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BIFURCATIONS OF FLUXON BOUND STATES IN NONUNIFORM LONG JOSEPHSON JUNCTIONS

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In long Josephson junctions with local inhomogeneities in which the Josephson current density $j(x)$ is locally decreasing, i.e. $j(x)=[1-\sum_{i=1}^{n}\mu_i\,\delta(x-x_i)]\,\sin\phi(x)$, $\mu_i>0$ x), there exist stable static distributions of the magnetic flux $\Phi(x)=\Phi_0\,\phi(x)/2\pi$, or fluxon bound states /1/ . For the zero external field the flux is localized near the inhomogeneities. As the values of the applied magnetic field, h_0 and h_L on the edges of the junction, $x_0=0$ and $x_{n+1}=L$, are changed, the flux distributions, generally, are deformed smoothly, but for some critical values of h_0 or h_L the bound states can be abruptly created or destroyed (a bifurcation). Similar bifurcation phenomena occur when the parameters x_i and μ_i are changed. Some special bifurcations have been treated in Ref.1. Here we show how to derive the bound states and their bifurcations in a more general case. A simple criterion for the stability of the bound states with respect to small fluctuations (local stability) is also proposed. Finally, some ideas concerning a direct experimental observation of the bound states and of transitions between them near the bifurcation points are briefly discussed.

Both the equation $\phi''(x)=j(x)$ (see Eq.(2.1)) and the boundary conditions $\phi'(0)=0$, $\phi(L)=h_L$ can be derived from the variational principle $\delta\mathcal{Y}=0$ for the generalized Hamiltonian

$$\mathcal{Y}=\sum_{i=0}^{n}\int_{x_i}^{x_{i+1}}dx\,[\phi'^2/2+2\sin^2(\phi/2)]-\sum_{i=1}^{n}2\mu_i\sin^2(\phi_i/2)-h_L\phi_L+h_0\phi_0. \qquad (1)$$

Here $\phi_i\equiv\phi(x_i)$, $\phi_0\equiv\phi(x_0)\equiv\phi(0)$, $\phi_L\equiv\phi(x_{n+1})\equiv\phi(L)$, and variations of $\phi(x)$ are free. The difference between $\mathcal{Y}$ and the energy $\mathcal{E}$, used in Ref.1 (see Eq. (3.1)), is easily interpreted for the uniform external magnetic field. If $h_0=h_L$, the quantity $(\phi_L-\phi_0)$ is proportional to the total magnetic flux trapped in the junction (the magnetization of the junction), and the relation between $\mathcal{Y}$ and $\mathcal{E}$

x) Here we use the units and notation of Ref.1 (see also Ref.2), and any references to the formulae given in Ref.1 are labelled with a mark I, e.g. Eq. (2.1) means Eq. (2) in Ref.1.

is similar to the relation between the Gibbs free energy (or the thermodynamic potential) and the Helmgoltz free energy (see Ref.2, p. 80) . Even for $h_0=h_L$ the generalized Hamiltonian (1) is conceptually distinct from the Gibbs free energy used in Ref.2; nevertheless, this analogy is useful for understanding the phenomena occurring in inhomogeneous Josephson junctions. Consider, for example, the criterion for the local stability of a static bound state $\phi(x)$. This state is stable against small fluctuations, $\delta\phi(x)$, if $\delta^2\mathcal{Y}>0$. As for this state $\delta\mathcal{Y}\{\phi\}=0$, we see that $\phi(x)$ minimizes $\mathcal{Y}\{\phi\}$. The condition $\delta^2\mathcal{Y}>0$ is equivalent to the requirement that the eigenvalues of the boundary value problem

$$-\psi''+[1-\sum_{i=1}^{n}\mu_i\,\delta(x-x_i)]\,\psi(x)\cos\phi(x)=\omega^2\psi(x),\quad \psi'(0)=\psi'(L)=0, \qquad (2)$$

be positive. If all eigenvalues ω^2 are positive, then the minimum one, ω_0^2, gives the fundamental frequency of the response of the system to small external perturbations (see Ref.2):

$$\phi(x,t)=\phi(x)+e^{-i\omega_0 t}\,\psi_0(x)+\ldots,\quad |\psi_0|\ll|\phi|.$$

