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EQUATION OF STATE IN THE QUANTAL CROSSOVER REGION FOR X-Y MODEL WITH LONGITUDINAL MAGNETIC FIELD

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The critical properties of the quantum-mechanical X–Y model in a longitudinal magnetic field were investigated by several authors/1–3/. As a finite temperature these properties correspond to those of the classical X–Y model/2,3/, but when the temperature is equal to zero they are characterized by new specifically quantum-mechanical exponents/1/. Considering the critical line $\Gamma_c = \Gamma_c(T)$ one can expect quantal-to-the-classical crossover behaviour in the vicinity of the multicritical point $[T = 0, \Gamma_c = 0]$. Such a crossover has been described by the Hartree-Fock approximation/1/ and recently by the modified field-theoretic renormalization-group (R.G.) method/4/. This method, previously used to the quantal crossover of the Ising model in a transverse field/5–8/, contains some additional renormalization, which removes singularities as $T \to 0$. In the papers treating quantal crossover in the X–Y model/1–4/, only the situation, when $\Gamma > \Gamma_c$, has been investigated and the order parameter, which is transverse magnetization, was found to be equal to zero.

In this letter we report some results concerning the equation of state in the crossover region for the spatial dimensions $2 < d < 4$ when $\Gamma = \Gamma_c$ and a nonzero perpendicular magnetic field B is applied. The spin operator Hamiltonian of our system is the following:

$$H = -\Gamma \sum_i S_i^z - \frac{1}{2}\sum_{ij} I_{ij} S_i^+ S_j^- - B^+ \sum_i S_i^- - B^- \sum_i S_i^+ , \tag{1}$$

where $S_i^{\pm} = S_i^x \pm i S_i^y$, S_i^{α} $(\alpha = x, y)$ is the α-th component of the spin operator referred to the i-th site of d-dimensional simple hypercubic lattice, $B^{\pm} = B_x \pm i B_y$, B_{α} $(\alpha = x, y)$ is the α-th component of the applied transverse field and I_{ij} denotes the exchange integral.

The Landau-Ginzburg-Wilson functional with the Matsubara frequencies can be written as follows:

$$\begin{aligned} S([\phi], H^+ H^-) = & \sum_{m=-\infty}^{\infty} \int_{|\vec{p}| < \Lambda} (\vec{p}^{\,2} - imt + r_0)\, \phi^*_{\vec{p},m}\, \phi_{\vec{p},m} \\ & + \frac{u_0 t}{4} \sum_{m_1} \cdots \sum_{m_4} \int_{|\vec{p}_1|<\Lambda} \cdots \int_{|\vec{p}_4|<\Lambda} \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\, \delta_{m_1+m_2,\, m_3+m_4} \\ & \times \phi^*_{\vec{p}_1 m_1} \phi^*_{\vec{p}_2 m_2} \phi_{\vec{p}_3 m_3} \phi_{\vec{p}_4 m_4} + H^+ \phi_{\vec{0},0} + H^- \phi^+_{\vec{0},0} , \end{aligned} \tag{2}$$

where

$$\int_{|\vec{p}|<\Lambda} \ldots = \frac{1}{(2\pi)^d} \int_{|\vec{p}|<\Lambda} d^d\vec{p}; \quad \delta(\vec{p}) = (2\pi)^d \delta^{(d)}(\vec{p}),$$

$\Lambda = \pi/a$ is momentum cut-off, a denotes the lattice constant, r_c is linear in the longitudinal field deviation, $h = (\Gamma - \Gamma_c)/\Gamma_c$, $t \sim T\Lambda^2$ $u_0 \sim \Lambda^{2-d}$ is a constant for small T, $H^{\pm} \sim t^{1/2}$ $B^{\pm}$ and $\phi_{\vec{p},m}$ is the Fourier transform of the classical spin field $\phi_i(\tau)$ depending on the Matsubara time $0 \le \tau \le \beta$.

