-energy operator in a self-consistent way, we have to express it approximatly by lower order Green functions. The IGF method has been applied recently in a number of solid state problems [19]. Christoph et al. [20] generalized this method to the calculation of elementary excitations with damping for the systems with a complex quasiparticle spectrum such as a generalized RKKY model of magnetism. The main point is that for the system with the complex spectrum all branches must be taken into account through performing damping calculations.

The paper is organised as follows: in §3 we derive the exact Dyson equation for one electron GF by the irreducible GF method. The self-consistent approximate formalism for the calculation of the electron self-energy operator is presented in §4. In §5 we derive the exact Dyson equation for the spin subsystem. The self-energy operator for this case is calculated in §6. In §7 we present our conclusions and possibilities of further developments. In the Appendix we give a simple but useful analysis of our truncation procedure on the basis of the moment conservation treatment.

2. HAMILTONIAN OF THE s-f MODEL

The total Hamiltonian of the s-f model is given by the following expression [1,2]

$$H = H_e + H_{ee} + H_f + H_{s-f}, \tag{1}$$

where H_e is the operator of kinetic energy of the itinerant band electrons

$$H_e = \sum_{ij\sigma} t_{ij} a^{+}_{i\sigma} a_{j\sigma} = \sum_{k\sigma} \epsilon_k a^{+}_{k\sigma} a_{k\sigma}. \tag{2}$$

Here $\epsilon_k = N^{-1} \sum_{ij} \exp[(-i\vec{k}(\vec{R}_i - \vec{R}_j)]$ is the band energy. Although the itinerant electrons (2) are predominantly d-electrons they are usually treated as s-electrons for mathematical simplicity. However, the retaining predominant d-character of the itinerant electrons may be very important for describing the heavy rare-earch metals and magnetic semiconductors [11-20]. For tight-binding electrons the band energy is given by

$$\epsilon_k = \sum_{\kappa} t(\vec{R}_\kappa) \cos(\vec{k}\vec{R}_\kappa) \tag{3}$$

for the lattices with the center of inversion. H_{ee} describes the Coulomb interaction of itinerant d-like electrons

$$H_{ee} = \frac{U}{2N} \sum_{kpq\sigma} a^{+}_{k+q\sigma} a_{k\sigma} a^{+}_{p-q,-\sigma} a_{p,-\sigma} . \tag{4}$$

Here U is the Hubbard Coulomb correlation integral. In the case of a pure semiconductor at low temperatures the "conduction" electron band is empty and the Coulomb term (4) therefore is not so important. A partial occupation of the band leades to increasing the role of the Coulomb correlation.

H_f describes the localised moments which are treated by the Heisenberg model

$$H_f = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j = -\frac{1}{2} \sum_q J_q \vec{S}_q \cdot \vec{S}_{-q} . \tag{5}$$

The two subsystems are coupled by a local spin-spin exchange interaction

$$H_{sf} = -I \sum_{i\sigma\sigma'} (\vec{S}_i \cdot \vec{\sigma})_{\sigma\sigma'} a^{+}_{i\sigma} a_{i\sigma'} =$$

$$= -\frac{I}{\sqrt{N}} \sum_{kq} \{ S^{+}_{-q} a^{+}_{k\downarrow} a_{k+q\uparrow} + S^{-}_{-q} a^{+}_{k\uparrow} a_{k+q\downarrow} + S^{z}_{-q} (a^{+}_{k\uparrow} a_{k+q\uparrow} - a^{+}_{k\downarrow} a_{k+q\downarrow}) \}. \tag{6}$$

The operator (6) describes the RKKI interaction of the localized spins of the 4f -shell with the spin density of itinerant electrons. In general the integral $I(\vec{k}, \vec{k} + \vec{q})$ depends on the quasimomentum $\vec{k}$. (A generalization to the nonlocal case can be done directly.)

3. DYSON EQUATION FOR THE ONE-ELECTRON GF

For the calculation of the electronic quasiparticle spectrum of the described model (1) let us consider the equation of motion for the one-electron GF [21]:

$$G_{k\sigma}(t-t') = -i\theta(t-t')\langle[a_{k\sigma}(t), a^{+}_{k\sigma}(t')]_{+}\rangle . \tag{7}$$

Performing the first-time (t) differentiation of (7) we get for the Fourier transform

$$\begin{aligned}(\omega-\epsilon_{k})G_{k\sigma}(\omega) &= 1 + \frac{U}{N}\sum_{pq}\langle\langle a^{+}_{p+q,-\sigma}a_{p,-\sigma}\,a_{k+q,\sigma}\,|a^{+}_{k\sigma}\rangle\rangle_{\omega} - \\ &- \frac{I}{\sqrt{N}}\sum_{q}\{\langle\langle S^{-\sigma}_{-q}a_{k+q,-\sigma}\,|a^{+}_{k\sigma}\rangle\rangle_{\omega} + z_{\sigma}\langle\langle S^{z}_{-q}a_{k+q,\sigma}\,|a^{+}_{k\sigma}\rangle\rangle_{\omega}\} ,\end{aligned} \tag{8}$$

where

$$S^{-\sigma}_{-q} = \begin{cases} S^{-}_{-q} & \text{if } \sigma = + \text{ or } \uparrow \\ S^{+}_{-q} & \text{if } \sigma = - \text{ or } \downarrow \end{cases} \qquad z_{\sigma} = \begin{cases} +1 & \text{if } \sigma = + \\ -1 & \text{if } \sigma = - \end{cases}$$

Following [15,17] we introduce by definition the irreducible Green functions

$$\begin{aligned}&\langle\langle S^{z}_{-q}\,a_{k+q\sigma}\,|a^{+}_{k\sigma}\rangle\rangle = \langle\langle(S^{z}_{-q}a_{k+q\sigma})^{ir}\,|a^{+}_{k\sigma}\rangle\rangle + \delta_{q,0}\langle S^{z}_{-q}\rangle\langle\langle a_{k+q\sigma}|a^{+}_{k\sigma}\rangle\rangle \\ &\langle\langle a^{+}_{p+q,-\sigma}a_{p-\sigma}\,a_{k+q\sigma}|a^{+}_{k\sigma}\rangle\rangle = \langle\langle(a^{+}_{p+q\sigma}a_{p-\sigma}\,a_{k+q\sigma})^{ir}\,|a_{k\sigma}\rangle\rangle + \\ &+ \delta_{q,0}\,\langle a^{+}_{p+q,-\sigma}a_{p,-\sigma}\rangle\langle\langle a_{k+q,\sigma}|a^{+}_{k\sigma}\rangle\rangle .\end{aligned} \tag{9}$$