If there is at least one negative eigenvalue, $\omega_1^2<0$, then the bound state $\phi(x)$ is unstable, and its lifetime is $\sim|\omega_1|^{-1}$. Consider now the dependence of some positive eigenvalue on the parameters $p=(h_0,h_L,x_i,\mu_i)$ (it is dependent on p because $\phi(x)$ changes with p, $\phi(x)=\phi(x;p)$). If the eigenvalue $\omega^2(p)$ vanishes for $p=p_c$, we call p_c the bifurcation point of the state $\phi(x;p)$. The set of all bifurcation points in the p-space forms the bifurcation surface (or, the catastrophe surface) which is the boundary of the stability domain for the bound states $\phi(x;p)$. When a trajectory in the p-space crosses the bifurcation surface, the number of the stable bound states abruptly changes. On the surface, when $p=p_c$, the stable state $\phi(x;p)$ merges with some unstable state, i.e. for $p=p_c$ there are at least two degenerated states $\phi(x;p_c)$). We see that the bifurcation surfaces are analogous to boundaries of stability domains for thermodynamic phases. In a sense, stable bound states $\phi(x;p)$ may be considered to be analogs of thermodynamic phases.

The bifurcation surfaces for long Josephson junctions with inhomogeneities can be derived by relatively simple means. For simplicity, consider a semi-infinite junction with one inhomogeneity: $x_0=0<x_1<+\infty$, $h_L\equiv h_\infty=0$, $\phi_L\equiv\phi_\infty=2\pi$.

The parameters k_0, ϕ_0, ϕ_1 defining a bound state $\phi(x)$ can be derived from the equations

$$-\mu_1 c_1 (1-c_1^2)(2+\mu_1 c_1) = 1-k_0^2 \;, \quad c_i \equiv \cos(\phi_i/2), \tag{3}$$

$$x_1 = \frac{1}{2}\int_{\phi_0}^{\phi_1} d\phi \left[k_0^2 - \cos^2(\phi/2)\right]^{-1/2} \;, \quad k_0^2 - c_0^2 = h_0^2/4 \;, \tag{4}$$

which follow from the first integral (see (4.1))

$$\phi'^2/4 = k_0^2 - \cos^2(\phi/2) \;,$$

and from the expression for the jump of the magnetic field ϕ' at $x = x_1$: $\Delta\phi' = -\mu_1 \sin\phi_1$ (see /1/; Eq. (4) coincides with Eq. (6.1)). By drawing in the plane $p = (h_0, x_1)$ the curves corresponding to constant values of k_0, one can easily find the domains where different solutions exist. The envelopes of these curves, which are the boundaries of the domains, give us all bifurcation points. The bound states (for small values of x_1) are represented on the (ϕ, ϕ') diagram (Fig.2), and the dependence of $\mathcal{Y}$ on h_0 for corresponding solutions $\phi(x)$ is schematically plotted in Fig.1. For $h < h_{c_1}$ there exists only one state , $E_1 D_1 D_1' O'$. Its total flux is $\leq \Phi_0/2$ and its generalized energy, $\mathcal{Y}(h_0)$, is represented by the curve OCB_2 in Fig.1. For $h \geq h_{c_1}$ two new solutions arise . The curve $B_1 C B_3$ gives the energy of the states $E_3 D_3 D_3' O'$, and the curve $B_1 B_2$ that of the states $E_2 D_2 D_2' O'$. The energy , $\mathcal{Y} = 8$, of a free fluxon, which is at rest in an infinite uniform function, as well as the critical value of the external field, $h_0 = 2$, for which the fluxons begin to enter a semi-infinite uniform junction (see Ref.2) are shown in Fig.1 . For $\mu_1 > 0$ we always have $h_{c_2} \leq 2$. The dependence of h_{c_2} on x_1 has a sharp minimum for $x_1 \sim 1$, and this minimum is always higher than the thermodynamical critical field $4/\pi$ (see Ref.2). The

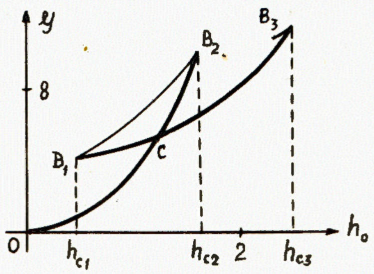

Fig.1

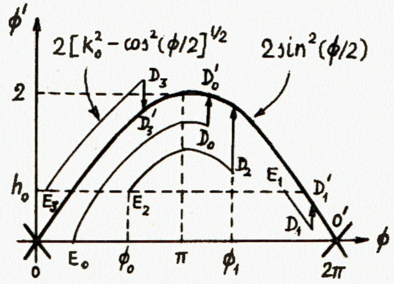

Fig.2

phenomenon of reduction of the critical field, for which the fluxons begin to penetrate the junction, has been discussed in Ref.3 for junctions with sharp edges. Here we observe a new phenomenon related to the creation of fluxon bound states on attractive microinhomogeneities: the formation of two new bifurcation points , B_1 and B_3 , and a resonance character of the dependence of h_{c_1}, h_{c_3} on x_1 . Finally, remark that h_{c_1} is diminising with growing x_1 , and for some value of x_1 ($x_1 = x_{min}(\mu_1)$) we have $h_{c_1} = 0$. For $x_1 \leq x_{min}(\mu_1)$ we therefore have a stable bound state in the zero external field (the curve $E_0 D_0 D_0' O'$ in Fig.2).

