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E17-83-293

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ELECTRON-PHONON INTERACTION AND THE GENERALIZED KINETIC EQUATION FOR SYSTEMS INTERACTING WITH HIGH-INTENSITY ELECTROMAGNETIC WAVE FIELDS

Submitted to "Physica Scripta"

1983

1. ON THE ACCOUNT OF THE ELECTRON-PHONON INTERACTION IN THE FRAMEWORK OF THE RANDOM PHASE APPROXIMATION

It is well known that the electron-phonon interaction plays an important role in may processes in solids (see, for example, [1]). In investigating such processes one usually takes into account this interaction in the framework of the random phase approximation (RPA), where all interparticle interactions are involved in the self-consistent field, and so the collision effects are neglected (see [2]). Consider, for instance, the situation in the theory of nonlinear action of high-power electromagnetic radiation fields on solids. Here, in the optical region of the spectrum, where the conduction electrons interact mainly with the optical phonons, the kinetic equation for the perturbation part $f(\vec p + \vec q, \vec p, t)$ of the quantum distribution function of electrons

$$\langle a^+_p a_{p+q} \rangle_t = \langle a^+_p a_p \rangle \delta_{\vec q, 0} + f(\vec p + \vec q, \vec p, t) \qquad (1)$$

contains the self-consistent electromagnetic field with the scalar $\phi(\vec r, t)$ and vector $\vec A(\vec r, t)$ potentials rather than the phonon operators b^+_q, b_q in an explicit form. In the formula (1) $a^+_p(a_p)$ is the creation (annihilation) operator of an electron with the canonical momentum $\vec p$ and mass m; the symbol $\langle \ldots \rangle_t$ means the quantum statistical averaging with the use of the Hamiltonian depending on time in a general case. The potentials $\vec A, \phi$ in their turn obey the Maxwell equations with the dielectric function corresponding to the solid considered with its concrete phonon structure. The mentioned equation for f is of the form [3]

$$\{-i\frac{\partial}{\partial t} + \epsilon_{p+q} - \epsilon_p\} f(\vec p + \vec q, \vec p, t) = \frac{e}{mc}(\vec A_0(t) \cdot \vec q)\, f(\vec p + \vec q, \vec p, t) + \{e\phi(\vec q, t) - \frac{e}{mc}(\vec p + \frac{1}{2}\vec q - \frac{e}{c}\vec A_0(t))\, \vec A(\vec q, t)\}[n_{p+q}(t) - n_p(t)]. \qquad (2)$$

Here ϵ_p is the kinetic energy of an electron, $\vec A_0(t)$ is the vector potential of the pumping field which is expressed in the dipole approximation by an oscillatory electric field $\vec E_0(t) = \vec E_0 \sin\omega_0 t$. The equilibrium distribution function $n_p(t) \equiv \langle a^+_p a_p \rangle_t$ in this approximation is set to be of the Fermi or Maxwell-Boltzmann

form in the quantum and classical cases, respectively. In the more general case of the interaction of electrons with phonons of arbitrary kinds, one has the following systems of equations for $f(\vec p + \vec q, \vec p, t)$ instead of (2) (the retardation effect is neglected, see [4]):

$$\{-i\frac{\partial}{\partial t} + \epsilon_{p+q} - \epsilon_p\} f(\vec p + \vec q, \vec p, t) + [\frac{4\pi e^2}{q^2}\delta n_q(t) + L_q Q_q(t)][n_p - n_{p+q}] + \frac{e}{mc\omega_0}(\vec q \cdot \frac{\partial \vec A_0(t)}{\partial t})\, f(\vec p + \vec q, \vec p, t) = 0, \qquad (3)$$

$$\{\frac{\partial^2}{\partial t^2} + \omega_q^2\} Q_q(t) = -L_q^* \delta n_q(t), \qquad (4)$$

from which the phonon coordinate $Q_q(t) \equiv b_q(t) + b^+_{-q}(t)$ can be easily eliminated. In equations (3), (4) $\delta n_q(t) \equiv \sum_p f(\vec p + \vec q, \vec p, t)$, ω_q is the phonon frequency; and L_q, the electron-phonon interaction coefficient.