In which mean-field contributions are removed. The choice of the IGF is determined by the conditions

$$\begin{aligned}&\langle[(S^{z}_{-q}a_{k+q,\sigma})^{ir}, a^{+}_{k\sigma}]_{+}\rangle = 0 , \\ &\langle[(a^{+}_{p+q,-\sigma}\,a_{p,-\sigma}\,a_{k+q,\sigma})^{ir}, a^{+}_{k\sigma}]_{+}\rangle = 0 .\end{aligned} \tag{10}$$

To calculate the IGF's, we will differentiate r.h.s. GF's in (8) with respect to the second-time variable (t'). Using the irreducible GF's, the equation of motions can be exactly transformed into Dyson equation [15,17]:

$$G_{k\sigma}(\omega) = G^{o}_{k\sigma}(\omega) + G^{o}_{k\sigma}(\omega)M_{k\sigma}(\omega)G_{k\sigma}(\omega), \tag{11}$$

where

$$G^{o}_{k\sigma}(\omega) = \{\omega - \epsilon^{o}(k\sigma)\}^{-1}, \quad \epsilon^{o}(k\sigma) = \epsilon_{k} - z_{\sigma}\frac{I}{\sqrt{N}}\langle S^{z}\rangle + \frac{U}{N}n_{-\sigma} \tag{12}$$

is the mean-field Green function. The self-energy operator $M_{k\sigma}(\omega)$ is connected with the scattering operator $P_{k\sigma}(\omega)$ by the relation

$$P_{k\sigma}(\omega) = M_{k\sigma}(\omega) + M_{k\sigma}(\omega)G^{o}_{k\sigma}(\omega)P_{k\sigma}(\omega), \tag{13}$$

$$M_{k\sigma}(\omega) = \{P_{k\sigma}(\omega)\}^{c}, \tag{14}$$

where the M is defined as the "connected" (or proper) part of the scattering operator P. The connected part of an irreducible GF does not contain inner parts connected by one G_0-line. Here the self-energy operator (14) has the following exact representation

$$M_{k\sigma}(\omega) = M^{ee}_{k\sigma}(\omega) + M^{e-m}_{k\sigma}(\omega), \tag{15}$$

where

$$M^{ee}_{k\sigma}(\omega) = \frac{U^{2}}{N^{2}}\sum_{pq}\sum_{p'q'}\langle\langle(a^{+}_{p+q,-\sigma}a_{p,-\sigma}a_{p+q,\sigma})^{ir}|(a^{+}_{p'+q',-\sigma}a_{p',-\sigma}\,a^{+}_{k-q',\sigma})^{ir}\rangle\rangle^{c}, \tag{16}$$

$$\begin{aligned}M^{e-m}_{k\sigma}(\omega) &= \frac{I^{2}}{N}\sum_{qq'}\{\langle\langle S^{-\sigma}_{-q}\,a_{k+q\sigma}\,|S^{\sigma}_{q'}a^{+}_{k+q',-\sigma}\rangle\rangle + \\ &+ \langle\langle(S^{z}_{-q}\,a_{k+q,\sigma})^{ir}\,|(S^{z}_{q'}\,a^{+}_{k+q',\sigma})^{ir}\rangle\rangle^{c}\} .\end{aligned} \tag{17}$$

In this paper, for the sake of simplicity we consider the wide-band (weak Coulomb correlation) case. A generalization to the narrow-band (strong Coulomb correlation) limit can be done following [17].

4. SELF-CONSISTENT APPROXIMATION CALCULATION OF THE SELF-ENERGY OPERATOR FOR ELECTRONS

To find explicit useful expressions for $M_{k\sigma}(\omega)$ (15), suitable approximations to evaluate the higher-order GF's in (16) and (17) should be used. The electron-electron part (16) was found earlier by Kuzemsky [17] considering the electron correlation in Hubbard model in the band limit. In the pair approximation for (16) we obtain

$$M^{ee}_{k\sigma}(\omega) = \frac{U^2}{N^2}\sum_{pq}\int_{-\infty}^{\infty}\frac{d\omega_1 d\omega_2 d\omega_3}{\omega+\omega_1-\omega_2-\omega_3}\{n(\omega_1)[1-n(\omega_2)-n(\omega_3)]+n(\omega_2)n(\omega_3)\}\times$$

$$\times\, g_{p+q,-\sigma}(\omega_1)g_{k+p,\sigma}(\omega_2)g_{p,-\sigma}(\omega_3), \tag{18}$$

$$g_{k\sigma}(\omega) = -\frac{1}{\pi}\operatorname{Im} G_{k\sigma}(\omega+i\epsilon) = \frac{1}{2\pi}(e^{\beta\omega}+1)A_{k\sigma}(\omega). \tag{19}$$

Let us consider now the spin-electron inelastic scattering. It is convenient to write down $M^{em}_{k\sigma}(\omega)$ in the form

$$M^{em}_{k\sigma}(\omega) = M^{\alpha\beta}_{k\sigma}(\omega) = \frac{I^2}{N}\sum_{qq'}\langle\langle S^{\alpha}_{-q}a_{k+q\sigma}\,|\,S^{\beta}_{q'}a^{+}_{k+q'\sigma}\rangle\rangle^{(ir,c)} =$$

$$= \frac{I^2}{N}\sum_{qq'}\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{\omega-\omega'}(e^{\beta\omega'}+1)\int_{-\infty}^{\infty}dt\,e^{i\omega' t}\langle S^{\beta}_{q'}a^{+}_{k+q'\sigma}S^{\alpha}_{-q}(t)a_{k+q\sigma}(t)\rangle^{(ir,c)}. \tag{20}$$

We use the following decoupling procedure

$$\langle S^{\beta}_{q'}a^{+}_{k+q'\sigma}S^{\alpha}_{-q}(t)a_{k+q,\sigma}(t)\rangle^{ir,c} \approx \langle S^{\beta}_{q'}S^{\alpha}_{-q}(t)\rangle\langle a^{+}_{k+q'\sigma}a_{k+q\sigma}(t)\rangle. \tag{21}$$