Now we introduce the functions $W(H^+, H^-)$ and $\Gamma(M^+, M^-)$ corresponding to the free energy and thermodynamic Gibbs potential per unit volume, respectively

$$W(H^+, H^-) = \frac{1}{V} \ln \int d(\phi)\, e^{-S([\phi], H^+, H^-)}, \tag{3}$$

$$\Gamma(M^+, M^-) = -W + H^+M^- + M^-H^+ = \Gamma(M), \tag{4}$$

where $\int d(\phi)\ldots$ denotes the integration over fields $\phi_{\vec{p},m}$,

$$M^{\pm} = \frac{\partial W}{\partial H^{\pm}} \sim \frac{1}{N} \sum_i \langle S_i^{\pm} \rangle \tag{5}$$

and $M = |M^{\pm}| = (M_x^2 + M_y^2)^{1/2}$. Equation of state

$$H = \frac{\partial \Gamma}{\partial M}, \tag{6}$$

where $H = |H^{\pm}|$, formulated in terms of nonrenormalized theory, cannot be considered in the limit $\Lambda \to \infty$ or $t \to 0$ since the singular terms arise. For that reason we pass to the renormalized theory introducing new fields $\phi^R_{\vec{p},m}$, dimensionless coupling constant g and renormalization momentum μ, cf., e.g. [4]. The set of transformations $(g, \mu) \to (\bar{g}, \bar{\mu})$ under which physical content of the theory remains invariant is called renormalization group (R.G.). We define $\phi_{\vec{p},m}$, g, μ and R.G. operation as follows

$$\phi^R_{\vec{p},m} = f^{-1/2}(t/\mu^2)\, Z_3^{-1/2} \phi_{\vec{p},m}, \tag{7}$$

$$u_0 t = \mu^{\epsilon} f(t/\mu^2) Z_1 Z_3^{-2} = \bar{\mu}^{\epsilon} f(t/\bar{\mu}^2) \bar{Z}_1 \bar{Z}_3^{-2}, \tag{8}$$

and

$$r - r_c = \mu^2 Z_2 h = \bar{\mu}^2 \bar{Z}_2 h e^{\ell}, \tag{9}$$

where $r_{oc} = r_o|_{h=0}$, $Z_i(g, t/\mu^2, \Lambda/\mu)$ (i = 1,2,3) are renormalization constants determined by conditions given in ref. [4], ℓ denotes the R.G. parameter, $\epsilon = 4-d$, $\bar{Z}_i = \bar{Z}_i(\bar{g}, t/\mu^2, \Lambda/\mu)$ (i = 1,2,3) and $\bar{f} = f(t/\mu^2)$, where f(x) is an arbitrary function which has the following asymptotic properties

$$\lim_{x\to 0} f(x) \sim x, \qquad \lim_{x\to\infty} f(x) = 1. \tag{10}$$

The introduction of $f(t/\mu^2)$ corresponds to the additional renormalization removing the singularities as $t \to 0$. In order to obtain the crossover scaling form of the equation of state, only asymptotic properties (10) of $f(t/\mu^2)$ are relevant. Now, let us define the renormalized dimensionless magnetization

$$M_R = Z^{-1/2} f^{1/2}(t/\mu^2)\, \mu^{1-\frac{d}{2}} M \tag{11}$$

and function

$$\Gamma_R = f(t/\mu^2)\, \mu^d \Gamma \tag{12}$$

with Γ defined by eq. (4). Using the R.G. operation we can write the equation of state as follows

$$H_R = \frac{\partial \Gamma_R(M_R)}{\partial M_R} = M_R F(gM_R^2, g, h, t/\mu^2) = (\bar{Z}_3/Z_3)^{-1/2} (\bar{f}/f)^{1/2} (\bar{\mu}/\mu)^{1+\frac{d}{2}} \bar{M}_R F(\bar{g}\bar{M}_R^2, \bar{g}, he^{\ell}, t/\bar{\mu}^2), \tag{13}$$

where

$$H_R = Z_3^{1/2} f^{1/2}(t/\mu^2)\, \mu^{-1-\frac{d}{2}} H \tag{14}$$

and

$$\bar{M}_R = M_R (\bar{f}/f)^{1/2} (\bar{Z}_3/Z_3)^{-1/2} (\bar{\mu}/\mu)^{1-\frac{d}{2}}. \tag{15}$$

