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TWO-PHOTON PROCESS IN THREE-LEVEL SYSTEM

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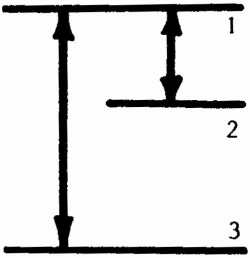
A number of recent papers (e.g., see [1-5]) was dedicated to a careful consideration of the problem of a single three-level "atom" interacting with two modes of electromagnetic field. Such a consideration is in a close connection with the problem of construction of a theory for a two-mode laser [2]. The exact Schrödinger wave function was obtained in ref.[2] for such a system with a special initial condition. In other papers [1-3,5] the so-called semiclassical expression for the Rabi frequency was used. It should be noted that a quantum expression for the Rabi frequency was obtained before for a two-level one-photon system in the rigorous investigation of Jaynes and Cummings [6]. Their result was generalized to the case of a two-level multiphoton system by Buck and Sukumar [7].



In the present paper we shall examine the model of a three-level atom with allowed transitions $|3\rangle \to |1\rangle$ and $|3\rangle \to |2\rangle$ and forbidden transition $|2\rangle \to |1\rangle$ (figure) interacting with two resonant modes ω_1, ω_2 of electromagnetic field. The exact dynamics will be obtained here for operators of the level filling and of occupation number of photon modes.

The system under consideration can be described by a Hamiltonian of the form

$$H = H_A + H_F + H_{AF}. \tag{1}$$

Here H_A is the energy of a free three-level atom $H_A = \sum_{j=1}^{3} h\Omega_j R_{jj}$.

Operator R_{jj} describes the filling of a j-th level with energy $h\Omega_j$. Operator H_F in (1) presents the energy of two resonant modes of a free electromagnetic field $H_F = h\omega_1 a_1^+ a_1 + h\omega_2 a_2^+ a_2$, where $\omega_1 = \Omega_3 - \Omega_1$, $\omega_2 = \Omega_3 - \Omega_2$ and $a_\alpha^+ (a_\alpha)$ is the creation (anihilation)operator for a photon of α-th mode. In the dipole approximation for the energy of atom-field interaction we have $H_{AF} = -ihg_1(a_1 R_{31} - a_1^+ R_{13}) - ihg_2(a_2 R_{32} - a_2^+ R_{23})$, where $g_\alpha = \mathrm{const}$ and operator R_{ij} describes the transition from state $|j\rangle$ to state $|i\rangle$ $(i \neq j)$. They obey the following rules

$$[R_{ij}, R_{k\ell}] = R_{i\ell}\delta_{kj} - R_{kj}\delta_{i\ell}, \quad R_{ij}R_{k\ell} = R_{i\ell}\delta_{kj} \tag{2}$$

and are connected with the generators of SU(3) group. The states $|j\rangle$ form the basis of state space $H_A|j\rangle = h\Omega_j|j\rangle$, $\langle i|j\rangle = \delta_{ij}$. It is obvious that

$$\sum_j R_{jj} = 1. \tag{3}$$

Let us consider the Heisenberg equation of motion for operators $R_{jj}(t)$:

$$\dot{R}_{11}(t) = g_1 A_1(t) \equiv g_1[a_1(t) R_{31}(t) + a_1^{+}(t) R_{13}(t)],$$
$$\dot{R}_{22}(t) = g_2 A_2(t) \equiv g_2[a_2(t) R_{32}(t) + a_2^{+}(t) R_{23}(t)]. \qquad (4)$$

The third equation follows from (3). Let $N_\alpha \equiv a_\alpha^{+} a_\alpha$ be the occupation number operator for α-th mode of the field. Then

$$\dot{N}_\alpha(t) = g_\alpha A_\alpha(t), \qquad \alpha = 1,2. \qquad (5)$$

From equations (4), (5) it follows that

$$N_\alpha(t) + R_{\alpha\alpha}(t) = M_\alpha, \qquad (6)$$

where operator M_α is independent of time t. Now the Heisenberg equations for operators $A_\alpha(t)$ can be obtained in the form

$$\dot{A}_1(t) = 2g_1(M_1 + 1)[1 - 2R_{11}(t) - R_{22}(t)] - g_2 B(t),$$
$$\dot{A}_2(t) = 2g_2(M_2 + 1)[1 - 2R_{22}(t) - R_{11}(t)] - g_1 B(t), \qquad (7)$$

where operator $B \equiv a_1 a_2^{+} R_{21} + a_1^{+} a_2 R_{12}$ obeys the following equation of motion

$$\dot{B}(t) = \sum_\alpha g_\alpha(M_\alpha + 1) A_\alpha(t). \qquad (8)$$

Equations (4), (5), (7), (8) form a closed system. They have yet another integral of motion

$$g_1 g_2 B(t) - g_1^2(M_1 + 1) R_{22}(t) - g_2^2(M_2 + 1) R_{11}(t) = K, \qquad (9)$$

where operator K is independent of time t. Operators M_α and K are commuting with each other.

