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V.K.Fedyanin, V.G.Makhankov

IDEAL GAS OF PARTICLE-LIKE EXCITATIONS AT LOW TEMPERATURES

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1. Due to the recent success of the "soliton science", there appears a possibility of studying with a good degree of rigour certain nonlinear effects in the condense matter physics and explain some experimental puzzles. In particular, problems of CP and of satellites in DSF of ferromagnetics at low temperature as well as of the systems undergoing structural phase transition are among them.

In what follows we consider a class of models described by the Klein-Gordon equation (KGE) in (1.1) space-time on the basis of the phenomenological approach, given apparently first in ref. /1/, calculate "longitudinal" and "transversal" DSF for these models defined by soliton and bion (breather) type excitations, and discuss possible generalizations.

To proceed further note that in the transfer-matrix approach usually employed /1-3/, the field variables $\dot{\phi}$ and ϕ and continual integral are used to calculate the partition function. The Gauss integral over $\dot{\phi}$ is first calculated, and then the complicated integral over ϕ is reduced to the solution of the "Schrödinger-type" differential equation /2/ with "mass" being constructed with the Hamiltonian (1) parameters and temperature: $(m^* = (A\omega_0 c_0 \beta)^2)$.

This procedure probably being natural when applied to nonintegrable systems seems to be awkward for complete integrable ones, for there is a transformation to new canonical variables in the latter case. In these variables the Hamiltonian of a system is "factorized", i.e., splits into the sum of contributions. For the sine-Gordon (SG) equation there are three types of excitations: phonon excitation continuous in the classical limit and discrete-bion and-kink ones. It means that in the KG-type models, excitations may be considered (in these variables) as a mixture of three noninteracting gases of phonon (magnon), bion and kink types. It is also clear that upon integrating over $\dot{\phi}$, we get the Hamiltonian which cannot be factorized, then phonon and kink excitations (bions disappeared!) are coupled strongly.

Thus, the language of angle-action variables appearing in the auxiliary linear problem is a natural one for complete integrable systems. Here again appears a difficulty related to the Jacobian of transformation from continuous field variables to spectral ones, part of which being continuous and another part discrete. This difficulty may be overcome by a technique analogous to that for quantizing integrable systems (i.e., proceeding into a lattice).

Nevertheless, emboldened by the confidence in that the "gas" approach mentioned is adequate to the discussed problem, we use it to calculate phenomenologically the dynamical structure factors and illustrate its action with the systems governed by the Hamiltonian

$$H = A a_0 \sum_j \left[\frac{1}{2}\dot{\phi}_j^2 + \frac{c_0^2}{2a_0^2}(\phi_{j+1} - \phi_j)^2 + \omega_0^2 V(\phi_j)\right] \tag{1}$$

(see, e.g.,[3]) here $\phi_j(t)$ is the one-component real dimensionless field which takes its value at the site j and moment t; the first term is the kinetic energy, the second one describes "gradient interaction", and the last term stands for the local potential. We shall discuss the potentials

a) $V(SG) = 1 - \cos\phi_j$,

b) $V(\phi_j^4) = \frac{1}{8}(1 - \phi_j^2)^2$

although another choice is possible. In eq. (1) c_0 and ω_0 are the characteristic parameters of velocity and frequency dimension, respectively, and defined by the physical features of the system studied; A is the constant of dimension (energy) x x (length)$^{-1}$ x (time)^2, giving the energetic scale. The characteristic parameter in (1) is $d = c_0/\omega_0$, so if $d \gg a_0$ then we have a small variation of $\phi_j(t)$ as j alters and one may proceed to the continual limit for H, that leads immediately to the differential equation for $\phi(x,t)$

$$\ddot{\phi}(x,t) - c_0^2 \phi_{xx}(x,t) = -\omega_0^2 \frac{dV}{d\phi}. \tag{2}$$

In what follows our concern will be with solutions of kink- and bion-types. In particular, kink solution for V(SG)

$$\cos\phi = 1 - 2\,\mathrm{sech}^2\left(\frac{x - vt + x_0}{d\gamma^{-1}}\right), \quad \gamma = \left(1 - \frac{v^2}{c_0^2}\right)^{-1/2}, \tag{3}$$

is localized in the region $d\gamma^{-1}$ and decreases with growing velocity. The solution (3) depends only on $s = x - vt + x_0$. The energy and momentum of kink are given by the "relativistic" formulae

$$E_k(v) = \gamma E_k^0 = [E_k^{0\,2} + p^2 c_0^2]^{1/2}, \quad p = M_k v\gamma,$$
$$E_k^0 = M_k c_0^2, \quad M_k = \frac{A\sqrt{2}}{d}\int_{\phi_1}^{\phi_2} |V|^{1/2} d\phi = \frac{A\sqrt{2}}{d}\int_{-\infty}^{\infty} V(\phi)\,dx. \tag{4}$$

The quantities E_k^0 and M_k are naturally thought of as the energy and rest mass of kink, ϕ_1 and ϕ_2 are the values of solutions at the minima of the local potential. Solutions (2) may be specified as kinks $(\frac{d\phi}{ds})_0 > 0$ and antikinks $(\frac{d\phi}{ds})_0 < 0$ and the topological charge $N = N_+ - N_-$ is introduced. Here regarding solutions (2) as particles of the mass and energy (4), we suppose $N_+ = N_-$, that is not necessary but gives rise to a simpler variant of thermodynamics and statistical mechanics of the one-dimensional lattice gas.

The bion-type solutions are given in the SG-model by the formula ($c_0 = 1$):

$$\phi_b = 4\,\mathrm{arctg}\left(\alpha\frac{\cos\theta}{\cos\kappa}\right), \quad \kappa = \alpha\Omega\gamma(x - vt + x_0),$$
$$\theta = \gamma\Omega(vx - t) + \theta_0, \quad E_b = 16m\sqrt{1 - \Omega^2}, \quad \alpha = \sqrt{1 - \Omega^2}. \tag{5}$$

In the framework of the ϕ^4 theory such solutions may be found but approximately in the small amplitude region (see, e.g.,[4] and the references cited).

$$\chi = -1 + \sqrt{\frac{8}{3}}\,\alpha\,\frac{\sin\theta}{\mathrm{ch}\kappa} - 2\alpha^2\left(1 + \frac{\cos\theta}{3}\right)\mathrm{ch}^{-2}\kappa + o(\alpha^2),$$
$$\Omega = \sqrt{2}\left(\frac{m^2}{1 + \alpha^2}\right)^{1/2} \tag{6}$$

(these solutions are constructed over vacuum $\phi_v = -1$).

