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CALCULATION OF THE BILLIARDS h-ENTROPY IN THE CLOSED PLANE REGION WITH THE SCATTERING BOUNDARY

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1. A region with the scattering boundary is the region with the piecewise smooth boundary which is at all points of smoothness either plane or convex into the region (see fig.1). For the billiards inside this region there is a formula for the entropy calculation [1]

$$h = \int_M B(x)\mu(dx)\,, \tag{1}$$

where h is the dynamic billiards entropy, M is the phase space, $x \in M$ are the points of the phase space, μ is the invariant measure (for the billiards)

$$\mu(dx) = \frac{1}{2\pi S} dx_1 dx_2 d\omega\,. \tag{2}$$

In (2) S is the region area, x_1 and x_2 are the linear coordinates on the plane, and ω is the velocity angular coordinate,

$$B(x) = \cfrac{1}{\tau_1 + \cfrac{1}{\cfrac{2k_1}{\cos\phi_1} + \cfrac{1}{\tau_2 + \cfrac{1}{\cfrac{2k_2}{\cos\phi_2} + \dots}}}} \tag{3}$$

is the continued fraction, τ_i are the time intervals between the subsequent reflections of the billiards trajectory originating at the point $x \in M$, ϕ_i are the angles between the vectors of velocity and the vector of normal to the region boundary at the i-th reflection (see fig.2), and k_i is the boundary curvature at the i-th reflection.

Below we shall develop a method of calculation of the entropy by this formula for a certain class of regions.

2. We consider on the plane a system of discs of the same r radius with the centers at the sites of square lattice, the coordinates of which are given by the pair of integers (m, n) (see fig.3). In this lattice one can cut out some figures, which

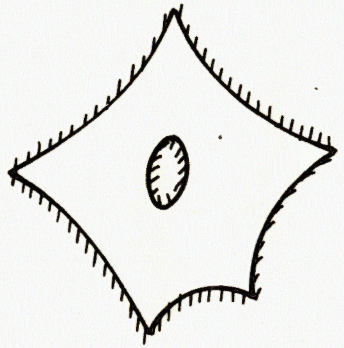

Fig. 1

under the multiple reflection from their boundaries will cover the whole lattice. These figures may be the following: a) square, b) rectangle, c) triangle (see fig.4). The billiards inside these figures (with the cut out discs on the boundaries) are obviously reduced to the motion of a free particle inside an unbounded lattice (fig.3) with an elastic reflection from the boundaries of the discs with radius r.

These billiards can be calculated by formula (1), where M is the phase space of the initial conditions inside the square fig.4a), all other systems of the billiards type (fig.4b,c) have just the same entropy.

We shall calculate this entropy assuming that r is sufficiently small.

It should be noted that in our case the quantities τ_i B(x)(3) can be considered to the time intervals between two subsequent reflections from the disc boundaries in the lattice, and $k_i = \frac{1}{r}$ since the curvature of circles is always constant.

Consequently

$$h = \frac{1}{2\pi S}\int_M \frac{1}{\tau_1 + \cfrac{1}{\cfrac{2}{r\cos\phi_1} + \cfrac{1}{\tau_2 + \dots}}}\, dx_1 dx_2\, d\omega . \qquad (4)$$

We approximate the fraction B(x) by an appropriate one $B_2(x)$, given by the formula

$$B_2(x) = \frac{1}{\tau_1 + \cfrac{1}{\cfrac{2}{r\cos\phi_1}}} = \frac{1}{\tau_1 + \cfrac{r\cos\phi_1}{2}} . \qquad (5)$$

It is obvious that $B_2(x) \le B(x)$, and

$$B(x) - B_2(x) = B(x)(B_2^{-1}(x) - B^{-1}(x))B_2(x) =$$

$$= \frac{1}{1 + \tau_1\left(\cfrac{2}{r\cos\phi_1} + \cfrac{1}{\tau_2 + \dots}\right)}\;\frac{1}{1 + \cfrac{2\tau_1}{r\cos\phi_1}}\;\frac{1}{\tau_2 + \cfrac{1}{\cfrac{2}{\cos\phi_2}}} \le \frac{1}{\left(1 + \cfrac{2\tau_1}{r\cos\phi_1}\right)^2} \cdot \qquad (6)$$