Thus, we see that the RPA considerably simplifies the procedure of solving the equations of motion for all the components of the particle system. On the other hand, it is obvious that a more adequate account of the electron-phonon interaction is really essential for the analysis of the behaviour of equilibrium distribution functions of particles as well as of the relaxation processes in the presence of external fields. In the next section an attempt is made to obtain the kinetic equation with such an account of the electron-phonon interaction.

2. THE GENERALIZED KINETIC EQUATION FOR ELECTRON-PHONON SYSTEMS INTERACTING WITH HIGH-INTENSITY ELECTROMAGNETIC WAVE FIELDS

We shall proceed from the method of elimination of boson operators in kinetic equations for the dynamical systems interacting with a phonon fields developed recently by N.N.Bogolubov and N.N.Bogolubov (Jr.) [5-7]. This method has proved to be effective in the treatment of a set of problems of solid state theory such as the polaron problem, the problem of electrical and heat conduction in metals and semiconductors (see [8]), etc.

In [5-7] it has been shown that the equation of motion for an arbitrary dynamical operator $\hat O(S_t)$ of the system S interacting with the boson field Σ can be written in the following form not containing the boson operators b_k^+, b_k:

$$\underset{(S)}{Sp}\{\hat{O}(S)\frac{\partial\rho_t(S)}{\partial t}+\frac{\Gamma(t,S)\,\hat{O}(S)-\hat{O}(S)\,\Gamma(tS)}{i}\rho_t(S)\}=$$

$$=\sum_{\vec{k}}\int_{t_0}^{t}d\tau\,\underset{(S,\Sigma)}{Sp}\exp[-i\omega_k(t-\tau)]\{N_k C_k^+(\tau,S_\tau)[\hat{O}(S_t),C_k(t,S_t)]+$$

$$+(1+N_k)[C_k^+(t,S_t),\hat{O}(S_t)]C_k(\tau,S_\tau)\}D_{t_0}+ \tag{5}$$

$$+\sum_{\vec{k}}\int_{t_0}^{t}d\tau\,\underset{(S,\Sigma)}{Sp}\exp[i\omega_k(t-\tau)]\{(1+N_k)C_k(\tau,S_\tau)[\hat{O}(S_t),C_k^+(t,S_t)]+$$

$$+N_k[C_k^+(t,S_t),\hat{O}(S_t)]C_k(\tau,S_\tau)\}D_{t_0}.$$

Here $N_k=\langle b_k^+b_k\rangle=[\exp(\omega_k/T)-1]^{-1}$ is the equilibrium boson distribution; D_t is the statistical operator of the total (S,Σ) -system (t_0 is the moment when the external fields and interactions are turned on), $\rho_t(S)$ is the reduced statistical operator defined as $\rho_t(S)=\underset{(\Sigma)}{Sp}D_t$. Note, that the dynamical system S with the partial Hamiltonian $\Gamma(t,S)$ can include the action of external fields but should not depend on the variables of the boson field Σ, the interaction between the subsystems Σ and S being expressed by the Hamiltonian of the form

$$H_{int}=\sum_{\vec{k}}\{C_k(t,S)b_k+C_k^+(t,S)b_k^+\}. \tag{6}$$

Now, following the notation of /5-7/, we can present the Hamiltonian of our electron-phonon system subjected to the action of a high-power electromagnetic radiation field and a weak d.c. electric field in the form