2. Following[5] and using as an example kinks we give a simple version of the statistical mechanics of such a system that is based on the introduction of a certain effective Hamiltonian. Given known liberty, one may consider even the interaction between kinks (and antikinks).

The system is as usually described in the phase space of momenta and coordinates (p, q) under the assumption that the volume in this space corresponding to a single quantum state of the system of N identical particles is $N!h^N$, $N = N_k + N_{\bar{k}}$. The integration regions over q and p are respectively $0 \div L$ and $(-\infty, +\infty)$. We consider below the equations

$$\Box\phi = \sin\phi$$

and

$$\Box\chi = -\chi + \chi^3.$$

Divide the one-dimensional "volume" into the elementary cells of size $\Delta(v)$, assuming the number of cells to be $\tilde{N} = L/\Delta \equiv \frac{N_0 a_0}{\Delta}$ with N_0 being the amount of sites of the initial system.

Let us suppose further that every cell is either occupied with the particle of mass m and velocity v or empty. Then we may describe the situation with the help of the operators having eigenvalues 1 and 0.

If the number of "particles" really existing in the system (denote it N) is less than the cell number $\tilde{N}$, we come to the one-component ideal lattice gas [6]. It should be stressed that the quantity of kinks and antikinks occurring in the system is in fact defined by external excitation conditions and temperature but when the ratio between the fixed number N in a canonical ensemble and the average one $\bar{N}$ in the grand canonical ensemble is definite, one may use either of two. Note, that the effective Hamiltonian implies this naturally.

Let us discuss possibilities of constructing effective Hamiltonians, that allow us to calculate in a simple way equilibrium characteristics of the soliton gas.

In the general case, when the particle energy is governed by (4), the effective Hamiltonian is given by a simple formula (summing is over all cells)

$$H_0^1 = \nu \sum_f n_f, \quad \nu = -\theta \ln\left(\frac{bq\Delta}{h}\right) + V \equiv -\nu_0 + V,$$
$$V = E_k^0 - \sigma \cdot \theta - \theta \ln(\beta h \omega_0), \quad q = \frac{E_k^0}{c_0} \int e^{\beta E_k^0 (1-\sqrt{1+y^2})} dy. \tag{7}$$

The operator $N = \sum_f n_f$, $n_f = 0,1$ "numbers" the total number of solitons in the system.

Expression (7) naturally formalizes the idea of the one-dimensional soliton gas (of kinks and antikinks). In getting (7) we have used the results for the free energy obtained via the transfer-matrix technique, and V describes that part of the energy which does not depend on momentum, and the augend stands for the "kinetic" energy. Since, according to [3] the Goldstone mode and bound modes of frequencies ω_b determine kink motion and oscillations of its form, we have $(-\sigma\cdot\theta - \theta \ln(\beta h\omega))$ and in (7) ν is multiplied by the operator $\sum_f n_f$.

The integral over y in (7) can be expressed through the McDonald function, so we get

$$q = 2\frac{E_k^0}{c_0} e^{\beta E_k^0} K_1(\beta E_k^0), \tag{8}$$

or in the $\beta E_k^0 \gg 1$ limit we have $q \simeq (2\pi M_k \theta)^{1/2} \left(1 + \frac{(\beta E_k^0)^{-1/2}}{2} + \dots\right)$.

Factor $b=2$ for the SG and $b=1$ for the ϕ^4 model is related to the fact that in the second case there is the ordered sequence of kinks and antikinks (or vice versa), in the first one it is not essential due to infinite-fold degeneracy of the ground state. Now if we ignore soliton-phonon interaction we arrive at

$$H_0^0 = -\nu_0 \bar{N} + E_k^0 \sum_f n_f, \quad \nu_0 = \theta \ln\left[\frac{b\Delta}{h}(2\pi M_k \theta)^{1/2}\right], \tag{7'}$$

i.e., only the rest energy of soliton contributes to its lattice term. So we have for the soliton partition function

$$Z = \mathrm{Sp}_{(f)}\, e^{-\beta H_0^0} = e^{\beta \gamma_0 \bar{N}} [1 + e^{-\beta E_k^0}]^{\tilde{N}}.$$

Then the free energy is

$$F_0 = -\theta \cdot \ln Z = \theta \cdot \bar{N} - \theta \tilde{N} \ln[1 + e^{-\beta E_k^0}] \simeq -\theta \bar{N},$$

where we use the fact that

$$\bar{N} = \frac{\mathrm{Sp}\left(\sum_f n_f \exp(-\beta H_0^0)\right)}{Z} = \tilde{N} \exp(-\beta E_k^0). \tag{9}$$

The free energy density is as follows:

$$f_s = \lim_{L\to\infty}\left(\frac{F_0}{L}\right) = -\frac{\theta \bar{N}}{L} = -\theta \frac{e^{-\beta E_k^0}}{\Delta} = -\theta \bar{n}. \tag{10}$$

The rest obtained coincides naturally with that of Krumhansl and Schrieffer [1]. It should be underlined that in the case of kinks we have "pure Boltzmanns" expression $\bar{n} = \frac{\exp(-\beta E_k^0)}{\Delta}$. For both models zero approximation of the transfer-matrix method (see ref. [3]) gives just the same in the framework of the model. There is however a correct temperature renormalization of $\bar{n}$ (see (11)) for the complete integrable SG model which is just improved by the numerical factor (≈1/8) in a further approximation. As it seems to us this shows once more that the zero approximation of the transfer matrix method for the SG-model already contains a part of kink-bion interaction. Therefore, in the orthodox formulation of the ideal lattice gas model [6] we have

$$\bar{n} = \frac{Z_1(\theta)}{\Delta(1+Z_1(\theta))} \simeq \frac{Z_1(\theta)}{\Delta} = \frac{2Mc_0}{h} K_1(\beta E_k^0) \equiv \frac{Mc_0}{\pi\hbar} K(\beta E_k^0). \qquad (9')$$

In a general case one has for the free energy density of solitons

$$f_s = -\theta\bar{n}, \quad \bar{n} = \frac{\langle \sum_f n_f \rangle}{N\cdot\Delta} = \frac{b e^{\sigma}}{\sqrt{2\pi}\, d} (\beta E_k^0)^{1/2} e^{-\beta E_k^0} \qquad (11)$$

that is the same as in ref.[3].

