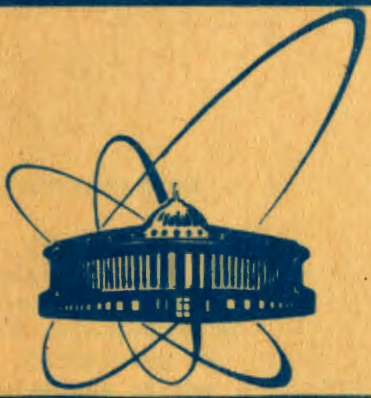

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PARAMAGNON EFFECT IN DISORDERED SUPERCONDUCTING ALLOYS

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1. INTRODUCTION

The theory of strong-coupling superconductivity in transition metal alloys encounters some peculiarities: narrow d-bands and Stoner-enhancement factors. Electrons, phonons, and paramagnons are affected by disorder.

Concerning the electron-phonon interaction microscopic calculations of the superconducting transition temperature T_c can be based on Eliashberg /1/ -type equations formulated in the Wannier representation. Without spin fluctuations there are several approaches /2-7/ which differ in performing the configurational averaging. The consequent application of the CPA (coherent potential approximation) to substitutionally random alloys $A_c B_{1-c}$ is necessary to treat the impurity scattering, where the standard weak-coupling T_c result /8/ must be obtained as a limit.

In nearly ferromagnetic systems paramagnons must be taken into account by the spin susceptibility, i.e., diagrammatically via electron-hole bubbles. The Berk-Schrieffer equations /9/ are the theoretical background for determining T_c. For pure metals (clean itinerant magnets) it has been investigated, e.g., in /10/ (for a review see /11/) as enhanced spin fluctuations counteract superconductivity. In this paper we are dealing with the influence of paramagnons on T_c in disordered alloys.

The present study of superconductivity in transition metal alloys is based on the following model Hamiltonian for the electron-phonon system at a fixed configuration of ions:

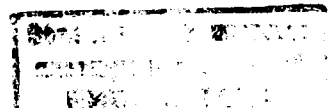
$$H = H_e + H_{ph} + H_{e\,ph}, \quad \text{where} \tag{1}$$

$$H_e = \sum_{i\sigma} \epsilon_i n_{i\sigma} + \sum_{\substack{ij\sigma \\ (i\neq j)}} t_{ij} c^+_{i\sigma} c_{j\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} \equiv H_V + H_{ee}, \tag{2}$$

$$H_{ph} = \sum_{\substack{i \\ \alpha}} \frac{p^2_{i\alpha}}{2M_i} + \frac{1}{2} \sum_{\substack{ij \\ \alpha\beta}} \Phi_{ij}^{\alpha\beta} u_{i\alpha} u_{i\beta}, \tag{3}$$

$$H_{eph} = \sum_{\substack{ij\sigma \\ (i\neq j)}} \vec{g}_{ij} (\vec{u}_i - \vec{u}_j) c^+_{i\sigma} c_{j\sigma}. \tag{4}$$

The random Hubbard model (2), the free phonon part (3), and the electron-phonon interaction (4) have been written down in the tight-binding representation /3,5,6/. Here the atomic poten-



tial ϵ_i, the hopping integrals t_{ij}, the intra-site Coulomb repulsion U_i, the ionic mass M_i, the harmonic force constants $\Phi^{\alpha\beta}_{ij}$, and the dominant two-centre integrals $\vec{g}_{ij} = \frac{q_0^i + q_0^j}{2} t_{ij} \frac{\vec{R}_j - \vec{R}_i}{|\vec{R}_j - \vec{R}_i|}$ with the Slater coefficient q_0^i are assumed to be configuration-dependent. $\vec{R}_i$ denotes the equilibrium position vector at the lattice site i.