Because we consider the equation of state for $\Gamma = \Gamma_c$ $(h = 0)$ it is convenient to use M_R as the basic scaling variable imposing the following condition

$$\bar{g}(\ell) \bar{M}_R(\ell) = 1, \tag{16}$$

which leads to the equation

$$M_R \frac{d\bar{\mu}}{dM_R} = \frac{2\bar{g}\bar{\mu}}{\bar{g}[2 - \epsilon(t/\bar{\mu}^2) + \eta(\bar{g}, t/\bar{\mu}^2)] - \beta(\bar{g}, t/\bar{\mu}^2)}, \tag{17}$$

where

$$\eta(\bar{g}, t/\bar{\mu}^2) = \bar{\mu}\frac{\partial \ln Z_3}{\partial \bar{\mu}} + \beta(\bar{g}, t/\bar{\mu}^2)\frac{\partial \ln \bar{Z}_3}{\partial \bar{g}} \qquad (18)$$

fulfills the following equations (cf. /4/):

$$\mu\frac{d\bar{g}}{d\bar{\mu}} = \beta(\bar{g}, t/\bar{\mu}^2) \qquad (19)$$

and

$$\epsilon(t/\bar{\mu}^2) = \epsilon - 2(t/\bar{\mu}^2)\frac{d\ln f(t/\bar{\mu}^2)}{d(t/\bar{\mu}^2)}. \qquad (20)$$

In our method $\epsilon(t/\bar{\mu}^2)$ can be considered as a formal expansion parameter assuming the following asymptotic values $\epsilon = 4-d$ when $t/\bar{\mu}^2 \to \infty$ and $\epsilon' = 2-d$ when $t/\bar{\mu}^2 \to 0$. The solution of eq. (17) for $\epsilon << 1$ has the following form in the scaling limit ($M_R \to 0$, $t/\bar{\mu}^2 \to 0$, $z = \text{const}$)

$$\bar{\mu}(M_R) = \mu M_R^{\nu_q/\beta_q} X(z), \qquad (21)$$

where $\nu_q = 1/2$ and $\beta_q = 1/2$ are multicritical exponents

$$X(z) = [1 + (1 - g/g^*_q)^{-1}(g/g^*_{cl})z]^{-1/2}, \qquad (22)$$

$$g^*_{cl} = \frac{4}{5}\frac{(2\pi)^4}{\Omega_4}\epsilon + O(\epsilon^2), \qquad (23)$$

Ω_d is solid angle in d-dimension.

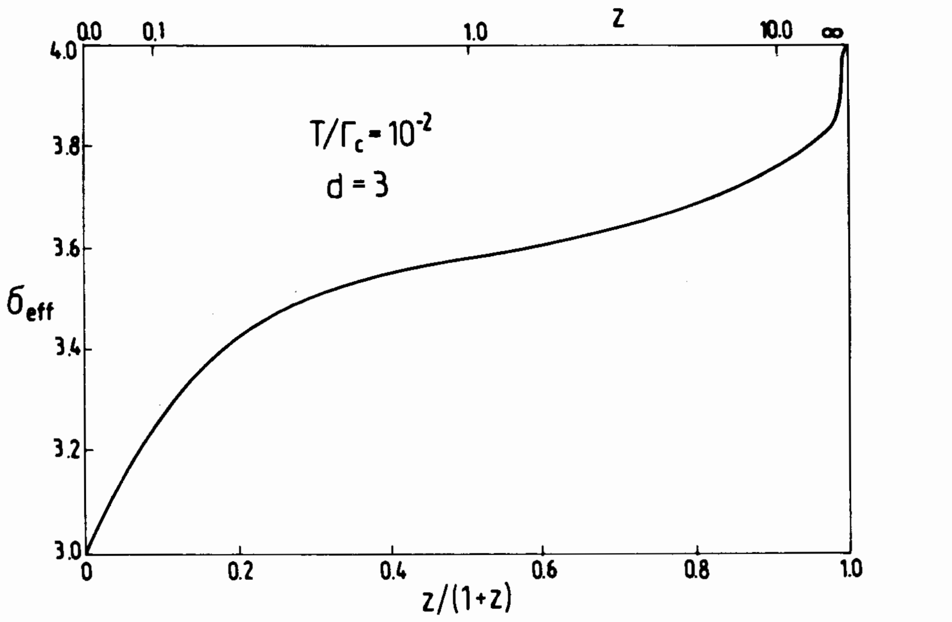
$$g^*_q = 4(\epsilon - 2) + O[(\epsilon - 2)^2], \qquad (24)$$