Upon taking $b=2, \sigma = \ln 2$ for SG and $b=1, \sigma = \ln(2\sqrt{3})$ for ϕ^4 model (see ref.[3]), we get the expression for $\bar{n}_s$ in which soliton-phonon interaction is allowed for.

The above model of the ideal lattice gas with effective Hamiltonians (7), (9) leads not only to simpler calculations but also admits, from our point of view, interesting generalizations.

Thus, in our case there is a mixture of three components: $\bar{N}_k$ kinks, $\bar{N}_{\bar{k}}$ antikinks and N_1 "empty places", then $\bar{N} = \bar{N}_k + \bar{N}_{\bar{k}} + N_1$. The fact that $\bar{N}_k \neq \bar{N}_{\bar{k}}$ may simply be included by introducing the chemical potentials μ_k, $\mu_{\bar{k}}$ and using equilibrium conditions. If $\bar{N}_k = \bar{N}_{\bar{k}}$ then $\mu_k = \mu_{\bar{k}}$. Taking into account that kink-kink and antikink-antikink interactions are repulsive whereas, kink-antikink one is attractive $\epsilon_{k\bar{k}} < 0$, $\epsilon_{kk} = \epsilon_{\bar{k}\bar{k}} > 0$ and for $\bar{N} \ll N_1$ $(\bar{N} = \bar{N}_k + \bar{N}_{\bar{k}})$ one can, using for H_0^{kk} formula of type (7), write the effective Hamiltonian for such a three-component system. Terms, with μ_k, $\mu_{\bar{k}}$ renormalize ν so that a certain number enters the effective Hamiltonian. Such a procedure may be done approximately by using the results of impurity problem in the Ising model[7,8] generalized for a two-impurity in ref.[9]. This yields a version of the lattice gas model with the nearest-neighbour interactions $(\epsilon_{kk}), (\epsilon_{\bar{k}\bar{k}}), (\epsilon_{k\bar{k}})$ which vanish rapidly with distance[10]. Parameters of the Hamiltonian are given by combinations of μ_k, $\mu_{\bar{k}}$, ϵ_{ij}, $i,j = k, \bar{k}$.

Now one can formulate the simplest generalization of the ideal lattice gas model.

It is more probable to meet kink-antikink pairs due to repulsive forces acting between identical particles (solitons), in all other respects we have a gas of particles "identical" with respect to their macroscopic properties. In the approximation of the nearest neighbour interaction one can according to ref.[10] set $\epsilon_{kk} = \epsilon_{\bar{k}\bar{k}} = 8m^{1/2}\,[E(m) + \frac{m-1}{2} K(m)]$, $K(m)$ and $E(m)$ being the complete elliptical integrals of the first and second type, $m = 4\gamma^2(b^2 + 4\gamma^2)^{-1}$, $b = \phi_k'(0)$ so that

$$\epsilon_{kk} = \epsilon_{\bar{k}\bar{k}} = -\epsilon_{k\bar{k}} \simeq 32\exp(-r) \quad \text{at} \quad r \to \infty$$

and

$$\epsilon_{kk} = \epsilon_{\bar{k}\bar{k}} \simeq \frac{2\pi^2}{r} \quad \text{at } r \to 0.$$

For the case $(k\bar{k})$ at $r \simeq \Delta$ there is a potential well of depth of the kink rest mass (see in ref.[10]) therefore, (7) may be generalized as follows:

$$H = E_0 + \nu \sum_f n_f - \frac{E_k^0}{2} \sum_{(f,g)} n_f n_g, \qquad (12)$$

here summing is over nearest cells. The one-dimensional system with Hamiltonian (12) is exactly soluable, and along with Z all correlation functions $\langle n_f \rangle$, $\langle n_f n_g \rangle, \ldots \langle n_f \prod_i n_i \rangle$ can be calculated. In particular, since our system is that with the attractive interaction $(E_k^0 > 0)$, the correlation functions describing probabilities of cluster formation of m kinks and $\bar{m}$ antikinks are especially of interest:

$$W(m,\bar{m}) = \langle n_1 n_{\bar{2}} \ldots n_{2m-1} n_{\overline{2m}} \rangle = \frac{\langle n_1 n_{\bar{2}} \rangle^m}{\langle n_f \rangle^{m-1}}, \qquad (13)$$

n_i means that there is a kink in the i-th cell, analogously $n_{\bar{j}}$ means for antikink. Upon using exact expressions for $\langle n_f \rangle$, $\langle n_f n_g \rangle$[7], we get at $\beta E_k^0 \gg 1$

$$W(m,\bar{m}) \simeq \bar{n}, \quad \bar{n} = \frac{b e^{\sigma}}{\sqrt{32\pi}\, d} (\beta E_k^0)^{1/2} e^{-\beta E_k^0}. \qquad (14)$$

Formulae (11) and (14) may be utilized in analysing the experimental data on neutron scattering by quasi-one-dimensional systems whose dynamical properties can be at low temperatures simulated in the framework of the SG and ϕ^4 models.

3. Some fine features of dynamical properties of the models considered (e.g., a central peak, peculiarities of transport processes and so on) will be defined just by soliton and bion response to an external action. This will manifest in the quasi-elastic scattering of neutrons (light). When energy transfer to the system is significant, excited soliton states can arise. All these questions may be analysed via studying the double differential scattering cross-section $\sigma_s(q,\omega)$ and the dynamical structure factor $S(q,\omega)$ (DSF). We need then a procedure for their calculations. It is just the point of this section. For this purpose we shall make use of the Kawasaki idea[11]. For the first time it was utilized by Mikeska in[12] to investigate neutron scattering, and then by one of the authors in[13]. Here we

develop it in a somewhat different way which allows us to get a general and simple formula (see (25)) for $S(q,\omega)$ and to avoid complicated intermediate calculations.

The scattering cross-section $\sigma_s(q,\omega)$ is expressed in the gaseous approximation through $S(q,\omega)$ as follows:

$$\sigma_s(q,\omega) = b^2 \frac{k'}{k} S(q,\omega) = b^2 \frac{k'}{k} \bar{N} S_1(q,\omega), \quad q = k' - k, \quad \omega = E' - E, \tag{15}$$

where the dynamical scattering formfactor of a separate scatterer $S_1(q,\omega)$ is the Fourier transform of $S_1(x,t)$ and b is the scattering length. According to [11] $S_1(x,t)$ is constructed of soliton characteristics $\Phi(x,t/x_0,p,\theta_0)$, $\Phi(0,0/x_0,p,\theta_0)$ (see below) averaged over all its possible positions $x_0 \in (-L,L)$, momenta p and initial phases θ_0. The phase averaging essential for bions gives rise to remarkable features that will be discussed in detail later.