$$H(t,S,\Sigma,\gamma)=H(t,S)+H(\Sigma)+H(\gamma)+$$

$$+\sum_{\vec{k}}\{C_k(t,S)\,b_k+C_k^+(t,S)\,b_k^+\}, \tag{7}$$

$$H(t,S)=\frac{1}{2m}\sum_{\vec{p},s}[\vec{p}-e^{\epsilon t}\frac{e}{c}\vec{A}_0(t)]^2a_{ps}^+a_{ps}-$$

$$-e^{\epsilon t}\frac{e}{mc}\sum_{\vec{p},\vec{q},s}[\vec{p}-\frac{e}{c}\vec{A}_0(t)]\,\vec{A}(q)\,a_{p+qs}^+a_{ps}+$$

$$+\frac{1}{2}e^{\epsilon t}\sum_{\substack{\vec{p},\vec{q},s\\ \vec{p}',s'}}{}'\phi_q\,a_{p+qs}^+a_{p'-qs'}^+a_{p's'}a_{ps}+\sum_{\vec{p},\vec{q},s}\phi_q^o\,a_{p+qs}^+a_{ps}, \tag{8}$$

$$H(\Sigma)=\sum_{\vec{k}}\omega_k b_k^+b_k\quad(\omega_k>0), \tag{9}$$

$$H(\gamma)=\sum_{\vec{k}',\nu}(k'c)\,c_{k'\nu}^+c_{k'\nu}, \tag{10}$$

$$C_k(t,S)=e^{\epsilon t}L_k\sum_{\vec{p},s}a_{p+ks}^+a_{ps},\quad C_k^+(t,S)=e^{\epsilon t}L_k^*\sum_{\vec{p},s}a_{ps}^+a_{p+ks}. \tag{11}$$

Here $c_{k\nu}^+$ ($c_{k\nu}$) is the creation (annihilation) operator of a "self-consistent" photon with the quasi-momentum $\vec{k}$ and polarization vector $\vec{e}_{k\nu}$; $\phi_q=4\pi e^2/q^2$ represents the Coulomb interaction between electrons;

$$\phi_q^o=(2\pi i)^3\,e(\vec{E}\cdot\frac{\partial}{\partial\vec{q}})\delta(\vec{q}),$$

where $\vec{E}$ is the d.c. electric field vector (this field is introduced for embracing the conductivity problem) and $\delta(\vec{q})$ is the Dirac delta-function;

$$\vec{A}(\vec{k})=\sum_{\nu}(\frac{2\pi c}{k})^{1/2}(c_{k\nu}+c_{-k\nu}^+)\,\vec{e}_{k\nu} \tag{12}$$

is the space Fourier component of the self-consistent field vector potential; the term $e^{\epsilon t}$ indicates the adiabatic switch on of all the external fields and interactions at the moment $t=-\infty$ (in the final results it should be set $\epsilon=+0$). The system of units with $\hbar=1$ is used.

Taking the dynamical operator $\hat{O}(S_t)$ in the form

$$\hat{O}(S_t)=\langle a_{ps}^+a_{p+qs}\rangle_t \tag{13}$$

(s is the electron spin) and using equation (5) with the Hamiltonian (7)-(11) (where $\Gamma(t,S)\equiv H(t,S)+H(\gamma)$), we can obtain after some algebraic procedure the quantum kinetic equation for the generalized distribution function $\langle a_{ps}^+a_{p+qs}\rangle_t$:

$$\{-i\frac{\partial}{\partial t}+W_{p+q}-W_p-e^{\epsilon t}\sum_{\vec{k},\ell,\ell'}L_k^2J_\ell(\lambda)J_{\ell'}(\lambda)[(\frac{N_k}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0-i0}+$$

$$+\frac{N_k+1}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0-i0})e^{i\omega_0(\ell'-\ell)t}-(\frac{N_{-k}+1}{\omega_k+\epsilon_p-\epsilon_{p+k}+\ell\omega_0+i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}+\ell\omega_0+i0})e^{-i\omega_0(\ell'-\ell)t}]\}<a^+_{ps}a_{p+qs}>_t=$$