2. ELECTRON SELF-ENERGY IN THE PRESENCE OF DISORDER

Define the electron Green function of the superconductor in Matsubara-Nambu technique as

$$\hat{G}_{ij}(\tau-\tau') = -\begin{pmatrix} \langle T_\tau c_{i\uparrow}(\tau)\bar{c}_{j\uparrow}(\tau')\rangle & \langle T_\tau c_{i\uparrow}(\tau)c_{j\downarrow}(\tau')\rangle \\ \langle T_\tau c_{i\downarrow}(\tau)c_{j\uparrow}(\tau')\rangle & \langle T_\tau c_{i\downarrow}(\tau)c_{j\downarrow}(\tau')\rangle \end{pmatrix}, \tag{5}$$

where "^" means 2x2 matrix, $\langle \ldots \rangle$ denotes the thermal average. The "temporal" Fourier transform $G(z)$ taken as a resolvent at $z = z_n = i(2n+1)\pi T$ obeys the Dyson equation

$$\hat{G}(z) = (z\tau_0 - H_V\tau_3 - \hat{\Sigma}(z))^{-1} \equiv \hat{G}[\hat{\Sigma}], \tag{6}$$

where τ_0 to τ_3 are the Pauli matrices; H_V stands for the random one-particle contribution of (2), i.e., $[H_V]_{ij} = \epsilon_i\delta_{ij} + t_{ij}(1-\delta_{ij})$.

The electron self-energy can be decomposed into

$$\hat{\Sigma} = \hat{\Sigma}^{ph} + \hat{\Sigma}^{m} \tag{7}$$

due to the electron-phonon and electron-paramagnon interactions, respectively. For simplicity, the static local Coulomb term is omitted. In perturbation theory the phonon contribution is given by

$$\hat{\Sigma}^{ph}_{ij}(z_n) = -T\sum_{n'}\sum_{\substack{\bar{i}\bar{j}\\ \alpha\beta}} g^{\alpha}_{i\bar{i}} g^{\beta}_{\bar{j}j} D^{\alpha\beta}_{i\bar{i},\bar{j}j}(z_n - z_{n'})\tau_3 \hat{G}_{\bar{i}\bar{j}}(z_{n'})\tau_3, \tag{8a}$$

$$\approx T\sum_{n'}\sum_{\substack{\bar{i}(\neq i)\\ \alpha}} g^{\alpha}_{i\bar{i}} g^{\alpha}_{\bar{i}i}(D_{ii}(z_n - z_{n'}) + D_{\bar{i}\bar{i}}(z_n - z_{n'}))\tau_3 \hat{G}_{\bar{i}\bar{i}}(z_{n'})\tau_3\delta_{ij}, \tag{8b}$$

where $D_{ij}(\tau-\tau') = -\langle T_\tau u_i(\tau)u_j(\tau')\rangle$ denotes the phonon Green function. Specially, (8b) involves the so-called contact approximation /3-7/. Cubic symmetry is presumed so that only $D = D^{\alpha\alpha}$ remains. The diagram analysis gives rise to the paramagnon part in the random site version (for pure systems cf. /11/)

$$\hat{\Sigma}^{m}_{ij}(z_n) = U_i U_j \sum_{n'} \chi_{ij}(z_n - z_{n'})\tau_1 \hat{G}_{ij}(z_{n'})\tau_1, \tag{9}$$

where the spin susceptibility reads $\chi_{ij}(\tau-\tau') = \langle T_\tau S_i^+(\tau) S_j^-(\tau')\rangle = \langle T_\tau \bar{c}_i(\tau)c_i(\tau')\bar{c}_j(\tau')c_j(\tau')\rangle$.

Introducing the spectral representations of the Green functions in (8b) and (9) in a standard way /11/ and performing analytical contibuation to retarded functions by letting $z_n = i\omega_n \to \omega + i\epsilon$ we obtain

$$\hat{\Sigma}^{ph}_{ij}(\omega+i\epsilon) = \sum_{i(\neq i)} \vec{g}^2_{ii} \iint_{-\infty}^{\infty} \frac{d\omega_1\, d\omega_2}{2\pi^2} \left(\frac{\mathrm{cth}\frac{\omega_1}{2T} + \mathrm{th}\frac{\omega_2}{2T}}{\omega - \omega_1 - \omega_2 + i\epsilon}\right)(\mathrm{Im}\, D_{ii}(\omega_1 + i\epsilon) + \mathrm{Im}\, D_{\bar{i}\bar{i}}(\omega_1 + i\epsilon))\tau_3 \mathrm{Im}\, \hat{G}_{\bar{i}\bar{i}}(\omega_2 + i\epsilon)\tau_3\delta_{ij}, \tag{10}$$