For $S_1(x,t)$ we have

$$S_1(x,t) = \frac{1}{Z_1 h} \int_{-L}^{L} dx_0 \int dp\, \Phi(x,t/x_0,p)\Phi(0,0/x_0,p)\, e^{-\beta E(p)},$$
$$Z_1 = \frac{1}{h}\int_{-L}^{L} dx_0 \int dp\, e^{-\beta E(p)} = \frac{2L}{h}\int e^{-\beta E(p)}\, dp. \tag{16}$$

The partition function of the separate scatterer Z_1 is calculated in the framework of either model by making use of the corresponding Hamiltonian and soliton-like solutions (see below), so that

$$S_1(q,\omega) = \frac{1}{(2\pi)^2}\iint dx\,dt\, S_1(x,t)\, e^{i(qx-\omega t)} =$$
$$= \frac{1}{hZ_1}\int_{-L}^{L} dx_0 \int dp\, \Sigma(q,\omega/x_0,p)\, e^{-\beta E(p)}; \tag{17}$$
$$\Sigma(q,\omega/x_0,p) = \frac{1}{(2\pi)^2}\iint dx\,dt\, e^{i(qx-\omega t)}\, \Phi(x,t/x_0,p)\Phi(0,0/x_0,p).$$

It is however convenient to obtain first the one-soliton function $\Sigma(q,\omega/x_0,p)$ and then to average over x_0 and p. Performing this we get

$$S_1(q,\omega) = \frac{p'(v_0)\Delta^2(v_0)}{2\pi q Z_1 h} f(-q\Delta(v_0))\, f(q\Delta(v_0))\, e^{-\beta E(v_0)} \tag{18}$$

here

$$f(\lambda) = \int \Phi(\rho)\, e^{i\lambda\rho}\, d\rho, \quad v_0 = \frac{\omega}{q}, \tag{18'}$$

and for the soliton gas

$$S(q,\omega) = \bar{N} S_1(q,\omega). \tag{19}$$

The physical quantity of the total system correlation function (in particular, a spin component) is as a rule given by

$$\Phi(x,t) = \Phi_0 - \Phi(x,t/x_0,p,\theta_0), \quad \Phi_0 = \text{const}. \tag{20}$$

so that for the overall formfactor (19) we have

$$S(q,\omega) = [\Phi_0^2 - 2\bar{n}\Delta\cdot\Phi_0]\,\delta(q)\delta(\omega) + \bar{N}S_1(q,\omega). \tag{21}$$

Thus, the scattering by solitons gives rise to a redistribution of Bragg peak intensity into the quasi-elastic component given by formulae (25), (27).

Notice here, that both equilibrium and dynamical properties of soliton gas have been described as in [3] in the momentum-coordinate space. In the first work devoted to this, Maxwell's gas has been considered, that led to some vagueness in the Z_1 normalization but did not effect the concrete dynamical formfactors.

4. Let us illustrate the usage of the above formulae by calculating a "parallel" response of the system $S^{\parallel}(q,\omega)$ related to the correlation function $\langle \cos\phi(x,t/x_0,p)\cos\phi(0,0/x_0,p)\rangle$. According to (8) the soliton contribution is given by averaging over the following product:

$$4\,\mathrm{sech}^2\left(\frac{x - vt + x_0}{\Delta(v)}\right)\mathrm{sech}^2\left(\frac{+x_0}{\Delta(v)}\right), \quad \Delta(v) = \gamma_0^{-1}\mu^{-1}, \quad \mu^{-1} = d,$$

and for $f(\lambda)$ one has

$$f(\lambda) = 2\int \frac{\exp(i\lambda\rho)}{\mathrm{ch}^2\rho}\, d\rho = \frac{4\lambda\pi}{\mathrm{sh}\frac{\lambda\pi}{2}},$$

or

$$f(q\Delta(v_0)) = f(-q\Delta(v_0)) = \frac{4\pi q}{\mu\gamma(v_0)}\left[\sinh\frac{\pi q}{2\mu\gamma(v_0)}\right]^{-1}. \tag{22}$$

Once $p(v) = M_k v\gamma(v)$, $E(v) = E^0_k \gamma(v)$ and $E^0_k = 8\mu$, $M_k = 8\mu c_0^{-2}$ we have

$$E(v_0) = E_k \gamma(v_0),\ p'(v_0) = M_k \gamma_0^3,\ \gamma_0 \equiv [1 - \frac{\omega^2}{q^2 c_0^2}]^{-1/2} \tag{23}$$

and

$$Z_1 = \frac{2L}{h} \int e^{-\beta E(p)} dp = \frac{4LE^0_k}{c_0 h} K_1(\beta E^0_k), \tag{24}$$

with $K_1(z)$ being the above-mentioned McDonald function. From (22-24) $S(q,\omega)$ is as follows:

$$S(q,\omega) = \frac{16\bar{n}\gamma_0}{\mu^2 c_0 \pi q} [\frac{x}{\operatorname{sh} x}]^2 \frac{e^{-\beta\gamma_0 E^0_k}}{K(\beta E^0_k)}, \qquad x = \frac{\pi q}{2\mu\gamma_0}, \tag{25}$$

where $\bar{n}$ is given by (11). In the "nonrelativistic" limit ($v \ll c_0$) $\gamma_0 = 1 + \omega^2/2q^2c_0$ and at $\beta E^0_k \gg 1$ this formula yields the Mikeska result[12]. Other response functions can be as simply obtained. Note that for analyzing the experimental data with the help of the formulae of type (25), it is necessary to fix exactly the system of units to be sure of the right coefficient in (25) (see, e.g.,[14]). We think that along with studied experimentally $CsNiF_3$ and (TMMC) $(CH_3)_4NMnCl_3$ systems, which display soliton contribution in quasi-elastic scattering with a large probability, there is another such quasi-one-dimensional system namely the $RbFeCl_3$ crystal (for its parameters J, A see ref.[15]). Formulae (18), (19) have been utilized in[16] for getting $S(q,\omega)$ of the isotropic Heisenberg ferromagnet, where the soliton mode is a solution of the S3-equation. In this case the crystal $[(CH_3)_4N][NiCl_3]$ with $S = 1$ in the temperature range $1.6°K \div 79°K$ can serve as a possible quasi-one-dimensional "candidate". A soliton solution has been found in ref.[17] for the anisotropic ferromagnet of the "easy axis"-type. By using this solution the authors of[18] have calculated $S(q,\omega)$ for "slow" magnetic solitons, i.e., when $|p| \ll p_0$, and one of the present authors in[5] has done it in the general case ($-p_0 \le p \le p_0$) on the basis of (18), (19).