The approximation (21) results from the neglect of the vertex corrections, i.e., the correlation between propagations of the electrons and the magnetic excitations. Taking into account the spectral theorem we obtain from (20), (21)

$$M^{em}_{k\sigma}(\omega) = \frac{I^2}{N}\sum_{q'}\int_{-\infty}^{\infty}d\omega_1 d\omega_2\frac{1+\nu(\omega_1)-n(\omega_2)}{\omega-\omega_1-\omega_2}m^{\alpha\beta}_{q}(\omega_1)g_{k+q,\sigma}(\omega_2) =$$

$$= \frac{I^2}{N}\sum_{q}\int d\omega_1 d\omega_2\frac{1+\nu(\omega_1)+n(\omega_2)}{\omega-\omega_1-\omega_2}\{m^{\sigma,-\sigma}_{q}(\omega_1)g_{k+q,-\sigma}(\omega_2)+ \tag{22}$$

$$+\,m^{zz}_{q}(\omega_1)g_{k+p,\sigma}(\omega_2)\},$$

where the following symbols are introduced:

$$m^{\alpha\beta}_{q}(\omega) = -\frac{1}{\pi}\operatorname{Im}\langle\langle S^{\alpha}_{-q}|S^{\beta}_{q}\rangle\rangle = \frac{1}{2\pi}(e^{\beta\omega}-1)K^{\alpha\beta}_{q}(\omega),$$

$$K^{\beta\alpha}_{q}(t) = \langle S^{\beta}_{q}S^{\alpha}_{-q}(t)\rangle. \tag{23}$$

Here $\nu(\omega)$ and $n(\omega)$ denote the Bose and Fermi distribution function, respectively.

Equations (11), (18), and (22) form a closed self-consistent system of equations for the one-electron Green function of ferromagnetic semiconductor. Electron-electron inelastic scattering is described in the pair approximation (18) and the spin-electron scattering is described by neglecting the vertex corrections (22). In principle, we may substitute in the r.h.s. of (18) and (22) any relevant initial Green function and solve it by iterations.

We choose for the first iteration step the following simple one-pole expressions for (19)

$$g_{k\sigma}(\omega) = \delta(\omega-\epsilon(k\sigma)). \tag{24}$$

Then we obtain from (18) and (22)

$$M^{ee}_{k\sigma}(\omega) = \frac{U^2}{N^2}\sum_{pq}\frac{n_{p+q,\sigma}(1-n_{k+p,\sigma}+\sigma n_{q,-\sigma})+n_{k+p,\sigma}n_{q,-\sigma}}{\omega-\epsilon(p+q,\sigma)-\epsilon(k+p,\sigma)+\epsilon(q,-\sigma)} =$$

$$= \frac{U^2}{N^2}\sum_{pq}\frac{N_{kpq}}{\omega-\Omega_{kpq}}, \tag{25}$$

$$M^{em}_{k\sigma}(\omega) = \frac{I^2}{N}\sum_{q}\int_{-\infty}^{\infty}d\omega_1\{\frac{1+\nu(\omega_1)-n(\epsilon(k+q,-\sigma))}{\omega-\omega_1-\epsilon(k+q,-\sigma)}m^{\sigma,-\sigma}_{q}(\omega_1)+$$

$$+\frac{1+\nu(\omega_1)-n(\epsilon(k+q,\sigma))}{\omega-\omega_1-\epsilon(k+q,\sigma)}m^{zz}_{q}(\omega_1). \tag{26}$$

The expression (25) was found by Kuzemsky (1978). It describes electron-electron pair scattering in the paramagnetic state of the electron subsystem. The expression (26) contains some results of papers /3-8/. To obtain the results of Woolsey and White /14/, one must neglect $m^{zz}_{q}(\omega)$ (that is reasonable at low temperatures) and use the following first iteration approximations

$$-\frac{1}{\pi}\operatorname{Im}\langle\langle S^{-\sigma}_{-q}|S^{\sigma}_{q}\rangle\rangle = 2\frac{\langle S^z\rangle}{\sqrt{N}}\delta(\omega+\sigma\omega(q)), \tag{27}$$

where the $\omega(q) = Dq^2$ is the magnon energy. Then we obtain perturbative result of /14/

$$M^{em}_{k+}(\omega) = \frac{2\langle S^z\rangle I^2}{N^{3/2}}\sum_{q}\frac{n(\epsilon(k+q,-))-\nu(\omega(q))}{\omega-\epsilon(k+q,-)+\omega(q)},$$

$$M^{em}_{k-}(\omega) = \frac{2\langle S^z\rangle I^2}{N^{3/2}} \sum_q \frac{1-n(\epsilon(k+q,+))+\nu(\omega(q))}{\omega-\epsilon(k+q,+)-\omega(q)}, \qquad (28)$$

In the static limit for $m^{\alpha\beta}_q(\omega) \to m^{\alpha\beta}_q(0)$ one can immediately obtain the Sinkkonen result [7]:

$$M^{em}_{k\sigma}(\omega) = \frac{I^2}{N} \sum_q \int_{-\infty}^{\infty} \frac{d\omega'}{\omega-\omega'}[K^{\sigma,-\sigma}_q \cdot g_{k+q,-\sigma}(\omega') + K^{zz}_q g_{k+q,\sigma}(\omega')] \qquad (29)$$

from which the Kuivalainen et al. [8] result can be found as a special case.

The renormalized electron energy is a self consistent solution of the equation

$$\epsilon - \epsilon^{\circ}(k\sigma) - \mathrm{Re}M_{k\sigma}(\epsilon) = 0 \qquad (30)$$

together with equations (18) and (22) or with (25) and (26). In this way energy shifts of electrons are to be calculated from the set of nonlinear integral equations (30), (25) and (26). For the electron line width one gets from (11):

$$\Gamma_{k\sigma}(\omega) = -\mathrm{Im}M_{k\sigma}(\omega+i\epsilon),$$

$$\frac{1}{\tau} = \bar{\Gamma}_{k\sigma} = \frac{\mathrm{Im}M_{k\sigma}(\epsilon(k\sigma))}{1-\frac{\partial}{\partial\omega}\mathrm{Re}M_{k\sigma}(\omega)|_{\omega=\epsilon(k\sigma)}}. \qquad (31)$$

One-electron density of states is defined as

$$D^e_\sigma(\omega) = -\frac{1}{\pi N}\sum_k \frac{\mathrm{Im}M_{k\sigma}(\omega)}{[\omega-\epsilon^{\circ}(k\sigma)-\mathrm{Re}M_{k\sigma}]^2+[\mathrm{Im}M_{k\sigma}(\omega)]^2}. \qquad (32)$$

On the basis of eqs. (31) and (32) we can obtain modified (due to the electron-electron correlation effect) expressions for the electron effective mass, electron mobility and electron specific heat found by Woolsey and White [14].