$$\hat{\Sigma}^{m}_{ij}(\omega+i\epsilon) = -\frac{U_i U_j}{2\pi^2} \iint_{-\infty}^{\infty} d\omega_1\, d\omega_2 \left(\frac{\mathrm{cth}\frac{\omega_1}{2T} + \mathrm{th}\frac{\omega_2}{2T}}{\omega - \omega_1 - \omega_3 + i\epsilon}\right) \mathrm{Im}\, \chi_{ij}(\omega_1 + i\epsilon)\tau_1 \mathrm{Im}\, \hat{G}_{ij}(\omega_2 + i\epsilon)\tau_1. \tag{11}$$

3. RANDOM BERK-SCHRIEFFER-TYPE EQUATIONS

The configuration-dependent self-energy (7) can be expressed in terms of the remormalization Z and the anomalous part Φ as

$$\hat{\Sigma}_{ij}(z) = z(1 - Z_{ij}(z))\tau_0 + \Phi_{ij}(z)\tau_1, \tag{12}$$

where the τ_3-term is neglected which may be included into H_V. Now we want to determine self-consistently an effective self-energy via the ansatz

$$\hat{\Sigma}_{eff}(z) = z(1 - \tilde{Z}(z))\tau_0 + \tilde{\Phi}(z)\tau_1 \tag{13}$$

which results in the following from two procedures, namely (i) configurational averaging and (ii) averaging over the Fermi surface (in $\vec{k}$-space).

By inserting Σ_{eff} in (6) instead of $\hat{\Sigma}$ we get /6,7/

$$\hat{G}[\hat{\Sigma}_{eff}] = \frac{1}{2}(G^0(\Omega) + G^0(-\Omega))\tau_3 + \frac{zZ}{2\Omega}(G^0(\Omega) - G^0(-\Omega))\tau_0 + \frac{\tilde{\Phi}}{2\Omega}(G^0(\Omega) - G^0(-\Omega))\tau_1 \tag{14a}$$

$$\approx \mathrm{Re}\, G^0(iO)\tau_3 + i\,\mathrm{Im}\, G^0(iO)\tau_0 + i\frac{\tilde{\Delta}(z)}{z}\mathrm{Im}\, G^0(iO)\tau_1 \tag{14b}$$

with $\Omega = \sqrt{(z\tilde{Z})^2 - \tilde{\Phi}^2}$. In deriving (14a) the identity $G^0(z_1)G^0(z_2) = -\frac{G^0(z_1) - G^0(z_2)}{z_1 - z_2}$ was used with the normal propagator

$$G^0(z) = (z - H_V)^{-1}. \tag{15}$$

Thus, (14a) holds for arbitrary randomness of H_V. Near T_c the linearization of (14a) with respect to the effective gap function $\Delta(z) = \frac{\tilde{\Phi}(z)}{\tilde{Z}(z)}$ leads to (14b), where $G^0(iO)$ is taken at the Fermi level being the energy origin.

In order to calculate T_c, we start with linearized equations for $\hat{\Sigma}_{ij}(z)$ according to (12). For this purpose we substitute (14b) into the r.h.s. of (10) and (11) and use (7) to find disordered linearised integral equations of the Berk-Schrieffer type

$$z(1 - Z_{ij}(z)) = -\int_{-\infty}^{\infty} [K_{ii}^{ph}(\omega_2, z)\,\delta_{ij} + K_{ij}^{m}(\omega_2, z)]\,d\omega_2, \tag{16}$$

$$\Phi_{ij}(z) = \int_{-\infty}^{\infty} [K_{ii}^{ph}(\omega_2, z)\,\delta_{ij} - K_{ij}^{m}(\omega_2, z)]\,\mathrm{Re}\,\frac{\tilde{\Delta}(\omega_2 + i\epsilon)}{\omega_2}\,d\omega_2, \tag{17}$$