5. Let us now proceed, following[21], to study the contribution of bion excitations to DSF in the framework of the SG and ϕ^4 theories. The fact that such a contribution can be very important at low temperatures has been pointed out by many authors (see, for example. refs.[14,20] for SG and[19] for both models).

In this case in addition to the kink degrees of freedom, there appears extra internal one over which averaging is to be done. The latter corresponds to averaging over the initial phases θ_0, and as a result we have $\langle F\rangle = Z_b^{-1} \int dM\, F \exp(-\beta E_b)$ with $Z_b = \int dM \exp(-\beta E_b)$ and $M = \{x_0, \theta_0, v, \alpha\}$ being the set of free bion parameters. For the SG-model we shall be interested in the correlators (see[14]):

$$S_1^{\parallel} = \langle(1-\cos\phi)(1-\cos\phi_0)\rangle, \quad S_1^{\perp} = \langle\sin\phi \sin\phi_0\rangle$$

and hence the functions

$$1 - \cos\phi = 8\frac{\epsilon^2}{(1+\epsilon^2)^2},$$

$$\sin\phi = \frac{4\epsilon}{1+\epsilon^2}[1 - (2\frac{\epsilon}{1+\epsilon^2})^2]^{1/2}.$$

Since at low temperatures ($\beta E^0_k \gg 1$) the main contribution will come from small amplitude bions $\epsilon \ll 1$ (or $\alpha \ll 1$), we have with an accuracy of $O(\epsilon^2)$

$$1 - \cos\phi \simeq 8\epsilon^2, \quad \sin\phi \simeq 4\epsilon.$$

Using the procedure described above one can do extra averaging over θ_0 to get $1/2\cos\theta$ in the correlator (for $\langle\sin\theta \sin\theta_0\rangle = 1/2\cos\theta$). After the Fourier transformation the following formula may be obtained instead of (18):

$$S_1(q,\omega) = Z_b^{-1} \sum_{i=1,2} \frac{p'(v_i)\Delta^2(v_i)}{2\pi q_i} f^2(q_i\Delta(v_i)) \exp(-\beta E(v_i)),$$

where

$$q_{1,2} = q \pm \Omega\cdot(v\gamma)_{1,2}, \quad v_i = \frac{\omega \pm \Omega\gamma_{1,2}}{q_i}$$

and $v_{1,2}$ are the solutions of equations obtained upon integration of appropriate δ-functions:

$$\delta(v - \frac{\omega + \Omega\gamma}{q + \Omega\gamma v}) \quad \text{and} \quad \delta(v - \frac{\omega - \Omega\gamma}{q - \Omega\gamma v}).$$

As a result, we have for the transversal response

$$S^{\perp}(q,\omega,\Omega) = \frac{A_{\perp}}{b_{\perp}} \operatorname{ch}^{-2} x_{\perp}[\exp(-\beta M_b\gamma_+) + \operatorname{sgn}(q^2-\omega^2)\exp(-\beta M_b\gamma_-)],$$

$$A_\perp = \frac{2\pi M_b}{\Omega^2}\left(\frac{\bar{n}_b}{Z_b}\right),\quad b_\perp = \sqrt{q^2-\omega^2+\Omega^2},\quad M_b=16m,\quad x_\perp = \frac{\pi b_\perp}{2m}, \tag{26}$$

$$y_\pm = \frac{q^2+\Omega^2}{|qb_\perp \pm \omega\Omega|}.$$

The longitudinal formfactor $S_1^{\parallel}$ contains several summands: terms of the order of $O(\alpha^2)$ only renormalize the coefficient in Bragg's term $\sim \delta(\omega)\delta(q)$ (to an analogous effect leads the $O(\alpha^4)$ term in the expansion of $\cos\phi$). Since $\sin\theta = \frac{1}{2}(1-\cos 2\theta)$ the bion part of the correlator consists of two summands (after averaging over θ_0), namely

$$\langle F\rangle_M = 16\alpha^4 \langle \mathrm{sech}^2\kappa\, \mathrm{sech}^2\kappa_0\rangle_{\kappa_0} + 16\alpha^2 \langle \cos 2\theta \cos 2\theta_0\rangle_{\theta_0} \langle \mathrm{sech}^2\kappa \cdot \mathrm{sech}^2\kappa_0\rangle_{\kappa_0}.$$

The augend coincides accurate to $4\alpha^4$ with the longitudinal kink correlator obtained above, if one puts in it $m=\alpha\Omega$ and $E_b^0 = 16\mu$ instead of μ and $E_s^0 = 8\mu$, respectively.

Denoting it by $S_{cp}^{\parallel}$, we have

$$S_{cp}^{\parallel}(q,\omega) = \frac{A_1^{\parallel}}{b_1}\left(\frac{x_1}{\mathrm{sh}\, x_1}\right)^2 \exp\left(-\beta M_b \frac{q}{b_1}\right), \tag{27}$$

$$A_1 = \frac{32\alpha^2 M_b^0}{\pi\mu^2\Omega^2}\left(\frac{\bar{n}_b}{Z_b}\right),\quad b_1 = \sqrt{q^2-\omega^2},\quad x_1 = \frac{\pi b_1}{2m}.$$

This bion contribution to the central peak is added to the kink one and dominates the latter at low temperatures $\beta M_k \gg 1$.

The addend (name is a satellite one) being of the form

$$S_{sat}^{\parallel} = \frac{A_2^{\parallel}}{b_2}\left(\frac{x_2}{\mathrm{sh}\, x_2}\right)^2 [\exp(-\beta M_b y_2^+) + \mathrm{sgn}(q^2-\omega^2)\exp(-\beta M_b y_2^-)],$$

$$A_2 = \frac{8\alpha^2 M_b}{\pi\mu^2\Omega^2}\left(\frac{\bar{n}_b}{Z_b}\right),\quad b_2 = (q^2-\omega^2+4\Omega^2)^{1/2},\quad x_2 = \frac{\pi b_2}{2m}, \tag{28}$$

$$y_2^\pm = \frac{q^2+4\Omega^2}{|qb_2 \pm 2\Omega\omega|}$$

describes the Raman scattering as well as $S^\perp$. Comparing (26) and (28) we see that they differ from one another by the pre-exponential factors and the change $\Omega\to 2\Omega$, that allows us to investigate one of them only.