Hence

$$-h \ge h_1 = \frac{1}{2\pi S}\int_M \frac{1}{\tau_1 + \cfrac{r\cos\phi_1}{2}}\, dx_1\, dx_2\, d\omega \qquad (7)$$

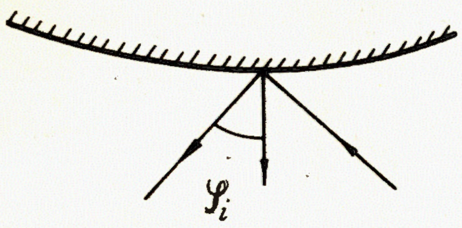

Fig. 2

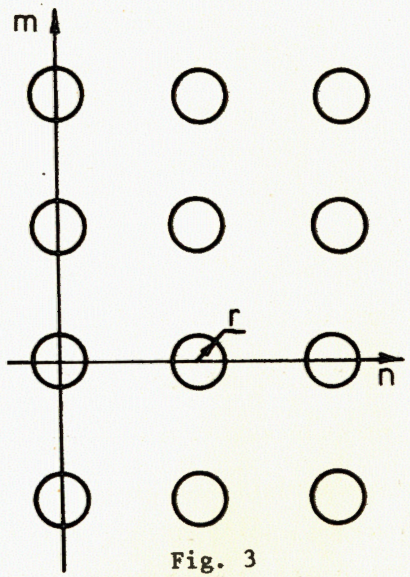

Fig. 3

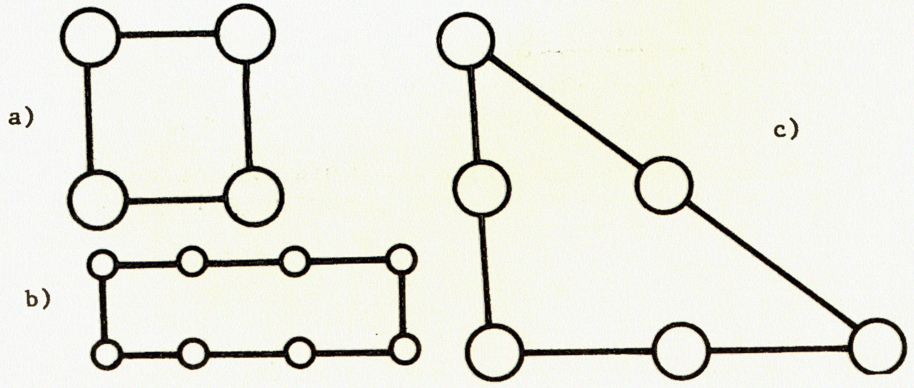

Fig. 4

and the difference between h and h_1 is given by the formula

$$h - h_1 \le \frac{1}{2\pi S} \int_M \frac{1}{\left(1 + \frac{2r_1}{r\cos\phi_1}\right)^2}\, dx_1\, dx_2\, d\omega. \qquad (8)$$

It is known /1/ that the invariant measure can be written in the polar coordinates: $dx_1 dx_2\, d\omega = \cos\phi_1 dr_1 d\phi_1\, d\rho$, where ρ is the linear coordinate on the circle at the point of the first reflection of the trajectory of the billiards.
Therefore

$$h - h_1 \le \frac{1}{2\pi S} \int_M \frac{\cos\phi\, dr\, d\phi d\rho}{\left(1 + \frac{2r}{r\cos\phi}\right)^2} = \frac{1}{2\pi S} \int_\partial \frac{r\cos^2\phi}{2}\left(1 - \frac{1}{1 + \frac{2r}{r\cos\phi}}\right) d\phi\, d\rho \le$$

$$\le \frac{1}{2\pi S} \int_\partial \frac{r\cos^2\phi}{2} d\phi d\rho = \frac{r}{2\pi S} \cdot 2\rho r \cdot \frac{\pi}{4} = \frac{\pi r^2}{4(1 - \pi r^2)}. \qquad (9)$$

Here ∂_0 is the circle with radius r, $\partial = \partial_0 \times [-\frac{\pi}{2}, \frac{\pi}{2}]$ in the set of unit vectors originating on the circle and directed to the outer part of the circle, and t is the time of "flight" of the ray between two circles.

The quantity $\Delta = \frac{\pi r^2}{4(1 - \pi r^2)}$ is rather small at small r. A concrete numerical calculation of the entropy will be performed choosing $r = \frac{1}{13}$ then $\Delta \le 0.0048$.