$z = (t/\mu^2)M^{-\phi/\beta_q}$ and $\phi = \epsilon/2$ is the crossover exponent /4/. Here g^*_d (23) and g^*_q (22) denote the coupling constants associated with nontrivial fixed points classical and quantum, respectively. Notice, that the multicritical behaviour for $2 < d < 4$ is governed by the Gaussian fixed point $g^*_q = 0$ and g^*_q (24) in eq. (22) plays only the role of some constant. We can obtain the equation of state evaluating $\Gamma_R(M_R)$ in the one-loop approximation. In the scaling limit we get

$$H_R = M^3\Psi(z), \qquad (25)$$

where

$$\Psi(z) = g(1 - g/g^*_q)^{-1}X^2(z)[1 + \frac{gS_d}{4}(1 - g/g^*_q)^{-1} \times \\ \times (5 - 7\ln 2 + 4.5\ln 3)zX^2(z)]. \qquad (26)$$



The effective magnetization exponent δ_{eff} vs $z/(1+z)$ for $t/\mu^2 \approx T/\Gamma_c = 10^{-2}$.

A convenient visual description of the crossover can be obtained by introducing the notion of an effective exponent δ_{eff} defined as follows:

$$H = M^{\delta_q}\Psi(z) = M^{\delta_{eff}}\Psi(0).$$

This gives

$$\delta_{eff}(z, t/\mu^2) = 3 + \frac{\ln\Psi(z)/\Psi(0)}{\ln M_R},$$

where $M_R = (t/\mu^2)^{\beta_q/\phi} z^{\beta_q/\phi}$.

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Люкерска-Валясек К. E17-83-857

Уравнение состояния в области квантового кроссовера для модели X-Y в продольном магнитном поле

Метод теоретико-полевой ренормализационной группы, использованный ранее для описания квантового кроссовера X-Y модели в продольном магнитном поле, предложен для исследования уравнения состояния этой модели. В случае $\Gamma=\Gamma_c$ квантовый кроссовер имеет форму $H_R=M_R^3\psi(z)$, где H_R и M_R означают поперечное поле и магнетизацию, соответственно, $z \sim TM_R^{-\phi/\beta_q}$, где ϕ и β_q есть кроссоверские и мультикритические экспоненты.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Lukierska-Walasek K. E17-83-857

Equation of State in the Quantal Crossover Region for X-Y Model with Longitudinal Magnetic Field

The field-theoretic renormalization-group method proposed recently to describe the quantum crossover behaviour in the X-Y model in a longitudinal magnetic field is now applied to the investigation of the equation of state for this model. For $\Gamma=\Gamma_c$ the quantal crossover behaviour of the form $H_R=M\ \psi(z)$ is obtained, where H_R and M_R denote transverse field and magnetization, respectively, $z \sim TM_R^{-\phi/\beta_q}$, while ϕ, β_q are the crossover and multicritical exponents.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983