From (26)-(28) it also follows that the preexponential factors and hence DSF depend on the mean bion density $\bar{n}_b$ or more precisely on $(\bar{n}_b/Z_b)$ which is less sensitive to the assumptions on $\bar{n}_b$. In our case

$$Z_b(\Omega) = \int_{-\infty}^{\infty} dp\, \exp(-\beta\sqrt{p^2+M_b^2}) = 2M_b K_1(\beta M_b)$$

or after averaging over the internal bion degree of freedom

$$Z_b = 2\int_0^\mu \frac{d\Omega}{\sqrt{\mu^2-\Omega^2}} M_b K_1(\beta M_b) = -\pi \frac{d}{d\beta}[I_0(8\beta\mu)K_0(8\beta\mu)] = \tag{29}$$

$$= 8\pi\mu[K_1(8\beta\mu)I_0(8\beta\mu) - K_0(8\beta\mu)I_1(8\beta\mu)]$$

that yields at large $z = (8\beta\mu \gg 1)$

$$Z_b \simeq \frac{\pi}{16\mu\beta^2}.$$

The quantity $\bar{n}_b$ may be also obtained from the mean kink gas density $\bar{n}_s$ by integrating over Ω and taking into account the change in the phase volume. For $\bar{n}_s$ there are at least two versions (mentioned about earlier), which differ by the preexponential factors

$$\bar{n}_s^M \simeq 4\mu\left(\frac{\beta\mu}{\pi}\right)^{1/2} \exp(-8\beta\mu) \text{ Mikeska, Curie et al.}^{/12,3/},$$

$$(s = k, b)$$

$$\bar{n}_s^T \simeq \frac{2\mu}{\pi}\left(\frac{\pi}{\beta\mu}\right)^{1/2} \exp(-8\beta\mu) \text{ Timomen and Bullough}^{/14/}.$$

The former may be obtained from the latter by renormalizing the kink energy related to their interaction with phonons. A formal basis for such a renormalization is the fact that it comes from the transfer matrix technique (see, however, above).

The second formula is simply a low temperature expansion of

$$\bar{n}_s = \frac{8\mu}{\pi} K_1(8\beta\mu)$$

which differs from Z_1^k by factor 2π (i.e., $(\bar{n}_k/Z_1^k) = (2\pi)^{-1}$) – the phase volume per a kink in the ideal kink gas. As a result, we get $(\bar{n}_k/Z_1^k) = (\Omega_{ph}^k)^{-1}$, and therefore for bions

$$\frac{\bar{n}_b}{Z_b} = \frac{1}{\Omega_{ph}^b} = \frac{1}{2\pi}\cdot\frac{1}{2\pi} = \frac{1}{4\pi^2} \tag{30a}$$

or in the Mikeska, Curie et al. version

$$\frac{\bar{n}_b^M}{Z_b} = \frac{\beta\mu}{2\pi^2} . \tag{30b}$$

In what follows we use (30a) realizing that the transformation to (30b) is straightforward. Let us evaluate the bion contribution to the CP at temperatures

$$2\beta M_k^0 \gg 1 . \tag{31}$$

In this case bions of mass $M_b \simeq \frac{b_1}{q\beta}$ give a finite contribution which upon averaging over the internal frequency or the bion mass becomes

$$S_{cp}(q,\omega) = a \int_0^{\mu} \frac{m\,dm}{\sqrt{\mu^2 - m^2}} \left(\operatorname{sech}\frac{\pi b_1}{2m}\right)^2 e^{-16\beta m \gamma}, \quad a = \frac{32 b_1}{\pi\mu^4} .$$

This integral can be evaluated via the saddle point method at the point $x_0 = (\pi b_1 / 16\beta\gamma\mu^2)^{1/2}$:

$$S_{cp}(q,\omega) = \frac{128 q}{\pi\mu^3}\left(\frac{\pi}{\kappa}\right)^{1/2} \frac{x_0^{3/2}\exp(-2\kappa x_0)}{(1 - \exp(-\kappa x_0))^2}, \quad \kappa = 16\beta\mu\gamma ;$$

or since $q = \gamma\mu\kappa x_0^2/\pi$

$$S_{cp}(q,\omega) = \frac{128}{\pi^{3/2}\mu^2} \frac{(\kappa x_0)^{7/2} e^{-2\kappa x_0}}{(1 - \exp(-\kappa x_0))^2} \frac{1}{\kappa^3},$$

$$\kappa x_0 = (16\pi\beta q)^{1/2} . \tag{32}$$

It follows from (32) that the main contribution to scattering is given by small momenta $\kappa x_0 \simeq 1.2$ or $(q_0/\mu) \simeq (4\beta M_k)^{-1}$, and at this point

$$S_{cp}(q,\omega) \simeq \frac{44.8}{\pi^{3/2}\mu^2}(2\beta M_k)^{-3}\left(1 - \frac{\omega^2}{q^2}\right)^{-3/2} .$$

The longitudinal and transversal satellite DSF are analogously obtained to give

$$S_{sat}^{\parallel}(q,\omega) = \int_0^{\mu} \frac{d\Omega}{\sqrt{\mu^2 - \Omega^2}} S^{\parallel}(q,\omega,\Omega); \quad S_{sat}^{\perp}(q,\omega) = \int_0^{\mu} \frac{d\Omega}{\sqrt{\mu^2 - \Omega^2}} S^{\perp}(q,\omega,\Omega).$$

Then using small b we expand $S_1^{\perp}$ and $S_2^{\parallel}$ with an accuracy of $O(b_1)$ to get

$$S^{\parallel}(q,\omega) = \frac{16}{\pi^{5/2}(\beta M_k)^4}\left(\frac{5}{2} + \frac{2\kappa\bar{x}}{1 - e^{-\kappa\bar{x}}}\right) \frac{(\kappa x)^{11/2} e^{-2\kappa\bar{x}}}{(1 - \exp(-\kappa\bar{x}))^2} \frac{\sqrt{\omega^2 - (2\bar{\Omega})^2}}{\omega^5}, \tag{33}$$

$$\kappa = 16\beta\mu\left(\frac{\omega}{2\Omega}\right), \quad \bar{x} = (2a/\mu\kappa)^{1/2}$$

whose peak is at the frequency

$$\omega_{\parallel} = \sqrt{\frac{5}{4}}(2\bar{\Omega}_{\parallel}), \quad \kappa\bar{x} \simeq 2.9 .$$