5. DYSON EQUATION FOR THE SPIN GREEN FUNCTIONS

To study the magnetic excitation spectrum of a localised-spin subsystem we need the GF

$$R(t) = \langle\langle S^+_k | S^-_{-k}\rangle\rangle = -i\theta(t-t')\langle[S^+_k(t), S^-_{-k}(t')]_-\rangle. \qquad (33)$$

In paper [20] it has been shown that for the composite quasi-particle excitation spectrum we must use the full generalized susceptibility of the system

$$\hat{R}(t) = \begin{bmatrix} \langle\langle S^+_k | S^-_{-k}\rangle\rangle & \langle\langle S^+_k | \sigma^-_{-k}\rangle\rangle \\ \langle\langle \sigma^+_k | S^-_{-k}\rangle\rangle & \langle\langle \sigma^+_k | \sigma^-_{-k}\rangle\rangle \end{bmatrix}, \qquad (34)$$

where

$$\sigma^+_k = \sum_q a^+_{q\uparrow} a_{k+q\downarrow}; \quad \sigma^-_{-k} = (\sigma^+_k)^+ = \sum a^+_{k+q\downarrow} a_{q\uparrow}.$$

Differentiating the GF $\langle\langle S^+_k|B\rangle\rangle$ with respect to the first time t and introducing the IGF following [20] (it is convenient to introduce the "irreducible" operators)

$$(S^z)^{ir} = S^z_q - \langle S^z_0\rangle\delta_{q,0},$$

$$(S^+_{k-q}(S^z_q)^{ir} - S^+_q(S^z_{k-q})^{ir})^{ir} = S^+_{k-q}(S^z_q)^{ir} - S^+_q(S^z_{k-q})^{ir} - (A_q - A_{k-q})S^+_k, \qquad (35)$$

$$A_q = \frac{2K^{zz}_q + K^{-+}_q}{2\langle S^z_0\rangle} \qquad (36)$$

we find the equation of motion

$$\Omega_1 \langle\langle S^+_k|B\rangle\rangle_\omega + \Omega_2\langle\langle\sigma^+_k|B\rangle\rangle = \begin{Bmatrix} \frac{\sqrt{N}}{I}\Omega_2 \\ 0 \end{Bmatrix} + \langle\langle A|B\rangle\rangle_\omega. \qquad (37)$$

Here

$$\Omega_1 = \omega - \frac{\langle S^z_0\rangle}{\sqrt{N}}(J_0 - J_k) - \frac{1}{\sqrt{N}}\sum_q (J_q - J_{q-k})\frac{2K^{zz}_q + K^{-+}_q}{2\langle S^z_0\rangle} - \frac{I}{N}(n_\uparrow - n_\downarrow), \qquad (38)$$

$$\Omega_2 = 2\frac{I\langle S^z_0\rangle}{N}, \qquad (39)$$

and B denotes the operator S^-_{-k} or σ^-_{-k}. The many-particle operator A has the form

$$A = \frac{1}{\sqrt{N}}\sum_q J_q\{S^+_{k-q}S^z_q - S^+_q S^z_{k-q}\}^{ir} + \frac{I}{N}\sum_{pq}\{S^+_{k-q}(a^+_{p\uparrow}a_{p+q\uparrow} - a^+_{p\downarrow}a_{p+q\downarrow})^{ir} - 2(S^z_{k-q})^{ir}a^+_{p\uparrow}a_{p+q\downarrow}\} \qquad (40)$$

and satisfies the condition

$$<[A, S^-_{-k}]_-> = <[A, \sigma^-_{-k}]_-> = 0 .$$

Now we consider the GF $\langle\langle\sigma_k^+|B\rangle\rangle$. Similarly to the equation (37) we have

$$-\sqrt{N}\cdot I\cdot\chi_0^{df}\langle\langle S_k^+|B\rangle\rangle_\omega + [1 - U\chi_0^{df}]\langle\langle\sigma_k^+|B\rangle\rangle_\omega = \left\{\begin{matrix} 0 \\ -N\chi_0^{df}\end{matrix}\right\} + \sum_p \frac{1}{\omega_{p,k}}\langle\langle B_p|B\rangle\rangle_\omega , \tag{41}$$

where

$$\omega_{p,k} = \omega + \epsilon_p - \epsilon_{p+k} - \Delta, \quad \Delta = \frac{2I}{\sqrt{N}}<S_0^z> + \frac{U}{N}(n_\uparrow - n_\downarrow) ,$$
$$\chi_0^{df} = \frac{1}{N}\sum_p \frac{n_{p+k\downarrow} - n_{p\uparrow}}{\omega_{pk}} . \tag{42}$$

The "irreducible" operator B_p is defined as

$$B_p = -\frac{I}{\sqrt{N}}\sum_{qq'}\{S^+_{-q'}(a^+_{p\uparrow}a_{q+q'\uparrow}\delta_{q,p+k} - a^+_{q\downarrow}a_{p+k\downarrow}\delta_{p,q+q'})^{ir} -$$
$$-(S^z_{-q'})^{ir}(a^+_{p\uparrow}a_{q+q'\downarrow}\delta_{q,p+k} + a^+_{q\uparrow}a_{p+k\downarrow}\delta_{p,q+q'})\} + \tag{43}$$
$$+\frac{U}{N}\sum_{q,q'}(a^+_{p\uparrow}a^+_{q+q'\uparrow}a_{q\uparrow}\cdot a_{p+k+q'\downarrow} - a^+_{p+q'\uparrow}a^+_{q-q'\downarrow}a_{q\downarrow}a_{p+k\downarrow})^{ir} .$$

The equations of motion (37) and (41) can be summarised in the matrix form

$$\hat{\Omega}\hat{G} = \hat{I} + \sum_p\hat{\Phi}_p\cdot\hat{G}_1 , \tag{44}$$

$$\hat{\Omega} = \begin{bmatrix}\Omega_1 & \Omega_2 \\ -I\sqrt{N}\chi_0^{df} & (1 - U\chi_0^{df})\end{bmatrix} ; \quad \hat{I} = \begin{bmatrix}\frac{\sqrt{N}}{I}\Omega_0 & 0 \\ 0 & -N\chi_0^{df}\end{bmatrix} \tag{45}$$

$$\hat{\Phi}_p = \begin{bmatrix}\frac{1}{N} & 0 \\ 0 & \frac{1}{\omega_{pk}}\end{bmatrix} ; \quad \hat{G}_1 = \begin{bmatrix}\langle\langle A|S^-_{-k}\rangle\rangle & \langle\langle A|\sigma^-_{-k}\rangle\rangle \\ \langle\langle B_p|S^-_{-k}\rangle\rangle & \langle\langle B_p|\sigma^-_{-k}\rangle\rangle\end{bmatrix} \tag{46}$$