$$=e^{\epsilon t}\sum_{\vec q'}\{[\frac{e}{mc}(\vec p+\vec q-\frac{e}{c}\vec A_0)\vec A(q')-\phi^{\circ}_{q'}]<a^+_{ps}a_{p+q-q's}>_t-$$

$$-[\frac{e}{mc}(\vec p-\frac{e}{c}\vec A_0)\vec A(q')-\phi^{\circ}_{q'}]<a^+_{p+q's}a_{p+qs}>_t]+e^{\epsilon t}\sum_{\vec p',s',\vec k}\{\phi_k-$$

$$-L^2_k\sum_{\ell,\ell'}J_\ell(\lambda)J_{\ell'}(\lambda)[\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p'}-\epsilon_{p'-k}-\ell\omega_0-i0}+\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_{-k}+\epsilon_{p'-k}-\epsilon_{p'}+\ell\omega_0+i0}]\}\times$$

$$\times[<a^+_{p+ks}a^+_{p'-ks'}a_{p's'}a_{p+qs}>_t-<a^+_{ps}a^+_{p'-ks'}a_{p's'}a_{p+q-ks}>_t]-$$

$$-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L^2_kJ_\ell(\lambda)J_{\ell'}(\lambda)\{e^{i\omega_0(\ell'-\ell)t}[\frac{N_{-k}+1}{\omega_{-k}+\epsilon_{p+q-k}-\epsilon_{p+q}-\ell\omega_0-i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}]-e^{-i\omega_0(\ell'-\ell)t}[\frac{N_{-k}+1}{\omega_{-k}+\epsilon_{p-k}-\epsilon_p+\ell\omega_0+i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}+\ell\omega_0+i0}]\}<a^+_{p-ks}a_{p+q-ks}>_t\,. \qquad (14)$$

Here $W_p=[\vec p-(e/c)\vec A_0(t)]^2/2m$; $J_\ell(\lambda)$ is the Bessel function of the first kind of the argument $\lambda=e(\vec E_0\cdot\vec q)/m\omega_0^2$; ℓ , ℓ' are integers. To give a more clear physical interpretation of equation (14), we write down its solution in the form (1) and perform the linearization procedure as a result of which we have the following equations for the equiliblium distribution function $n_p(t)$ and its perturbation $f(\vec p+\vec q,\vec p,t)$ under the action of the pumping field:

$$i\{\frac{\partial}{\partial t}+e\vec E\frac{\partial}{\partial\vec p}\}n_p(t)=e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L^2_kJ_\ell(\lambda)J_{\ell'}(\lambda)\{[N_k+1)n_{p+k}-N_kn_p]\times$$

$$\times(\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0-i0}-\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0+i0})+[N_kn_{p-k}-(N_k+1)n_p]\times$$

$$\times(\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}-\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0+i0})\}, \qquad (15)$$

$$\{-i\frac{\partial}{\partial t}+\epsilon_{p+q}-\epsilon_p-\frac{e}{mc}(\vec A_0(t)\cdot\vec q)-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L^2_kJ_\ell(\lambda)J_{\ell'}(\lambda)[e^{i\omega_0(\ell'-\ell)t}\times$$

$$\times(\frac{N_k}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0-i0}+\frac{N_k+1}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0-i0})-e^{-i\omega_0(\ell'-\ell)t}\times$$

$$\times(\frac{N_k+1}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0+i0}+\frac{N_k}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}-\ell\omega_0+i0})]\}f(\vec p+\vec q,\vec p,t)=$$

$$=e^{\epsilon t}\{\frac{e}{mc}(\vec p-\frac{e}{c}\vec A_0(t))\vec A(\vec q,t)+\sum_{\vec p',s'}\phi_qf(\vec p'+\vec q,\vec p',t)-L^2_q\sum_{\substack{\vec p',s'\\ \ell,\ell'}}J_\ell(\lambda)J_{\ell'}(\lambda)\times$$