In the same manner, we get

$$S_{sat}^{\perp} = \frac{8/\sqrt{\pi}}{(\beta M_k)^2}\left(\frac{5}{2} + \frac{2\kappa\bar{x}}{1 + e^{-\kappa\bar{x}}}\right) \frac{(\kappa\bar{x})^{1/2} e^{-2\kappa\bar{x}}}{(1 + \exp(-\kappa\bar{x}))^2} \frac{\sqrt{\omega^2 - \bar{\Omega}^2}}{\omega^3} \tag{34}$$

with the peak being at the frequency $\omega_{\perp} = \sqrt{3/2}\,\Omega_{\perp}$, $\kappa\bar{x} \simeq 3.3$. Frequencies $\bar{\Omega}_{\perp} \simeq \bar{\Omega}_{\parallel}$ are evaluated with the help of the relation κx to be $\bar{\Omega}_{\parallel} \simeq \mu(1 - 0.8(\beta M_k)^{-2})$. Formulae (32)-(34) mean also the bion contributions to both CP and satellites (situated at the main frequency harmonics) to be of nonactivation types.

Suppose the situation studied to keep qualitatively up to temperatures $\beta M_k \lesssim 1$ then the satellites will move towards CP (simultaneously broadening) with growing temperature till full fusion at $\bar{\Omega} \to 0$. This picture takes place in a narrow temperature range near $\beta M_k \simeq 1$.

A few words are now about the small amplitude bions in the framework of ϕ^4 theory*

$$\Box\chi + \mu^2\chi - \chi^3 = 0 .$$

Such solutions have been found in a series of works (see review [4] and references cited therein)

$$\chi = -1 + 2a\sqrt{\frac{2}{3}}\,\frac{\sin\theta}{\operatorname{ch}\kappa} - 2a^2\left(1 + \frac{1}{3}\cos 2\theta\right)\operatorname{ch}^{-2}\kappa \tag{35}$$

here now $\Omega = \mu\left(\frac{2}{1 + a^2}\right)^{1/2} \simeq \mu(2(1 - a^2))^{1/2}$. This solution is constructed over vacuum $\chi = -1$.

Inserting (35) into $S^{\perp}$ and $S^{\parallel}$, we see that in addition to the Bragg term, there appear terms of the type (26) proportional

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*We should notice that the results obtained below are valid for every field theory massive in a small amplitude limit.

to α^2 as well as (27) and (28) proportional to α^4. It means that in the case of ϕ^4 model, used in ref.[1], to describe structural phase transitions we shall have the picture considered above for SG, and now $S = S_{cp.} + S_1 + S_2$:

$$S_{cp} = \frac{1}{4} S_1^{\parallel}(q,\omega,\Omega),\ S_1 = \frac{1}{18} S^{\perp}(q,\omega,\Omega),\ S_2 = \frac{1}{36} S_2^{\parallel}(q,\omega,2\Omega)$$

with $\Omega = \sqrt{2}\,\Omega(SG)$, $M_k = \frac{2\sqrt{2}}{3}\mu^3$ and possible changing of the pre-exponential factor.

At sufficiently low temperatures quantum effects will apparently play a role, e.g., will set a lower limit for bion mass in the SG model: M_b (quantim) $= (16\mu/\bar{g})\sin(\bar{g}/16)$ if

$$\bar{g} = \frac{g/\mu^2}{1 - g/8\pi\mu^2} < 8\pi .$$

6. Let us consider possible generalizations of the above models in space $R_{D,1}$. Let

$$\mathcal{L} = \bar{\psi}_\mu \psi^\mu + \epsilon V(\bar{\psi}\psi) \tag{36}$$

with ψ being the complex or real scalar field in the simplest case. Suppose now that we have the following expansion for $V(\bar{\psi}\psi)$:

$$m^2\bar{\psi}\psi - g(\bar{\psi}\psi)^2 + \dots = V, \tag{37}$$

i.e., a field theory free and massive in the $\psi \to 0$ limit.

There are now two possibilities

1) $\epsilon > 0$, (38a)

2) $\epsilon < 0$. (38b)

In the first case a stable "vacuum" state of the system, over which soliton solutions may be constructed, is trivial: $\psi_v = 0$*. In the latter, a nontrivial stable vacuum state is possible: $|\psi_v|^2 = m^2/g$ therefore, to avoid the appearance of Goldstone's bosons and gauge fields at this stage, we take the field ψ to be real. Earlier, in the series of computer experiments [22,23] the existence of stable neutral ($Q = 0$) oscillating localized solutions (pulsons) has been demonstrated for the models of both the first and the second type. For the former models there were also found stable charged ($Q \neq 0$) pulsons[23]. Moreover, it follows from these works that the evolution of various initial perturbations over stable vacuum goes, as a rule, through the pulson phase, i.e., either stable solitons (if the system has them) or weak radiating pulsons of large or small amplitudes appear as a result of the decay of initial perturbations*.

*Our concern will be only with classical finite energy solutions well localized in space.

In ref.[23] there have been stated and verified the hypothesis: uncharged pulsons exist in a certain model in those phase space regions where appropriate charged ($Q \neq 0$) solitons are stable (models are supposed non-integrable).

Letting this hypothesis be valid we consider a qualitative picture of Q-soliton behaviour. To proceed further let us reformulate Derrick's theorem as follows: in relativistic (and not) classical field theories (without gradient interactions) there are no stable localized solutions (soliton-like solutions SLS) without an internal structure (inner symmetries, time dependence and so on) in space of dimensions more than two, i.e., (2.1) and (3.1).