To obtain a Dyson equation, we have to use equations of motion for the r.h.s. Green functions in (46) and introduce the

irreducible parts as discussed above. Thus, we obtain

$$\hat{R} = \hat{R}_0 + \hat{R}_0\hat{P}\hat{R}_0 , \tag{47}$$

where

$$\hat{P} = \hat{I}^{-1}\{\sum_{pq}\hat{\Phi}_p\hat{P}(p,q)\hat{\Phi}_q\}\hat{I}^{-1} , \tag{48}$$

$$\hat{P}(pq) = \begin{bmatrix}\langle\langle A|A^+\rangle\rangle & \langle\langle A|B_q^+\rangle\rangle \\ \langle\langle B_p|A^+\rangle\rangle & \langle\langle B_p|B_q^+\rangle\rangle\end{bmatrix} ; \quad \hat{R}_0 = \hat{\Omega}^{-1}\cdot\hat{I} . \tag{49}$$

Using the definition (13) eq. (47) can be transformed into the exact Dyson equation

$$\hat{R} = \hat{R}_0 + \hat{R}_0\hat{\Pi}\hat{R} , \tag{50}$$

$$\hat{\Pi} = \{\hat{P}\}^c . \tag{51}$$

The solution of (50) can be written in the form

$$\hat{R} = \{\hat{R}_0^{-1} - \hat{\Pi}\}^{-1} . \tag{52}$$

Hence, the determination of $\hat{R}$ has been reduced to the determination of the mean-field GR $\hat{R}_0$ and self-energy operator $\hat{\Pi}$.

The mean-field GF R_0 has the explicit form

$$\hat{R}_0 = \frac{1}{\det\hat{G}_0}\begin{bmatrix}(1 - U\chi_0^{df})\frac{\sqrt{N}}{I}\Omega_2 & N\Omega_2\chi_0^{df} \\ N\Omega_2\chi_0^{df} & -N\Omega_1\chi_0^{df}\end{bmatrix} , \tag{53}$$

where

$$\det\hat{R}_0 = (1 - U\chi_0^{df})\Omega_1 + \sqrt{N}I\Omega_2\chi_0^{df} . \tag{54}$$

For the localized-spin Green function $\langle\langle S_k^+|S_{-k}^-\rangle\rangle_\omega^\circ$ we find

$$\langle\langle S_k^+|S_{-k}^-\rangle\rangle_\omega^\circ = \frac{2<S_0^z>}{\sqrt{N}}\{\omega - \frac{<S_0^z>}{\sqrt{N}}(J_0 - J_k) - \frac{I}{N}(n_\uparrow - n_\downarrow) -$$
$$-\frac{1}{\sqrt{N}}\sum_q(J_q - J_{k-q})\frac{2K_q^{zz} + K_q^{-+}}{2<S_0^z>} + 2\frac{I^2<S_0^z>}{\sqrt{N}}\chi_0^{df}(1 - U\chi_0^{df})^{-1}\}^{-1} . \tag{55}$$

As follows from the expression (55), both the interacting subsystems (localized spins and itinerant electrons) are described in the Generalized Hartree-Fock approximation which can be considered as a good starting point to study wide-band ferromagnetic semiconductors. The magnetic excitation spectrum following from the Green function (55) consists of three branches - the acoustic spin waves, the optical spin waves, and the Stoner-like continuum of excitations. Our consideration generalises the first-order theory given by Babcenco and Cottam [18].

In the limit $k \to 0$, $\omega \to 0$ the Green function (55) can be written as

$$\langle\langle S^+_k | S^-_{-k} \rangle\rangle^{0}_{\omega} = \frac{2\langle S^z_0\rangle}{\sqrt{N}} \, \frac{1}{\omega - \omega(k)}\,, \qquad (56)$$

where the acoustic spin-wave energies $\omega(k) = D_{ac}k^2$ are determined by the stiffness constant

$$D_{ac} = \frac{\langle S^z_0\rangle}{2\sqrt{N}} \Big\{ \Psi_0 + \sum_q \Psi_q \frac{2K^{zz}_q + K^{-+}_q}{2\langle S^z_0\rangle} + \frac{\sqrt{N}}{2\langle S^z_0\rangle} \cdot \frac{1}{N} \sum_q (n_{q\uparrow} + n_{q\downarrow}) \frac{1}{2} (\hat{k}\cdot\nabla_q)^2 \epsilon_q + \frac{\sqrt{N}}{2\langle S^z_0\rangle} \cdot \frac{1}{N\Delta} \sum_q (n_{q\downarrow} - n_{q\uparrow}) (\hat{k}\cdot\nabla_q \epsilon_q)^2 \Big\}. \qquad (57)$$

Here

$$\Psi_q = \sum_\alpha (\hat{k}\cdot\vec{R}_\alpha)^2 J(|\vec{R}_\alpha|)\, e^{-i\vec{q}\vec{R}_\alpha}$$

and the sum is taken over the lattice sites denoted by R_α; $J(|\vec{R}_\alpha|)$ is the exchange integral, and $\hat{k} = \vec{k}/|\vec{k}|$. The stiffness constant D_{ac} can be expressed by the parameters of the Hamiltonian if one evaluates the band splitting Δ by a selfconsistent solving of the RPA equation

$$n_\sigma = \sum_k n_{k\sigma} = \sum_k \Big\{ \exp\cdot\beta\Big(\epsilon_k + \frac{U}{N} n_{-\sigma} - \frac{I}{\sqrt{N}}\langle S^z_0\rangle - \epsilon_f\Big) + 1 \Big\}^{-1}. \qquad (58)$$

6. SPIN SELF-ENERGY OPERATOR AND MAGNON DAMPING

To calculate the self-energy operator $\hat{\Pi}$ (50) in a self-consistent way, we must approximate it by the lower-order Green functions.