$z = \omega + i\epsilon$, with the kernels

$$\begin{Bmatrix} K_{ii}^{ph}(\omega_2, z) \\ K_{ij}^{m}(\omega_2, z) \end{Bmatrix} = \int_0^{\infty} \frac{d\omega_1}{2} \begin{Bmatrix} \Gamma_i^{ph}(\omega_1) \\ \Gamma_{ij}^{m}(\omega_1) \end{Bmatrix} \left\{ \frac{\mathrm{cth}\frac{\omega_1}{2T} + \mathrm{th}\frac{\omega_2}{2T}}{\omega_1 + \omega_2 - z} - \frac{\mathrm{th}\frac{\omega_2}{2T} - \mathrm{cth}\frac{\omega_1}{2T}}{\omega_2 - \omega_1 - z} \right\} \tag{18}$$

involving the random spectral functions

$$\Gamma_i^{ph}(\omega) = \frac{1}{\pi^2} \sum_{i(\neq i)} \vec{g}_{ii}^{\,2} (\mathrm{Im}\, D_{ii}(\omega + i\epsilon) + \mathrm{Im}\, D_{\bar{i}\bar{i}}(\omega + i\epsilon))\, \mathrm{Im}\, G_{\bar{i}\bar{i}}^{0}(iO), \tag{19}$$

$$\Gamma_{ij}^{m}(\omega) = -\frac{U_i U_j}{\pi^2} \mathrm{Im}\, \chi_{ij}(\omega + i\epsilon)\, \mathrm{Im}\, G_{ij}^{0}(iO). \tag{20}$$

The locality of the phonon kernel K_{ii}^{ph} comes from the contact approximation; Γ_i^{ph} is the configuration-dependent analog of the spectral function $\alpha^2(\omega)F(\omega)$ [11]. However, the off-diagonality of K_{ij}^{m} must be retained to maintain the paramagnon concept. It should be pointed out that the r.h.s. of (17) already contains the effective pairing $\tilde{\Delta}(z)$ which will be determined self-consistently.

4. AVERAGING PROCEDURE

The question is how to evaluate $\hat{\Sigma}_{eff}(z)$. The averaging procedure used here does not consist in a simple replacing Σ_{ij} by $\bar{\hat{\Sigma}}_{ij}$, where the bar "-" denotes the configuration average.

Within a single-site averaging scheme we approximate

$$\hat{\Sigma}_{ij} = \hat{\Sigma}_{ii}\,\delta_{ij} + \bar{\hat{\Sigma}}_{ij}(1 - \delta_{ij}), \tag{21}$$

whereby the local character of the impurity scattering is emphasized. After inserting (21) into (6) (with (7)) we impose the self-consistency condition

$$\overline{\hat{G}[\hat{\Sigma}]} = \overline{(z\tau_0 - H_V\tau_3 - \hat{\Sigma}^{ph}(z) - \delta\hat{\Sigma}^{m}(z) - \langle\hat{\Sigma}^{m}(z)\rangle_{FS})^{-1}} =$$

$$= (z\tau_0 - H_V\tau_3 - \hat{\Sigma}_{eff}(z))^{-1} \equiv \hat{G}[\hat{\Sigma}_{eff}] \tag{22}$$

with the local perturbation $\delta\hat{\Sigma}_{ij}^{m} = (\hat{\Sigma}_{ii}^{m} - \bar{\hat{\Sigma}}_{ii}^{m})\delta_{ij}$, and $\langle\ldots\rangle_{FS}$ refers to the average over the Fermi surface.