$$\times[\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_q+\epsilon_{p'}-\epsilon_{p'-q}-\ell\omega_0-i0}+\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_{-q}+\epsilon_{p'-q}-\epsilon_{p'}+\ell\omega_0+i0}]f(\vec p'+\vec q,\vec p',t)\}[n_{p+q}-n_p]-$$

$$-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L^2_kJ_\ell(\lambda)J_{\ell'}(\lambda)\{e^{i\omega_0(\ell'-\ell)t}[\frac{(N+1)f(\vec p+\vec k+\vec q,\vec p+\vec k,t)}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}-\ell\omega_0-i0}-$$

$$-\frac{N_kf(\vec p-\vec k+\vec q,\vec p-\vec k,t)}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}]-[\frac{N_kf(\vec p-\vec k+\vec q,\vec p-\vec k,t)}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0+i0}+$$

$$+\frac{(N_k+1)f(\vec p+\vec k+\vec q,\vec p+\vec k,t)}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0+i0}]e^{-i\omega_0(\ell'-\ell)t}\}. \qquad (16)$$

The right-hand side of equation (15) defines the deviation of the distribution function from the usual Fermi (or Maxwell-Boltzmann) form. In the approximation with $\ell=\ell'$ that we call "the slow oscillation approximation" the collision integral is considerably simplified, and we obtain the known equation /9/ for $n_p(t)$:

$$+\frac{N_k+1}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0-i0}\Big)e^{i\omega_0(\ell'-\ell)t}-\Big(\frac{N_{-k}+1}{\omega_k+\epsilon_p-\epsilon_{p+k}+\ell\omega_0+i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}+\ell\omega_0+i0}\Big)e^{-i\omega_0(\ell'-\ell)t}\Big]\Big\}\langle a^+_{ps}a_{p+qs}\rangle_t=$$

$$=e^{\epsilon t}\sum_{\vec q'}\{[\frac{e}{mc}(\vec p+\vec q-\frac{e}{c}\vec A_0)\vec A(q')-\phi^\circ_{q'}]\langle a^+_{ps}a_{p+q-q's}\rangle_t-$$

$$-[\frac{e}{mc}(\vec p-\frac{e}{c}\vec A_0)\vec A(q')-\phi^\circ_{q'}]\langle a^+_{p+q's}a_{p+qs}\rangle_t]+e^{\epsilon t}\sum_{\vec p',s',\vec k}\{\phi_k-$$

$$-L_k^2\sum_{\ell,\ell'}J_\ell(\lambda)J_{\ell'}(\lambda)\Big[\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p'}-\epsilon_{p'-k}-\ell\omega_0-i0}+\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_{-k}+\epsilon_{p'-k}-\epsilon_{p'}+\ell\omega_0+i0}\Big]\}\times$$

$$\times[\langle a^+_{p+ks}a^+_{p'-ks'}a_{p's'}a_{p+qs}\rangle_t-\langle a^+_{ps}a^+_{p'-ks'}a_{p's'}a_{p+q-ks}\rangle_t]-$$

$$-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L_k^2J_\ell(\lambda)J_{\ell'}(\lambda)\{e^{i\omega_0(\ell'-\ell)t}\Big[\frac{N_{-k}+1}{\omega_{-k}+\epsilon_{p+q-k}-\epsilon_{p+q}-\ell\omega_0-i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}\Big]-e^{-i\omega_0(\ell'-\ell)t}\Big[\frac{N_{-k}+1}{\omega_{-k}+\epsilon_{p-k}-\epsilon_p+\ell\omega_0+i0}+$$

$$+\frac{N_k}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}+\ell\omega_0+i0}\Big]\}\langle a^+_{p-ks}a_{p+q-ks}\rangle_t\,. \qquad (14)$$