Consider first theories with the trivial vacuum. In a small amplitude region ($\omega \to m$) radially symmetric solutions for all models of the type (37) are given by the expansions

$$\psi = \exp(-i\omega t) \sum_{n=1} R_n(r) \qquad \text{Q-solitons,}$$

$$\phi = \sum_{n=1} \cos(n\omega t) R_n(r) \qquad \text{pulsons,}$$

and the radial function R_1 subjects to the known nonlinear boundary problem

$$(d^2/dr^2 + \frac{D-1}{r} d/dr - \kappa^2) R_1 + \xi R_1^3 = 0,$$

$$R_1(0) = \text{const},\ R_1(\infty) = 0,$$

ξ is a numerical coefficient depending on a model. It may be easily checked the system energy and "charge" (or the appropriate adiabatic invariant) are proportional

$$Q \sim E \sim \int R_1^2(\kappa, r) d^D r = \text{const}\, \alpha^{2-D}, \tag{39}$$

where again $\alpha^2 = m^2(1-\nu^2)$, $\nu = \omega/m$. It follows from (39) that the sign of $dQ/d\omega$ (or $dE/d\omega$) depends at $\alpha \ll 1$ on the number of space dimensions D:

$D = 1$: $dQ/d\omega < 0$

*Pulsons may also be created by the soliton interactions.

$D = 2$: $dQ/d\omega \simeq 0$ at $\omega \to m$.

$D = 3$: $dQ/d\omega > 0$

Therefore, small amplitude Q-solitons (and pulsons) are stable only in one-space dimension geometry; when $D = 2,3$ the sign of $dQ/d\omega$ can alter for finite amplitudes, say, at $\omega = \omega_c$ and $E = E_c$, i.e., Q-solitons of mass $M \geq E_c$ get stable. The values of ω_c and E_c are model dependent (e.g., in ϕ^4 theory $dQ/d\omega > 0$ everywhere in the SLS existence region); they have been found by computer for various field[23,26] as well as spin (magnetic)[17] models. Analogous results can be also obtained for the second class of models (38b) with nontrivial vacuum (including models with broken symmetry describing, particularly, structural phase transitions). Here small amplitude solutions are constructed over one of the vacua $\psi = \psi_v$, and the expansions look less symmetrical (see (35), review[4] and refs. cited there). In the region $a \ll 1$ the dependence of the integral $\int(\phi - \phi_v)^2 d^D x$ on a (and then on pulson mass) is like that obtained above for Q-solitons

$$M \sim a^{2-D},$$

i.e., small amplitude stable pulsons exist but in the one-dimensional systems. In more dimensional ones, stable pulsons (if exist) have a lower critical mass M_{cr} and an upper critical frequency ω_{cr}, which are model dependent too (for details, see[21]).

According to the results of various authors[22-24] the objects of pulson-type occur in the systems under perturbation of energy greater than M_{cr} (we would like to mention work[25] very interesting in this sense).

7. The above technique for calculating structural soliton factors may be applied to study more dimensional systems.

For not too large SLS amplitudes, we come to the formula of form (26)*

$$S(q,\omega) \simeq \frac{\bar{n}_b}{Z_b}\left(e^{-\beta E_0 \cdot \gamma_0^+} + \mathrm{sgn}(q^2 - \omega^2)\, e^{-\beta E_0 \cdot \gamma_0^-}\right) \qquad (40)$$

in which $(\bar{n}_b / Z_b) = (2\pi)^{-(D+1)}$ and $F_b = \frac{1}{\Delta}\int \Phi(\frac{x_0}{\Delta}) e^{i\vec{q}\vec{x}_0} d^D x_0$.

*Getting (40) we assumed again $\phi \sim \cos(kx - \bar{\Omega}t)$, i.e., small amplitude approximation.

Taking the asymptotic behaviour of the function Φ as $\Phi \to \frac{1}{r}e^{-mr}$, one can calculate F_b as follows:

$$F_b = \int d\vec{x}_\perp\, dx_3 (x_\perp^2 + x_3^2)^{-1/2} \exp(-\bar{m}\sqrt{x_3^2 + x_\perp^2} + iqx_3) \to \frac{4\pi}{q^2 + \bar{m}^2}$$

with $\bar{m} = m(1 - \Omega^2)^{1/2}$ being the effective mass.

Let us formulate briefly the results obtained:

1) The numerical studies within various field theories show that the appearance of soliton (bion)-type excitations is not a privilege of integrable two space-time dimensional systems but is a natural behaviour of the systems possessing stable SLS.

2) The lower energy limit E_{cr} for the existence of such objects grows with the number of system dimensions D in the framework of the models considered and vanishes at $D = 1$ (in the classical limit).

3) Using the hypothesis of dilute soliton (bion) gas being in the thermodynamic equilibrium, the dynamic structure factor $S(q,\omega)$ may be calculated, which describes scattering of light, neutrons and so on.

4) The functions $S(q,\omega,T)$ of T are different for various D in the framework of the same model. When $D = 1$ stable solitons (bions) arise at every low temperature T, as a result S contains the central peak and satellites (red and blue).

In the $D > 1$ case stable solitons (bions) and hence CP and satellites in DSF appear only when temperature is greater than a certain critical value $T_c \simeq E_{cr}$. The satellites move towards CP with growing temperature for any D.

5) Such a behaviour of the system may be regarded as a phase transition with respect to its clusterization. Temperature of this transition (clusterization) vanishes in the one-space dimensional systems and is finite in more dimensional ones growing with D.

Note ultimately, that an analogous picture has been observed in paper[27] in which the authors investigated the structural factor of a certain model by means of molecular dynamic method and computer.

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Федянин В.К., Маханьков В.Г. E17-83-30

Идеальный газ частицеподобных возбуждений
при низких температурах

Предложен полуфеноменологический подход к описанию равновесных характеристик "кинков", базирующийся на модели решеточного газа. Получены и исследованы динамические структурные факторы, определяемые солитонными возбуждениями в рамках уравнения Клейна-Гордона. Показано, что возбуждения бионного типа при низких температурах вносят основной вклад в центральный пик /конкурируя с вкладом кинков/, а также приводят к появлению сателлитов на гармониках основной частоты бионов /комбинационное рассеяние/.

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Fedyanin V.K., Makhankov V.G. E17-83-30

Ideal Gas of Particle-Like Excitations
at Low Temperatures

A phenomenological approach to the description of the equilibrium characteristics of "kink" gas is proposed, which is based on model of lattice gas. A general simple technique for obtaining the dynamical structure factors (DSF) of solitons and bions in the "gas " approximation is developed. The DSF defined by soliton excitations in the framework of the Klein-Gordon equation are got and discussed. It is shown that at low temperatures the bion type excitations dominate the contribution to the central peak (CP) as well as give rise to the satellites to appear at harmonics of the main bion frequency.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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