On CPA basis the prescription (22) can be realized by ($P_i = |i\rangle\langle i|$)

$$\overline{\hat{G}[\hat{\Sigma}_{eff}]} = \overline{\hat{G}_i[\hat{\Sigma}_{eff}]\,(\tau_0 - (\hat{\Sigma}^{ph} + \delta\hat{\Sigma}^{m} - \hat{\Sigma}_{eff}^{ph} - \delta\hat{\Sigma}_{eff}^{m})P_i\,\hat{G}_i[\hat{\Sigma}_{eff}])^{-1}} \tag{23a}$$

or its linearized version

$$\overline{\hat{G}_i[\hat{\Sigma}_{eff}]\,(\hat{\Sigma}^{ph} + \delta\hat{\Sigma}^{m} - \hat{\Sigma}_{eff}^{ph} - \delta\hat{\Sigma}_{eff}^{m})\,P_i\,\hat{G}_i[\hat{\Sigma}_{eff}]} = 0 \tag{23b}$$

appropriate near T_c. Here the conditionally averaged propagator $\overline{G_i[\hat{\Sigma}_{eff}]}$ fulfils (14) with $G_i^0 = \mathcal{G} + \mathcal{G}T_i^0\mathcal{G}$ instead of G^0, where $\mathcal{G}(z) = \overline{G^0(z)}$, and T_i^0 is the single-site scattering operator. Altogether, we have $\hat{\Sigma}_{eff} = \hat{\Sigma}_{eff}^{ph} + \delta\hat{\Sigma}_{eff}^{m} + \langle\hat{\Sigma}^{m}\rangle_{FS}$ being site- and $\vec{k}$-independent, but z-dependent. Furthermore, the Ward-like identity

$$G_i^0(z_1)[z_1 - z_2 - \Sigma(z_1) + \Sigma(z_2) + \Sigma_i(z_1) - \Sigma_i(z_2)]\,G_i^0(z_2) = G_i^0(z_2) - G_i^0(z_1) \tag{24}$$

is employed to carry out averages resulting from (23). The normal coherent potential $\Sigma(z) = \sum_i \Sigma_i(z)$ enters into $\mathcal{G}(z) = (z - H_B^0 - \Sigma(z))^{-1}$, where H_B^0 is the periodic part of H_V.

4.1. Averaged Berk-Schrieffer-Type Equations

By means of (24) we find from (23) (for details see [7]) the site-diagonal contributions to $\hat{\Sigma}_{eff}(z)$ as the weighted averages

$$\tilde{Z}_{ii}(z) = F^{-1}(iO) \sum_{\nu=A,B} c_\nu F_\nu(iO)\, Z_{ii,i=\nu}(z), \tag{25}$$

$$\tilde{\Phi}_{ii}(z) = \rho^{-1}(0) \sum_{\nu} c_\nu \rho_\nu(0) \Phi_{ii,i=\nu}(z) \qquad (26)$$

with the notation: $F(z) = \mathcal{G}_{ii}(z)$, $F_\nu(z) = G^0_{i=\nu,ii}(z)$; $\rho(0) = -\frac{1}{\pi}\mathrm{Im}F(i0)$ and $\rho_\nu(0) = -\frac{1}{\pi}\mathrm{Im}F_\nu(i0)$ as the alloy and component (i.e., totally and partially averaged) densities of states at the Fermi level; $c_A = c$ and $c_B = 1 - c$. Further, $Z_{ii,i=\nu}$ and $\Phi_{ii,i=\nu}$ are available from (16) and (17) provided that a ν-atom occupies the i-th site in the sense of the terminal point approximation.

Consequently, we arrive at averaged counterparts to (16) and (17):

$$z(1 - \tilde{Z}(z)) = -\int_{-\infty}^{\infty} [\tilde{K}^{ph}_{I}(\omega_2, z) + \overline{K^m}(\omega_2, z) + \delta\tilde{K}^m_{I}(\omega_2, z)]\, d\omega_2, \qquad (27)$$

$$\tilde{\Delta}(z)\,\tilde{Z}(z) = \int_{-\infty}^{\infty} [\tilde{K}_{II}(\omega_2, z) - \overline{K^m}(\omega_2, z) - \delta\tilde{K}^m_{II}(\omega_2, z)] \frac{\mathrm{Re}\,\tilde{\Delta}(\omega_2 + i\epsilon)}{\omega_2}\, d\omega_2 . \qquad (28)$$