The analysis of local stability can be greatly simplified by using the following piecewise-linear (or "saw-like") approximation for the Josephson current

$$\sin\phi \simeq \lambda^2 (-1)^N (\phi - \pi N) \;, \quad \lambda = \frac{2\sqrt{2}}{\pi} \simeq 0.9 \tag{5}$$

on the intervals $I_N : |\phi - \pi N| \leq \pi/2$. In this approximation the energy of the Josephson currents is continuously-differentiable, quadratic function of ϕ and the potential in Eq. (2), $\cos\phi$, is approximated on I_N by the simple step-function: $\cos\phi \simeq \lambda^2(-1)^N$. For solving the problem, Eq. (2), now it is sufficient to find the points $\bar{x}_N$, for which $\phi(\bar{x}_N) = (2N+1)\pi/2$. By using this approximation (Eq. (5)) we can also greatly simplify the process of deriving the bound states; the equations for the parameters ϕ_i, k_i and $\bar{x}_N$ are now elementary though rather complicated. The error induced by the approximation does not exceed several per cent for quantities of physical interest.

The solution of the problem, Eq.(2), in this approximation confirms an intuitive guess that the states corresponding to the curves OCB_2 and $B_1 C B_3$ (Fig.1) are locally stable while those corresponding to the curve $B_1 B_2$ are unstable. The form of the curve $OCB_2 B_1 C B_3$ suggests an interesting analogy with the dependence of the Gibbs free energy of the Van der Waals gas on the pressure at $T < T_c$ (see Ref.4). Remembering that our $\mathcal{Y}$ is an analogue of the Gibbs free energy a suspiction may arise that the states corresponding to the curves $B_1 C$ and CB_2 are in fact metastable (i.e. unstable with respect to some large fluctuations). However, in transitions from the level $B_1 C$ to the level OC (or, from CB_2 to CB_3) the total flux of the state must be changed by an amount $\sim \Phi_0/2$; mechanisms of such transitions are yet poorly understood, a discussion will be given in another paper. Here we only mention that to change one stable state into another it is apparently sufficient to cross the critical values of the applied field (h_{c1} for decreasing h_0 and h_{c2} for increasing h_0). Details of such transitions are now under investigation.

The bound states and their bifurcations in the case of a junction of finite length L and with several inhomogeneites can be derived by similar methods, though the calculations tend to become formidably complicated, even with the approximation (5). An interesting application of this approximation is possible for the two-dimensional Josephson junction with a one-dimensional inhomogeneity. For axially symmetric junctions we have succeeded in obtaining some stable static bound states localized near attractive inhomogeneities.

The phenomena described above can occur in other systems, for example, in magnetics where the Bloch walls play the role of the fluxons. However, the Josephson junctions seem to be most convenient for the experimental studies of these phenomena due to a wide choice of available observational means in conjunction with a feasibility of changing the parameters (h_0, h_L, x_i, μ_i).

A simplest direct way of observing the bound states and their bifurcations is provided by a technique of scanning a focused weak laser beam. This allows one to measure local current distributions along the junction. (See, e.g. Ref.5). By using an additional, stronger laser beam an attractive inhomogeneity with controllable position can be created and thus bifurcations, depending on the magnetic field and on the position of the inhomogeneity, are in principle observable.

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Бифуркации связанных состояний флюксонов в неоднородных длинных джозефсоновских переходах

Изучены бифуркации во внешнем магнитном поле связанных состояний флюксонов в длинных джозефсоновских контактах с локальными притягивающими неоднородностями. Предложен простой критерий локальной устойчивости этих состояний. Отмечена возможность прямого экспериментального наблюдения связанных состояний и их бифуркаций при изменении внешнего поля и положения неоднородности.

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Bifurcations of Fluxon Bound States in Nonuniform Long Josephson Junctions

Bifurcations of fluxon bound states in long Josephson junctions with local attractive inhomogeneities are analyzed. A simple criterion for local stability of these bound states is proposed. A possibility of a direct experimental observation of the bound states and of their bifurcations depending on an external magnetic field and/or the position of inhomogeneity is pointed out.

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