Therefore, the h-entropy can be calculated with a good accuracy

$$h \approx h_1 = \frac{1}{2\pi S} \int_M \frac{1}{r_1 + \frac{r\cos\phi}{2}}\, dx_1 dx_2\, d\omega = \frac{1}{2\pi S} \int_M \frac{\cos\phi\, dr_1 d\phi d\rho}{r_1 + \frac{r\cos\phi}{2}}. \qquad (10)$$

To achieve a higher accuracy, one should use the n-fold rather than the two-fold fraction. Then, an error will not exceed

$$\frac{\pi}{4} \frac{r^n}{(1 + r)^n} \le \frac{\pi r^n}{4}.$$

3. Here we shall calculate the entropy in the approximation discussed in sec.2 by formula (10). We have

$$h \approx h_1 = \frac{1}{2\pi S} \int_M \frac{\cos\phi\, dr_1 d\phi\, d\rho}{r + \frac{r\cos\phi}{2}} = \frac{1}{2\pi S} \int_\partial \cos\phi \ln\frac{2t + r\cos\phi}{r\cos\phi} d\phi d\rho =$$

$$= \frac{1}{2\pi S} \int_\partial \cos\phi \ln(2t + r\cos\phi)\, d\phi d\rho - \frac{1}{2\pi S} \int_\partial \cos\phi \ln r\, d\phi\, d\rho - \qquad (11)$$

$$- \frac{1}{2\pi S} \int_\partial \cos\phi \ln\cos\phi\, d\phi\, d\rho = A_1 - A_2 - A_3,$$

$$A_1 = \frac{1}{2\pi S} \int_\partial \cos\phi \ln(2t + r\cos\phi)\, d\phi d\rho \le \frac{1}{2\pi S} \int_\partial \cos\phi\left(\ln 2t + \frac{r\cos\phi}{2t}\right) d\phi\, d\rho \le$$

$$\le \frac{1}{2\pi S} \int_\partial \cos\phi \ln 2t\, d\phi\, d\rho + \frac{1}{2\pi S} \int_\sigma \frac{r\cos^2\phi}{2} d\phi d\rho =$$

$$= \frac{1}{2\pi S} \int_\partial \cos\phi \ln t\, d\phi\, d\rho + \frac{2\ln 2 \cdot r}{1 - \pi r^2} + \frac{\pi r}{2(1 - \pi r^2)} = \bar{A}_1,$$

$$A_2 = \frac{1}{2\pi S} \int_\partial \cos\phi \cdot \ln r\, d\rho\, d\phi = \frac{\ln r}{2\pi S} \int_{\partial_0} 2\, d\rho = \frac{\ln r}{\pi S} 2\pi r = -\frac{2r}{1 - \pi r^2} \ln\frac{1}{r}, \qquad (12)$$

$$A_3 = \frac{1}{2\pi S} \int_\partial \cos\phi \ln\cos\phi\, d\phi\, d\rho = \frac{2(\ln 2 - 1)r}{1 - \pi r^2}.$$

The error, which we have committed changing A_1 by $\bar{A}_1$ will be the same as under the change of h by h_1 $\Delta = \frac{\pi r^2}{4(1 - \pi r^2)}$. Moreover, in (11) we have diminished the true value of the entropy ($h > h_1$), under the change of A_1 by $\bar{A}_1$ we have increased, it, i.e., these errors compensate each other to a certain extent and the true error is expected to be small.

Thus, using (12) we have improved approximation (11). Collecting $\bar{A}_1$, A_2, and A_3 we have

$$h \approx \bar{A}_1 - A_2 - A_3 = \frac{1}{2\pi S} \int_\partial \cos\phi \ln t\, d\phi\, d\beta + \frac{2r}{1 - \pi r^2} \ln\frac{1}{r} + \qquad (13)$$

$$+ \frac{(4 + \pi) r}{2(1 - \pi r^2)}.$$

4. To obtain an analytic formula for the h-entropy we have to calculate the integral

$$I = \frac{1}{2\pi S} \int_{\partial} \cos\phi \, \ln t \, d\phi \, d\rho .$$

Let us estimate the value of this integral from above and below. We have

$$I = \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} \cos\phi \, d\phi \int_{\partial_0} \ln t(\rho, \phi) \, d\rho . \qquad (14)$$