The phonon-mediated kernels $\tilde{K}^{ph}_{I}$ and $\tilde{K}^{ph}_{II}$ are given by (18) if Γ^{ν}_{i} is replaced by $\tilde{\Gamma}^{ph}_{I}(\omega) = F^{-1}(i0) \sum_{\nu} c_\nu F_\nu(i0)\, \Gamma^{ph}_{\nu}(\omega)$ and $\tilde{\Gamma}^{ph}_{II}(\omega) = \rho^{-1}(0) \sum_{\nu} c_\nu \rho_\nu(0)\, \Gamma^{ph}_{\nu}(\omega)$, respectively. The "fluctuating" kernel $\delta\tilde{K}^m_{I,II} = \tilde{K}^m_{ii} - \overline{K^m_{ii}}$ is caused by local spin fluctuations, where $\tilde{K}^m_{ii}$ is defined in analogy to $\tilde{K}^{ph}_{I,II}$, and $\overline{K^m_{ii}} = \sum_{\nu} c_\nu K^m_{ii,i=\nu}$.

4.2. Paramagnon Kernel

Next we are looking for the paramagnon kernel $\overline{K^m}$ as a functional of $\overline{\Gamma^m}$ in the sense of (18). Corresponding to the Green function decoupling inherent in (9), we choose the chain factorization to get from (20) the average

$$\overline{\Gamma^m_{ij}}(\omega) = -\frac{U^2}{\pi^2}\mathrm{Im}\,\overline{\chi_{ij}}(\omega + i\epsilon)\,\mathrm{Im}\,\overline{G^0_{ij}}(i0), \qquad (29)$$

where we have assumed uniform exchange $U_i = U$ for simplicity. The spatial Fourier transform of (29)

$$\overline{\Gamma^m}(\vec{k}, \omega) = -\frac{U^2}{\pi^2 N}\sum_{\vec{k}'} \mathrm{Im}\,\bar{\chi}(\vec{k} - \vec{k}', \omega + i\epsilon)\,\mathrm{Im}\,\mathcal{G}_{\vec{k}'}(i0) \qquad (30)$$

suggests to define the average over the Fermi surface as

$$\overline{\Gamma^m}(\omega) = \langle \overline{\Gamma^m}(\vec{k}, \omega)\rangle_{FS} = \frac{-\frac{1}{\pi N}\sum_{\vec{k}} \mathrm{Im}\,\mathcal{G}_{\vec{k}}(i0)\,\overline{\Gamma^m}(\vec{k}, \omega)}{-\frac{1}{\pi N}\sum_{\vec{k}} \mathrm{Im}\,\mathcal{G}_{\vec{k}}(i0)}, \qquad (31)$$

yielding

$$\overline{\Gamma^m}(\omega) = \frac{U^2}{\pi^3 N^2}\frac{1}{\rho(0)}\sum_{\vec{k},\vec{k}'} \mathrm{Im}\,\mathcal{G}_{\vec{k}}(i0)\,\mathrm{Im}\,\mathcal{G}_{\vec{k}'}(i0)\,\mathrm{Im}\,\bar{\chi}(\vec{k} - \vec{k}', \omega + i\epsilon), \qquad (32)$$

where N is the number of lattive sites. Note that $\overline{\Gamma^m}(\omega)$ from (32) enters into $\overline{K^m}$ of (27) and (28).

As for an impure metal we adapt the diffusion-pole propagator

$$\bar{\chi}(\vec{q}, \omega + i\epsilon) = \bar{\chi}_{\vec{q}}\frac{Dq^2}{Dq^2 - i\omega} \qquad (33)$$

being valid in the long-wavelength limit $q\ell \ll 1$, where ℓ is the electron mean free path. $\bar{\chi} = \bar{\chi}(\vec{q}, 0)$ is the static susceptibility. D denotes the spin diffusion constant, which can be typified, e.g., by $D = D_0(1 - U\rho(0))$ /13,14/, where $D_0 = \frac{1}{3}\ell v_F$ with the Fermi velocity v_F. From (33) one obtains the Lorentzzian /15,16/