Here $W_p=[\vec p-(e/c)\vec A_0(t)]^2/2m$; $J_\ell(\lambda)$ is the Bessel function of the first kind of the argument $\lambda=e(\vec E_0\cdot\vec q)/m\omega_0^2$; ℓ , ℓ' are integers. To give a more clear physical interpretation of equation (14), we write down its solution in the form (1) and perform the linearization procedure as a result of which we have the following equations for the equiliblium distribution function $n_p(t)$ and its perturbation $f(\vec p+\vec q,\vec p,t)$ under the action of the pumping field:

$$i\{\frac{\partial}{\partial t}+e\vec E\frac{\partial}{\partial\vec p}\}n_p(t)=e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L_k^2J_\ell(\lambda)J_{\ell'}(\lambda)\{[(N_k+1)n_{p+k}-N_kn_p]\times$$

$$\times\Big(\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0-i0}-\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0+i0}\Big)+[N_kn_{p-k}-(N_k+1)n_p]\times$$

$$\times\Big(\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}-\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0+i0}\Big)\}, \qquad (15)$$

$$\{-i\frac{\partial}{\partial t}+\epsilon_{p+q}-\epsilon_p-\frac{e}{mc}(\vec A_0(t)\cdot\vec q)-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L_k^2J_\ell(\lambda)J_{\ell'}(\lambda)[e^{i\omega_0(\ell'-\ell)t}\times$$

$$\times\Big(\frac{N_k}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0-i0}+\frac{N_k+1}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0-i0}\Big)-e^{-i\omega_0(\ell'-\ell)t}\times$$

$$\times\Big(\frac{N_k+1}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0+i0}+\frac{N_k}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}-\ell\omega_0+i0}\Big)]\}f(\vec p+\vec q,\vec p,t)=$$

$$=e^{\epsilon t}\{\frac{e}{mc}(\vec p-\frac{e}{c}\vec A_0(t))\vec A(\vec q,t)+\sum_{\vec p',s'}\phi_q f(\vec p'+\vec q,\vec p',t)-L_q^2\sum_{\substack{\vec p',s'\\ \ell,\ell'}}J_\ell(\lambda)J_{\ell'}(\lambda)\times$$

$$\times\Big[\frac{e^{i\omega_0(\ell'-\ell)t}}{\omega_q+\epsilon_{p'}-\epsilon_{p'-q}-\ell\omega_0-i0}+\frac{e^{-i\omega_0(\ell'-\ell)t}}{\omega_{-q}+\epsilon_{p'-q}-\epsilon_{p'}+\ell\omega_0+i0}\Big]f(\vec p'+\vec q,\vec p',t)\}[n_{p+q}-n_p]-$$

$$-e^{\epsilon t}\sum_{\vec k,\ell,\ell'}L_k^2J_\ell(\lambda)J_{\ell'}(\lambda)\{e^{i\omega_0(\ell'-\ell)t}\Big[\frac{(N+1)f(\vec p+\vec k+\vec q,\vec p+\vec k,t)}{\omega_k+\epsilon_{p+q+k}-\epsilon_{p+q}-\ell\omega_0-i0}-$$

$$-\frac{N_kf(\vec p-\vec k+\vec q,\vec p-\vec k,t)}{\omega_k+\epsilon_p-\epsilon_{p-k}-\ell\omega_0-i0}\Big]-\Big[\frac{N_kf(\vec p-\vec k+\vec q,\vec p-\vec k,t)}{\omega_k+\epsilon_{p+q}-\epsilon_{p+q-k}-\ell\omega_0+i0}+$$

$$+\frac{(N_k+1)f(\vec p+\vec k+\vec q,\vec p+\vec k,t)}{\omega_k+\epsilon_{p+k}-\epsilon_p-\ell\omega_0+i0}\Big]e^{-i\omega_0(\ell'-\ell)t}\}. \qquad (16)$$