At each fixed ϕ the integral over ∂_0 can be expanded into the sum

$$\int_{\partial_0} \ln t(\rho, \phi) \, d\rho = \sum_i \int_{\partial_0^{(i)}} \ln t \, d\rho , \qquad (15)$$

where the circle is divided into the segments, ∂_0 -ray under the angle ϕ to the normal falling onto only one circle of the lattice (see fig.5). The quantity $\ln t$ changes inside each circular arc $\partial_0^{(i)}$ slightly, and it can be assumed that $t = \ell_i$, where ℓ_i is the distance from the center of the initial circle to the center of the i-th circle. An error will not be more than r^2, and the formula (15) will acquire the form

$$\int_{\partial_0} \ln t \, d\rho \approx \sum_i r_i \ln r_i , \qquad (16)$$

where r_i is the $\partial_0^{(i)}$ segment length.

If the i-th circle is "seen" from the initial circle wholly, i.e., any its part is not shaded by another circle, then geometrically

$$r_i = \frac{2r^2}{\ell_i}$$

at any value of ϕ.

Having applied the methods of analytic geometry, one can prove that the i-th circle is wholly seen from the initial one if and only is $\ell_i \le \frac{1}{2r}$ and the coordinates of the center are simple.

Therefore, the sum in (16) can be divided into two sums

$$\int_{\partial_0} \ln t \, d\rho \approx \sum_{i:\ell_i < 1/2r} \frac{2r^2}{\ell_i} \ln \ell_i + \sum_{i:\ell_i > 1/2r} r_i \ln \ell_i = S_1 + S_2 . \qquad (17)$$

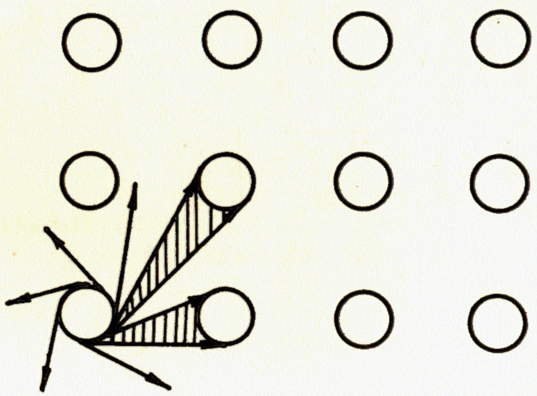

Fig. 5

Under a concrete choice of r the sum S can be calculated exactly. For instance, at $r = \frac{1}{13}$ the region $\ell_i \le \frac{1}{2r} = 6.5$ includes 88 discs with the center coordinates (±1,0), (0,±1), (±1,±1), (±2,±1), (±1,±2), (±3,±1), (±1, ±3), (±4.±1), (±1, ±4), (±5, ±1), (±4,±5), (±1., ±6), (±3,±2), (±2,±3), (±4,±3), (±3,±4), (±5,±3), (±3,±5), (±5,±4), (±4, ±5).

If the center coordinates are (m, n), then $\ell_i = \sqrt{m^2 + n^2}$. Therefore,

$$S = \frac{8}{169}\left(\frac{\ln 2}{2\sqrt{2}} + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 10}{\sqrt{10}} + \frac{\ln 17}{\sqrt{17}} + \frac{\ln 26}{\sqrt{26}} + \frac{\ln 13}{\sqrt{13}} + \frac{\ln 25}{\sqrt{25}} + \frac{\ln 34}{\sqrt{34}} + \frac{\ln 37}{\sqrt{37}} + \frac{\ln 41}{\sqrt{41}}\right). \qquad (18)$$

Hence

$$I = \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi \int_{\partial_0} \ln t \, d\rho = \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi \cdot S_1 + \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi = \frac{1}{\pi S} S_1 + \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi \cdot S_2 . \qquad (19)$$