$$\mathrm{Im}\,\bar{\chi}(\vec{q}, \omega + i\epsilon) = \bar{\chi}_{\vec{q}}\frac{Dq^2\omega}{(Dq^2)^2 + \omega^2} = \bar{\chi}_{\vec{q}}\frac{\omega_{\vec{q}}\,\omega}{\omega^2_{\vec{q}} + \omega^2} \qquad (34)$$

with $\omega_{\vec{q}} \propto \ell v_F q^2 \bar{\chi}^{-1}_{\vec{q}}$ for the diffusive model. The opposite (Landau-damped) case $q\ell \gg 1$ for a "clean" system with $\omega_{\vec{q}} \propto v_F q \chi^{-1}_{\vec{q}}$ is reflected by (34), too /16/.

The static $\bar{\chi}_{\vec{q}}$ can be deduced on a microscopic footing from a Ginzburg-Landau expansion for alloys /17/ giving rise to the Ornstein-Zernicke form

$$\bar{\chi}_{\vec{q}} \propto \frac{1}{1 - U\rho(0) + c^m q^2}, \qquad (35)$$

where c^m was found to be proportional to the stiffness constant affected by disorder.

5. RESULT AND CONCLUSION

The Berk-Schrieffer-type equations (27) and (28) are the basis to determine T_c. In contrast to the pure case two different phonon-mediated kernels appear. For simplicity we omit the fluctuating terms $\delta\tilde{K}^m_{I,II}$ hereafter. Adopting the trial-function approach /18/ we can parametrize the integral equation (28). The crucial point is the complexity of the scaling factor

$$\tilde{Z}(i0) = 1 + \lambda^{ph}_{I} + \lambda^m \qquad (36)$$

due to the complex λ^{ph}_{I} unlike the pure system. Hence, the strong-coupling result becomes

$$T_c \propto \exp\left\{-\frac{1}{\lambda_{II}^{ph}-\lambda^m}\left[\frac{(1+\lambda^m+\mathrm{Re}\,\lambda_I^{ph})^2+(\mathrm{Im}\,\lambda_I^{ph})^2}{1+\lambda^m+\mathrm{Re}\,\lambda_I^{ph}}\right]\right\} \quad (37)$$

with the electron-phonon coupling constants

$$\lambda_I^{ph} = \frac{2}{F(i0)}\sum_\nu c_\nu F_\nu(i0)\int_0^\infty d\omega\,\frac{\Gamma_\nu^{ph}(\omega)}{\omega}, \quad (38)$$

$$\lambda_{II}^{ph} = \frac{2}{\rho(0)}\sum_\nu c_\nu \rho_\nu(0)\int_0^\infty d\omega\,\frac{\Gamma_\nu^{ph}(\omega)}{\omega}, \quad \text{where} \quad (39)$$

$$\Gamma_\nu^{ph}(\omega) = \aleph\sum_{\nu'}\vec{g}^{\,2}_{\nu\nu'}c_{\nu'}(b_\nu(\omega)+b_{\nu'}(\omega))\,\rho_{\nu'}(0) \quad (40)$$

with the conditionally averaged phonon Green function $b_\nu(\omega) = -\frac{1}{\pi}\mathrm{Im}\,D_{ii,i=\nu}(\omega+i\epsilon)$, the number $\aleph$ of nearest-neighbours, and the electron-paramagnon coupling parameter

$$\lambda^m = 2\int_0^\infty d\omega\,\frac{\overline{\Gamma^m}(\omega)}{\omega} = \frac{U^2}{\rho(0)}\frac{1}{N}\sum_{\vec q}\bar I_{\vec q}\,\bar\chi_{\vec q}, \quad \text{where} \quad (41)$$

$$\bar I_{\vec q} = \frac{1}{\pi^2 N}\sum_{\vec k}\mathrm{Im}\,\mathcal{G}_{\vec k}(i0)\,\mathrm{Im}\,\mathcal{G}_{\vec k-\vec q}(i0). \quad (42)$$