Let us consider the approximate calculation of the Green function $\langle\langle A|A^+\rangle\rangle$ appearing in $\hat{\Pi}$. Using the spectral theorem and neglecting the higher-order-correlation effects vertex corrections between the magnetic excitations and charge density fluctuations we obtain

$$\begin{aligned}
\langle\langle A|A^+\rangle\rangle^{ir,c}_{\omega} &= \frac{1}{N}\sum_q (J_q - J_{k-q})^2 \int_{-\infty}^{\infty} d\omega_1\, d\omega_2\, \frac{1+\nu(\omega_1)+\nu(\omega_2)}{\omega-\omega_1-\omega_2} \times \\
&\times \Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle S^+_{k-q}|S^-_{-(k-q)}\rangle\rangle_{\omega_1}\Big)\Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle (S^z_q)^{ir}|(S^z_{-q})^{ir}\rangle\rangle_{\omega}\Big) + \\
&+ \frac{I^2}{N^2}\sum_{pq}\sum_{p'q'\sigma}\int_{-\infty}^{\infty} d\omega_1 d\omega_2\, \frac{e^{\beta(\omega_1+\omega_2)}-1}{(e^{\beta\omega_1}-1)(e^{\beta\omega_2}-1)}\cdot\frac{1}{\omega-\omega_1-\omega_2} \times \\
&\times \Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle S^+_{k-q'}|S^-_{-(k-q)}\rangle\rangle_{\omega_1}\Big)\Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle a^+_{p'\sigma'}a_{p'+q'\sigma}|a^+_{p\sigma}a_{p+q\sigma}\rangle\rangle_{\omega_2}\Big) + \\
&+ \frac{4I^2}{N^2}\sum_{pq\,p'q'}\int_{-\infty}^{\infty} d\omega_1 d\omega_2\, \frac{e^{\beta(\omega_1+\omega_2)}-1}{(e^{\beta\omega_1}-1)(e^{\beta\omega_2}-1)}\cdot\frac{1}{\omega-\omega_1-\omega_2} \times \\
&\times \Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle (S^z_{k-q'})^{ir}|(S^z_{-(k-q)})^{ir}\rangle\rangle_{\omega_1}\Big) \times \\
&\times \Big(-\frac{1}{\pi}\mathrm{Im}\langle\langle a^+_{p'\uparrow}a_{p'+q'\downarrow}|a^+_{p\downarrow}a_{p+q\uparrow}\rangle\rangle_{\omega_2}\Big).
\end{aligned} \qquad (59)$$

Using the same arguments the Green functions $\langle\langle A|B^+_q\rangle\rangle$, $\langle\langle B_p|A^+\rangle\rangle_\omega$ and $\langle\langle B_p|B^+_q\rangle\rangle$ can be represented in a similar form. The equations (50), (59) and three equations for other Green functions give a self-consistent system of equations for $\hat{R}$ and $\hat{\Pi}$.

For the simplest first iteration approximations

$$-\frac{1}{\pi}\mathrm{Im}\langle\langle S^+_{k-q'}|S^-_{-(k-q)}\rangle\rangle = \frac{2\langle S^z_0\rangle}{\sqrt{N}}\,\delta(\omega-\omega(k+q))\,\delta_{q',-q}\,, \qquad (60)$$

$$-\frac{1}{\pi}\mathrm{Im}\langle\langle a^+_{p'\sigma}a_{p'+q'\sigma}|a^+_{p\sigma}a_{p+q\sigma}\rangle\rangle = (n_{p'\sigma} - n_{p\sigma})\,\delta(\omega+\epsilon(p'\sigma)-\epsilon(p\sigma))\,\delta_{p',p+q}\cdot\delta_{p,p'+q'} \qquad (61)$$

in the r.h.s. of (59) we get (cf. [20]):

$$\langle\langle A|A^{+}\rangle\rangle_{\omega}^{ir,c} = \frac{2\langle S_0^z\rangle}{N^{3/2}} \sum_q (J_q - J_{k-q})^2 \int_{-\infty}^{\infty} d\omega' \frac{1+\nu(\omega')+\nu(\omega(k-q))}{\omega-\omega'-\omega(k-q)} \times$$

$$\times\left[\frac{-1}{\pi} \mathrm{Im}\langle\langle (S_q^z)^{ir} | (S_{-q}^z)^{ir}\rangle\rangle_{\omega'}\right] +$$

$$+ \frac{2\langle S_0^z\rangle}{\sqrt{N}} \frac{I^2}{N^2} \sum_{pq\sigma} \frac{[1+\nu(\omega(k+q))]\, n_{p+q,\sigma}(1-n_{p\sigma}) - \nu(\omega(k+q))\, n_{p\sigma}(1-n_{p+q,\sigma})}{\omega-\omega(k+q)+\epsilon(p+q\sigma)-\epsilon(p\sigma)} +$$

$$+ \frac{4I^2}{N^2} \sum_{pq} \int_{-\infty}^{\infty} d\omega' \frac{[1+\nu(\omega')]\, n_{p+q\uparrow}(1-n_{p\downarrow}) - \nu(\omega')\, n_{p\downarrow}(1-n_{p+q\uparrow})}{\omega-\omega'+\epsilon(p+q\uparrow)-\epsilon(p\downarrow)} \times$$

$$\times\left\{-\frac{1}{\pi} \mathrm{Im}\langle\langle (S_{k+q}^z)^{ir} | (S_{-(k+q)}^z)^{ir}\rangle\rangle_{\omega'}\right\}, \tag{62}$$

where the first term describes the magnon-magnon inelastic scattering; and others, the magnon-electron scattering. For the concrete calculations we need a suitable approximate form of the longitudinal spin susceptibility. For this aim we may use the result of paper /13/.

To calculate the damping of the magnetic excitations for the system with composite spectrum, we must take into account all matrix elements of the self-energy operator

$$M_{11} = \frac{I^2}{N\Omega_2^2} \langle\langle A|A^{+}\rangle\rangle_{\omega}^{ir,c}, \tag{63}$$

$$M_{12} = M_{21}^{*} = -\frac{I}{N^{3/2}\Omega_2 \chi_0^{df}} \sum_q \frac{1}{\omega_{q,k}} \langle\langle A|B_q^{+}\rangle\rangle_{\omega}^{ir,c}, \tag{64}$$

$$M_{22} = \frac{1}{N^2(\chi_0^{df})^2} \sum_{pq} \frac{1}{\omega_{p,k}\cdot\omega_{q,k}} \langle\langle B_p|B_q^{+}\rangle\rangle_{\omega}^{ir,c}. \tag{65}$$