Let us now differentiate each of equations (4) with respect to time. Then taking into account expressions (7) and (8) we receive

$$\ddot{R}_{11}(t) + [4g_1^2(M_1 + 1) + g_2^2(M_2 + 1)] R_{11}(t) + 3g_1^2(M_1 + 1) R_{22}(t) = 2g_1^2(M_1 + 1) - K,$$
$$\ddot{R}_{22}(t) + [4g_2^2(M_2 + 1) + g_1^2(M_1 + 1)] R_{22}(t) + 3g_2^2(M_2 + 1) R_{11}(t) = 2g_2^2(M_2 + 1) - K. \qquad (10)$$

One can consider these expressions as a system of differential equations for bounded quantum oscillators.

At first, let us consider a simple single-photon case with $g_2 = 0$. Then from (4), (5), (6), and (9) we have that R_{22} is independent of t and that $K = -g_1^2(M_1 + 1) R_{22}$. Therefore the

first equation in (10) takes the form $\ddot{R}_{11}(t) + 4g_1^2(M_1 + 1)R_{11}(t) = 2g_1^2(M_1 + 1)(1 - R_{22})$. Then for the operator filling difference of levels one and three $S_{31}(t) \equiv [R_{33}(t) - R_{11}(t)]/2$ we get $\ddot{S}_{31}(t) + 4g_1^2(M_1 + 1)S_{31}(t) = 0$. Its solution is $S_{31}(t) = S_{31}(0)\cos 2\lambda_1 t + (\dot{S}_{31}(0)/2\lambda_1)\sin 2\lambda_1 t$, where $\lambda_1 = g_1(M_1 + 1)^{1/2}$. This is the so-called quantum expression for the Rabi frequency obtained by Jaynes and Cummings /6/. Thus, our system of equations (10) for a three-level two-photon system leads to the known result for a two-level single-photon system in the special case of one resonant photon.

Now we return to the consideration of the general case of a three-level atom with two-phonot interaction. Taking into account the commutativity of operators M_α and K we can present the system (10) in the following form:

$$R_{11}(t) = \mu_1 \cos\lambda t + \beta_1 \sin\lambda t + \lambda_1^2[\mu_2 \cos 2\lambda t + \beta_2 \sin 2\lambda t] + P_{11}(0), \qquad (11)$$
$$R_{22}(t) = -\mu_1 \cos\lambda t - \beta_1 \sin\lambda t + \lambda_2^2[\mu_2 \cos 2\lambda t + \beta_2 \sin 2\lambda t] + P_{22}(0).$$

Here λ_α can be considered as the quantum expression for the Rabi frequency in the system (1). They are defined as the fundamental values for the matrix of linear coefficients of system (10). So $\lambda_\alpha = g_\alpha\sqrt{M_\alpha + 1}$, $\lambda \equiv \sqrt{\sum_\alpha \lambda_\alpha^2}$. Operators $P_{\alpha\alpha}$ define the "quantum point of equilibrium": $P_{\alpha\alpha} = [\lambda_\alpha^2 \lambda^2 + (3\lambda_\alpha^2 - 2\lambda^2)K]/(2\lambda^4)$. And operators μ_α, β_α are

$$\mu_1 = \{\lambda^2[\lambda_2^2 R_{11}(0) - \lambda_1^2 R_{22}(0)] + (\lambda_2^2 - \lambda_1^2)K\}/\lambda^4,$$

$$\mu_2 = \{\lambda^2[1 - 2R_{33}(0)] + K\}/(2\lambda^4),$$

$$\beta_1 = [\lambda_2^2 \dot{R}_{11}(0) - \lambda_1^2 \dot{R}_{22}(0)]/\lambda^3,$$

$$\beta_2 = [\dot{R}_{11}(0) + \dot{R}_{22}(0)]/(2\lambda^3).$$

Now from the conservation laws (3) and (6) one can obtain

$$R_{33}(t) = -\lambda^2[\mu_2(\cos\lambda t - 1) + \beta_2 \sin 2\lambda t] + R_{33}(0),$$
$$N_1(t) = \mu_1(\cos\lambda t - 1) + \beta_1 \sin\lambda t + \lambda_1^2[\mu_2(\cos 2\lambda t - 1) + \beta_2 \sin 2\lambda t] + N_1(0), \qquad (12)$$
$$N_2(t) = -\mu_2(\cos\lambda t - 1) - \beta_1 \sin\lambda t + \lambda_2^2[\mu_2(\cos 2\lambda t - 1) + \beta_2 \sin 2\lambda t] + N_2(0).$$

Expressions (11), (12) present the exact result for operators of the level filling and of the occupation number of photon modes. Some conclusions of paper /2/ can also be obtained on the basis of expressions (11), (12). We intend to examine some consequences of our result in a subsequent more detailed paper.

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Боголюбов Н.Н. /мл./, Фам Ле Киен, Шумовский А.С. E17-83-829
О двухфотонном процессе в трехуровневой системе

Точно исследовано динамическое поведение населенностей уровней и числа заполнения фотонных мод. Получено квантовое выражение для частоты Раби.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Two-Photon Process in Three-Level System

The dynamical behaviour of level filling and occupation numbers of photon modes is examined rigorously. A "quantum expression" for the Rabi frequency is obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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