By setting $\bar I_{\vec q} \approx \bar I_0$, which is finite due to the damping ($\sim|\mathrm{Im}\,\Sigma(i0)|$) at the Fermi level, we get from (35) and (41) with the momentum cut-off q_c:

$$\lambda^m \propto \frac{U^2\bar I_0 q_c}{2\pi^2\rho(0)c^m}\left(1-\frac{\sqrt{1-U\rho(0)}}{q_c\sqrt{c^m}}\arctan\frac{q_c\sqrt{c^m}}{\sqrt{1-U\rho(0)}}\right), \quad (43)$$

i.e., λ^m remains finite at the ferromagnetic instability $U\rho(0)=1$. For comparison we quote the result for a pure system /11/. Then (42) turns out to be $I_{\vec q} = \frac{1}{N}\sum_{\vec k}\delta(\epsilon_F-\epsilon_{\vec k})\,\delta(\epsilon_F-\epsilon_{\vec k-\vec q}) = \frac{\rho(0)}{2k_F}\frac{1}{q}$ with the parabolic band $\epsilon_{\vec k} = \frac{k^2}{2m}$, giving rise to singular $\lambda^m \propto \ln\left(\frac{1}{1-U\rho(0)}\right)$ for $U\rho(0)\to 1$.

Let us discuss some limiting cases of the modified T_c-formula (37).

(i) Without paramagnons: At $\lambda^m=0$ the result of /7/ is obtained; additionally, in /7/ the Coulomb pseudopotential was included.

(ii) Without disorder: The weighted means in (38) and (39) are cancelled so that $\lambda_I^{ph}=\lambda_{II}^{ph}$ and $\tilde Z(i0)$ become real. Thus the T_c result tends to the relation given in /11/. Moreover, if $\lambda^m=0$ we get McMillan's form /18/.

(iii) Weak-coupling limit: For $\tilde Z=1$ the second factor in the exponent of (37) originated from $\frac{|\tilde Z(i0)|^2}{\mathrm{Re}\,\tilde Z(i0)}$ goes to unity. Thus we are left with the pairbreaking situation as formulated in /11/, but here the parameters in $T_c \propto \exp\left\{-\frac{1}{\lambda_{II}^{ph}-\lambda^m}\right\}$ are concentration dependent. Finally, at $\lambda^m=0$ the T_c expression of Weinkauf-Zittartz /8/ is reached.

As is proved experimentally /19/ it is necessary to incorporate spin fluctuations in reducing T_c. For example, T_c in Nb and V came out to be a factor 2 too large /10/ if paramagnons were ignored. Disordered materials such as binary or ternary (with two transition metals included) alloys and metallic glasses /19/ seem to be more appropriate as pure samples for extracting the effect of paramagnons on superconductivity.

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18.	Applied researches
19.	Biophysics

Коллей Е., Коллей В. E17-83-705

Парамагнонный эффект в сверхпроводящих разупорядоченных сплавах

На основе ванье-представления уравнений Берка-Шриффера исследуется влияние парамагнонов на сверхпроводимость в разупорядоченных сплавах $A_c B_{1-c}$. Самосогласованное усреднение по конфигурациям выполнено в приближении когерентного потенциала. Получена формула зависимости критической температуры сверхпроводящего фазового перехода T_c от концентрации, которая зависит от двух эффективных констант электрон-фононного взаимодействия. Электрон-парамагнонная константа связи, определяемая диффузией спинов, остается конечной в противоположность ее логарифмической расходимости в чистых системах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kolley E., Kolley W. E17-83-705

Paramagnon Effect in Disordered Superconducting Alloys

The influence of paramagnons (enhanced spin fluctuations) or superconductivity in substantionally disordered alloys $A_c B_{1-c}$ is investigated on the basis of random Berk-Schrieffer-type equations formulated in Wannier representation. The configurational averaging is performed self-consistently by means of the CPA. A concentration dependent formula for the superconducting transition temperature T_c is derived which involves two different phonon-mediated coupling parameters. The electron-paramagnon coupling constant affected by spin diffusion remains finite in contrast to the logarithmic singularity for a clean system.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983