Then the Green function $\hat{R}$ becomes

$$\hat{R} = (\hat{R}_0^{-1} - \hat{\Pi})^{-1} =$$

$$= \frac{1}{\det(\hat{R}_0^{-1} - \hat{\Pi})} \begin{bmatrix} \dfrac{\langle\langle S_k^{+}|S_{-k}^{-}\rangle\rangle}{\det\hat{R}_0} - M_{22}\ ; & -\left\{\dfrac{\langle\langle S_k^{+}|\sigma_{-k}^{-}\rangle\rangle}{\det\hat{R}_0} + M_{12}\right\} \\ -\left\{\dfrac{\langle\langle \sigma_k^{+}|S_{-k}^{-}\rangle\rangle}{\det\hat{R}_0} + M_{21}\right\} & \dfrac{\langle\langle \sigma_k^{+}|\sigma_{-k}^{-}\rangle\rangle}{\det\hat{R}_0} - M_{11} \end{bmatrix}. \tag{66}$$

As an example consider the Green function

$$\langle\langle S_k^{+}|S_{-k}^{-}\rangle\rangle_{\omega} = \frac{1}{(\langle\langle S_k^{+}|S_{-k}^{-}\rangle\rangle_{\omega}^{0})^{-1} - \Sigma(k,\omega)}, \tag{67}$$

where the $\Sigma(k,\omega)$ is given by

$$\Sigma(k,\omega) = M_{11} - \left(\frac{I}{\sqrt{N}\det\cdot\hat{R}_0} - M_{12}\right)\left(\frac{I}{\sqrt{N}\cdot\det\hat{R}_0} - M_{21}\right) \times$$

$$\times\left(\frac{1-U\chi_0^{df}}{N\chi_0^{df}\det\hat{R}_0} + M_{22}\right)^{-1} + \frac{I^2\chi_0^{df}}{(1-U\chi_0^{df})\det\hat{R}_0}. \tag{68}$$

The Green function (67) contains acoustic and optical magnon excitations as well as the Stoner continuum of excitations damped by magnon-magnon, electron-magnon, and electron-electron inelastic scattering processes. For brevity we calculate only the acoustic magnon damping. Condidering only the linear terms in M_{ij} we find for small k and ω

$$\langle\langle S_k^{+}|S_{-k}^{-}\rangle\rangle_{\omega} = \frac{\frac{2}{\sqrt{N}}\langle S_0^z\rangle}{\omega-\omega(k)-\frac{2\langle S_0^z\rangle}{\sqrt{N}}\Sigma(k,\omega)}, \tag{69}$$

$$\Sigma(k,\omega) = M_{11} + (M_{12}+M_{21})\frac{I\sqrt{N}\chi_0^{df}}{1-U\chi_0^{df}} + \frac{I^2N(\chi_0^{df})^2}{(1-U\chi_0^{df})^2} M_{22}. \tag{70}$$

Then the spectral density of the spin-wave excitations with the wave vector k reads

$$-\frac{1}{\pi}\mathrm{Im}\langle\langle S_k^{+}|S_{-k}^{-}\rangle\rangle = \frac{\frac{2}{\sqrt{N}}\langle S_0^z\rangle\Gamma(k,\omega)}{[\omega-\omega(k)-\Delta(k,\omega)]^2+\Gamma^2(k,\omega)}, \tag{71}$$

where

$$\Delta(k,\omega) = \frac{2\langle S_0^z\rangle}{\sqrt{N}} \operatorname{Re}\Sigma(k,\omega), \tag{72}$$

$$\Gamma(k,\omega) = -\frac{2\langle S_0^z\rangle}{\sqrt{N}} \operatorname{Im}\Sigma(k,\omega) \tag{73}$$

describes the shift and damping of the acoustic magnons, respectively.

Finally as an example we shall estimate the temperature dependence of $\Gamma(k,\omega)$ due to electron-magnon scattering at low temperatures. We have

$$\operatorname{Im} M_{11} \sim \sum_{pq\sigma} \nu(\omega(k+q))(n_{p+q\sigma} - n_{p\sigma})\delta(\omega - \omega(k+q) + \epsilon(p+q\sigma) - \epsilon(p\sigma)). \tag{74}$$

If we substitute

$$\sum_{pq} \to \frac{\Omega^2}{(2\pi)^6} \int d\vec{p} \int d\vec{q} \tag{75}$$

and expand in powers of q, we obtain

$$\operatorname{Im} M_{11} \sim \frac{\Omega^2}{(2\pi)^6} 2\pi \int dp \int_0^{q_{max}} q^2 dq \int d\cos\theta\, \Phi(n_{p\sigma}, \nu(\omega(q))) \times$$

$$\times \frac{\delta(\cos\theta - \cos\theta_0)}{q \left|\frac{\partial\epsilon(p\sigma)}{\partial p}\right|} \sim \frac{1}{2\beta D_{ac}} \cdot \int_0^{\beta\omega_{max}} dx \frac{1}{e^x - 1} \sim T. \tag{76}$$

The other contributions to $\Sigma(k,\omega)$ may be treated in the same way. So, the electron-magnon low-temperature dependence of $\Gamma(k,\omega)$ is

$$\Gamma(k,\omega) \sim \Gamma_1 \cdot T, \tag{77}$$

$\Gamma_1 \to 0$ in the limit $k,\omega \to 0$.

7. CONCLUSIONS

The s-f model Hamiltonian, as given in the text, is the simplest theoretical model for a study of magnetically ordered semiconductors. In this paper we have shown that the IGF method gives a unified and self-consistent formalism for the complete description of the electronic and magnetic spectra including electron-magnon, magnon-magnon, and electron-electron inelastic scattering processes for the wide-band fm semiconductors. The importance of the presented results is certainly not in the application to any concrete substance. Their interest is of a more fundamental nature, for they give the complete picture of inelastic scattering of the quasiparticles in the systems with composite spectrum of excitations and can be applied to other models and systems. We point out also that the s-f model, extended by an additional Hubbard interaction term can be useful for some transition and rare-earth metals and their compounds (cf. [20]). The formalism developed in this paper can be extended to antiferromagnetic semiconductors and can be applied to the magnetic-polaron problem.