The right-hand side of equation (15) defines the deviation of the distribution function from the usual Fermi (or Maxwell-Boltzmann) form. In the approximation with $\ell=\ell'$ that we call "the slow oscillation approximation" the collision integral is considerably simplified, and we obtain the known equation /9/ for $n_p(t)$:

$$i\{\frac{\partial}{\partial t} + e\vec{E}\frac{\partial}{\partial \vec{p}}\} n_p(t) = 2\pi e^{\epsilon t} \sum_{\vec{k},\ell} L_k^2 J_\ell^2(\lambda) \{[(N_k+1) n_{p+k} - N_k n_p] \times$$
$$\times \delta(\epsilon_{p+k} - \epsilon_p - \omega_k - \ell\omega_0) + [N_k n_{p-k} - (N_k+1) n_p] \delta(\epsilon_{p-k} - \epsilon_p + \omega_k + \ell\omega_0)\}. \tag{17}$$

Now the relaxation time τ can be introduced and determined as a function of the frequency and intensity of the pumping field. Following the calculations performed in [9] we can write down the formula for τ as follows (see also [10]):

$$\tau(\vec{p}, E_0, \omega_0) = \frac{\tau^{o,a}(\vec{p})}{F(2eE_0 p/m\omega_0^2)}, \tag{18}$$

where $\tau^{o,a}$ is the relaxation time in the absence of external fields for the cases of interaction of electrons with optical (o) and acoustical (a) phonons. The function $F(x)$ is defined by the formula

$$F(x) = \frac{4}{3}\int_0^1 (1-y^3) J_0^2(xy)\, dy = 2\sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(n+2)(2n+1)(n!)^3} \left(\frac{x}{2}\right)^{2n}. \tag{19}$$

In obtaining (18) the average over all polarization directions of the field $E_0(t)$ was taken and the approximation with $\ell = 0$ was used, which is valid for the value of E_0 and ω_0 satisfying the condition $\lambda \ll 1$ and also for the narrow-band semiconductors with the conduction band width $\Delta \ll \omega_0$. It is interesting to note that the use of such an expression (18) for the renormalized electron relaxation time τ directly introduces some correction in determining the threshold fields for the parametric excitation of the plasmon-phonon eigenmodes of a solid under the action of electromagnetic waves (see [3]). Indeed, the threshold field E_{oth} must be calculated from the equality

$$\gamma(E_{oth}, \omega_0) = (\tau_1 \cdot \tau_2)^{-1/2}, \tag{20}$$

where τ_1 is the plasmon damping coefficient which is connected with the electron relaxation time as $\tau_1 = 2\tau$, and τ_2 is the damping coefficient of the phonon mode (or of the mode of other nature); γ is the parametric growth rate of the given pair of modes.

Equation (17) has been considered in connection with the conductivity problem in crystals in the high- and low-frequency limits in [9,11]. An attempt has been made in [12,13] to obtain solutions for $n_p(t)$ in particular cases of a nondegenerate so-

lid state plasma interacting with equilibrium acoustical phonons.

The analysis of the dispersion properties of an electron-phonon system placed under the action of a strong electromagnetic wave field on the basis of equation (16) and the system of Maxwell equations will be made elswhere. Here, in conclusion, we present the form of this equation that is convenient for the above-mentioned purpose. Thus, introducing the new variable $\tilde{f} = f \exp[-i\lambda \sin\omega_0 t]$ and transiting to the Fourier representation, we come to the following equation:

$$\sum_{\vec{p}} \tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega) + P(\vec{q}, \omega)\{\phi_q \sum_{\vec{p}} \tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega) + L_q^2 \sum_{\vec{p},\ell,\ell'} J_\ell(\lambda) J_{\ell'}(\lambda) \times$$
$$\times [\frac{\tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega - \omega_0\tilde{\ell})}{\omega_q + \epsilon_{p-q} - \epsilon_p + \ell\omega_0 + i0} + \frac{f(\vec{p}+\vec{q}, \vec{p}, \omega + \omega_0\tilde{\ell})}{\omega_{-q} + \epsilon_p - \epsilon_{p-q} - \ell\omega_0 - i0}]\} + \sum_{\substack{\vec{p},\vec{k} \\ \ell,\ell'}} L_k^2 J_\ell(\lambda) J_{\ell'}(\lambda) \times$$
$$\times \{\frac{N_k \tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega - \omega_0\tilde{\ell})}{\omega_k + \epsilon_p - \epsilon_{p+k} + \ell\omega_0 + i0} [\frac{1}{\omega + \epsilon_{p+k} - \epsilon_{p+k+q} + i0} - \frac{1}{\omega + \epsilon_p - \epsilon_{p+q} + i0}] +$$
$$+ \frac{(N_k+1)\tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega - \omega_0\tilde{\ell})}{\omega_k + \epsilon_{p+q-k} - \epsilon_{p+q} + \ell\omega_0 + i0} [\frac{1}{\omega + \epsilon_{p-k} - \epsilon_{p-k+q} + i0} - \frac{1}{\omega + \epsilon_p - \epsilon_{p+q} + i0}] +$$
$$+ \frac{(N_k+1)\tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega + \omega_0\tilde{\ell})}{\omega_k + \epsilon_{p-k} - \epsilon_p + \ell\omega_0 - i0} [\frac{1}{\omega + \epsilon_p - \epsilon_{p+q} + i0} - \frac{1}{\omega + \epsilon_{p-k} - \epsilon_{p-k+q} + i0}] +$$
$$+ \frac{N_k \tilde{f}(\vec{p}+\vec{q}, \vec{p}, \omega + \omega_0\tilde{\ell})}{\omega_k + \epsilon_{p+q} - \epsilon_{p+q+k} + \ell\omega_0 - i0} [\frac{1}{\omega + \epsilon_p - \epsilon_{p+q} + i0} - \frac{1}{\omega + \epsilon_{p+k} - \epsilon_{p-k+q} + i0}]\} = 0. \tag{21}$$

Here $P(\vec{q}, \omega) = \sum_{\vec{p}} [(n_{p+q} - n_p)/(\omega + \epsilon_p - \epsilon_{p+q} + i0)]$ is the electron polarizability; $\tilde{\ell} = \ell' - \ell$; the summation over $\vec{p}$ was introduced and the terms with $\vec{A}(\vec{q}, t)$ representing the retardation interaction in the system were omitted for simplicity.

ACKNOWLEDGEMENT

The author is deeply grateful to Academician N.N.Bogolubov for paying her attention to the electron-phonon interaction problem in the systems discussed and the stimulating interest in a range of questions related.

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Received by Publishing Department
on May 4,1983.

Во Хонг Ань E17-83-293

Электрон-фононное взаимодействие и обобщенное кинетическое уравнение для систем, взаимодействующих с сильными полями электромагнитных волн

На основе метода исключения бозонных операторов Н.Н.Боголюбова выводится обобщенное кинетическое уравнение для системы взаимодействующих электронов и фононов, находящихся под воздействием сильного поля электромагнитной волны, с учетом эффектов электрон-фононных столкновений. Рассматриваются условия, при которых интеграл столкновений допускает введение времени релаксации, и вычисляется последнее как функция частоты и интенсивности поля накачки.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Vo Hong Anh E17-83-293

Electron-Phonon Interaction and the Generalized Kinetic Equation for Systems Interacting with High-Intensity Electromagnetic Wave Fields

The generalized kinetic equation for an electron-phonon system subjected to the action of an intense electromagnetic wave field is derived on the basis of the Bogolubov method of elimination of boson operators, taking into account the electron-phonon collision effects. The conditions are considered under which the collision integral allows the introduction of the relaxation time, and the latter is calculated as a function of the frequency and intensity of the driving field.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983