APPENDIX

Let us write the one-electron spectral density (19) in the form [17]:

$$g_{k\sigma}(\omega) = -\frac{1}{\pi} \frac{\Gamma_{k\sigma}(\omega)}{(\omega - \epsilon(k\sigma))^2 + \Gamma^2_{k\sigma}} \approx$$

$$\approx (1 - a_{k\sigma})\delta(\omega - \epsilon(k\sigma)) + \frac{1}{\pi} \frac{\Gamma_{k\sigma}(\omega)}{(\omega - \epsilon(k\sigma))^2}, \tag{A.1}$$

where the unknown constant $a_{k\sigma}$ is defined by the condition

$$\int_{-\infty}^{\infty} g_{k\sigma}(\omega)\, d\omega = 1. \tag{A.2}$$

Then, within this approximation for the average electron occupation numbers we get

$$n_\sigma = \frac{1}{N} \sum_k n(\epsilon(k\sigma)) = \frac{I^2}{N^2} \sum_{kq} \frac{K_q^{zz}}{(\epsilon^\circ(k+q\sigma) - \epsilon(k\sigma))^2} \times$$

$$\times \{ n(\epsilon^\circ(k+q\sigma) - n(\epsilon(k\sigma)) \} +$$

$$+ \frac{I^2}{N^2} \sum_{kq} \frac{K_q^{-\sigma\sigma} + z_\sigma \frac{2\langle S_0^z\rangle}{\sqrt{N}} n[\epsilon^\circ(k+q,-\sigma) - z_\sigma \omega(q)]}{(\epsilon^\circ(k+q,-\sigma) - z_\sigma \omega(q) - \epsilon(k\sigma))^2} \times \tag{A.3}$$

$$\times \{ n[\epsilon^\circ(k+q,-\sigma) - z_\sigma \omega(q)] - n(\epsilon(k\sigma))] \}.$$

As follows from eq. (A.3), the occupation numbers are determined in a self-consistent way. The first term in the right-hand side describes the effects of renormalizing the particle energies, and subsequent terms take into account the particle scattering by fluctuations of the magnetic moment in second order of I and explicitly the daming of the electronic states. It is interesting to note here that the approximation (A.1) conserves first four moments in second order of I and in the low-concentration limit. We have

$$M^{(0)} = \int_{-\infty}^{\infty} g_{k\sigma}(\omega)\, d\omega = 1\,, \tag{A.4}$$

$$M^{(1)} = \int_{-\infty}^{\infty} \omega g_{k\sigma}(\omega)\, d\omega = \epsilon^{\circ}(k\sigma), \tag{A.5}$$

$$M^{(2)} = \int_{-\infty}^{\infty} \omega^2 g_{k\sigma}(\omega)\, d\omega = (\epsilon^{\circ}(k\sigma))^2 +$$

$$+ \frac{I^2}{N} \sum_q \{ K_q^{zz} + K_q^{-+} + \frac{2 < S_0^z >}{\sqrt{N}} [\delta_\sigma + z_\sigma n(\epsilon^{\circ}(k+q, -\sigma))]\}. \tag{A.6}$$

Here

$$K_q^{-\sigma,\sigma} = \frac{2 < S_0^z >}{\sqrt{N}} [\delta_\sigma - \nu(\omega(q))]\,,$$

$$D_2 = S^2 - < S_0^z >^2\,; \quad D_{1\sigma} = S - z_\sigma < S_0^z >\,, \tag{A.7}$$

$$M^{(3)} = \int_{-\infty}^{\infty} \omega^3 g_{k\sigma}(\omega) d\omega = (\epsilon^{\circ}(k\sigma))^3 + I^2 [2\epsilon_k D_\sigma + \frac{I^2}{N} \sum_q \epsilon_{k+q} (K_q^{zz} + K_q^{-\sigma,\sigma})] +$$

$$+ I^3 \{2 z_\sigma < S_0^z > [(< S_0^z >^2) - R_0 - \frac{1}{2} D_{1\sigma}] - z_\sigma < S_0^z > D_2\,. \tag{A.8}$$

where

$$D_\sigma = \frac{1}{N} \sum_q (K_q^{zz} + K_q^{-\sigma,\sigma})\,, \tag{A.9}$$

$$M^{(4)} = \int_{-\infty}^{\infty} \omega^4 g_{k\sigma}(\omega)\, d\omega = (\epsilon^{\circ}(k\sigma))^4 +$$

$$+ I^2 \{ 3\epsilon_k^2 D_\sigma + 2\epsilon_k \frac{1}{N} \sum_q (K_q^{zz} + K_q^{-\sigma,\sigma}) \epsilon_{k+q} + \frac{1}{N} \sum_q (K_q^{zz} + K_q^{-\sigma,\sigma}) \epsilon_{k+q}^2 \} - \tag{A.10}$$

$$- I^3 \{ 4 z_\sigma < S_0^z > \epsilon_k D_\sigma + 2 z_\sigma \frac{1}{N} \sum (K_q^{zz} + K_q^{-\sigma,\sigma}) \epsilon_{k+q} +$$

$$+ 2 z_\sigma < S_0^z > \frac{1}{N} \sum_q (K_q^{zz} - K_q^{-\sigma,\sigma}) \epsilon_{k+q} \} + I^4 \{ 4 < S_0^z >^2 D_\sigma \}\,.$$

Thus, the first four moments calculated with the spectral density (A.1) coincide with the exact moments in second order of I in the low-concentration limit.

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Самосогласованная теория элементарных возбуждений с затуханием в системах со сложным спектром. Ферромагнитные полупроводники

Развита последовательная самосогласованная теория взаимного влияния электронной и спиновой подсистем в рамках s–f модели магнитного полупроводника. Вычисления проведены с использованием метода неприводимых функций Грина, в рамках которого выводятся уравнение Дайсона и точное выражение для массового оператора. Развитый подход позволяет естественным образом вычислить квазичастичный спектр и затухание в случае, когда система имеет сложный многоветвевой спектр. Описаны процессы электрон-электронного, электрон-магнонного и магнон-магнонного рассеяния, причем учитывается наличие акустической и оптической магнонных ветвей, а также континуум стонеровских возбуждений. Недавний результат работы /13/ следует из нашей теории в низшем приближении.

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Self-Consistent Theory of Elementary Excitations with Damping in the Systems with Many-Branch Spectrum. Ferromagnetic Semiconductors

A unified self-consistent theory is given of the mutual influence of electronic and spin subsystems for the s-f model of ferromagnetic semiconductors. The calculations are based on the novel approach of the thermodynamic two-time Green function method. This approach consists in the introduction of the "irreducible" Green functions and derivation of the exact Dyson equation and exact self-energy operator. We show that the IGF method gives a unified and natural approach for the calculation of elementary excitations and their damping in the systems with composite, many-branch quasiparticle spectra. By this method we calculate the full electronic and magnetic quasiparticle spectra of the s-f model of the magnetic semiconductor by taking explicitly into account magnon-magnon, electron-magnon, and electron-electron lifetimes including the acoustic and optical branches to the magnon spectrum as well as a Stoner-like continuum of excitations. The recent Babcenco and Cottam /13/ results follow from our theory in the lowest-order approximation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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