The second integral in (19)

$$J = \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi \cdot S_2 .$$

Let us estimate it. In our case

$$S_2 = \sum_{i:\ell_i > 1/2r} r_i \ln \ell_i . \qquad (20)$$

a) Estimation from below. We divide the whole planes with $\ell > \frac{1}{2r}$ (i.e., $\sqrt{x^2+y^2} > \frac{1}{2r}$) into the concentric rings of width 1:

$$\frac{1}{2r} < \sqrt{x^2+y^2} \le \frac{1}{2r} + 1; \quad \frac{1}{2r} + 1 < \sqrt{x^2+y^2} \le \frac{1}{2r} + 2, \ldots$$

and so on.
Then

$$S_2 \ge \sum_{k=0}^{\infty} \ln\left(\frac{1}{2r} + k\right) x_k, \qquad (21)$$

where $x_k = \sum_{i:\, 1/2r + k - 1 < \ell_i < 1/2r + k} r_i$. It has been proved in the theory of numbers[2] that the number of pairs of simple numbers (m,n), where $m^2 + n^2 < R^2$, will not exceed $\pi \cdot R^2$. Moreover

$$x_k \le \frac{2r^2}{\frac{1}{2r} + k} A_k .$$

where A_k is the number of "visible" discs in the k-th ring. Therefore, from the probability reasons

$$S_2 \ge \sum_{k=0}^{[Q]} \ln\left(\frac{1}{2r} + k\right) \frac{2r^2}{\frac{1}{2r} + k} 4\left(\frac{1}{2r} + k\right) = \qquad (22)$$

$$= \sum_{k=0}^{[Q]} \ln\left(\frac{1}{2r} + k\right) 8r^2 ,$$

where $Q = \dfrac{2\pi - \sum_{i:\,\ell_i \le 1/2r} \frac{2r^2}{\ell_i}}{8r^2}$. We denote

$$\sum_{i:\,\ell_i < 1/2r^2} \frac{2r^2}{\ell_i} = 2r^2 \sum_{i:\,\ell_i < 1/2r^2} \frac{1}{\ell_i} = 2r\epsilon_0 , \qquad (23)$$

then

$$Q = \frac{\pi - \epsilon_0 r}{4r} = \frac{\pi}{4r} - \frac{\epsilon_0}{4}$$

and

$$S_2 \ge 8r^2 \sum_{k=0}^{[Q]} \ln\left(\frac{1}{2r} + k\right) = 8r \ln\left[\frac{1}{2r}\left(\frac{1}{2r} + 1\right) \ldots \left(\frac{1}{2r} + Q\right)\right] = 0.45. \qquad (24)$$

For each concrete r one can perform these calculations and get an estimate from below for S_2. For instance, for $r = 1/13$

$$\sum_{i:\,\ell_i < 6,5} \frac{1}{\ell_i} = 4 + \frac{4}{\sqrt{2}} + \frac{8}{\sqrt{5}} + \frac{8}{\sqrt{10}} + \frac{8}{\sqrt{13}} + \frac{8}{\sqrt{17}} + \qquad (24a)$$

$$+ \frac{8}{\sqrt{25}} + \frac{8}{\sqrt{26}} + \frac{8}{\sqrt{34}} + \frac{8}{\sqrt{41}} \approx 23 .$$

Therefore

$$\epsilon_0 \approx 1.769, \quad Q = 9.768; \quad [Q] \le 10. \qquad (25)$$

Hence

$$I \ge \frac{S_1}{\sqrt{2\pi S}} + J \ge 0.242.$$

A general estimate (more rough) can be obtained without dividing 1 into S_1 and S_2 by using the same theorem from the theory of numbers[2]

$$I \ge \frac{1}{2\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cos\phi \left(\sum_{k=1}^{Q} 2r^2 \frac{\ln k}{k} 4k \right),$$

$$Q = \left[\frac{2\pi r - 8r^2}{8r^2}\right] = \left[\frac{\pi}{4r} - 1\right],$$

That is,

$$I \ge \frac{4 \cdot 2r^2}{4\pi S} \int_{-\pi/2}^{\pi/2} d\phi \cdot \cos\phi \cdot \ln\left[\frac{\pi}{4r} - 1\right]! \ge \qquad (26)$$

$$\ge \frac{8r^2}{\pi S} \ln\frac{\pi^2}{4r} + \frac{2r}{S} \ln\frac{\pi}{4\ell r} .$$

b) Estimation from above. To estimate from above we use the relation

$$\frac{1}{2\pi S} \int_{\partial} t \cos\phi \, d\phi \, d\rho = 1. \qquad (27)$$

On the other hand, this integral can be divided as in (17) into S_1' and S_2':

$$\int_{\partial_0} t \, d\rho = \sum_{i:\,\ell_i < 1/2r} \frac{2r^2}{\ell_i} + \sum_{i:\,\ell_i > 1/2r} r_i \ell_i = S_1' + S_2' .$$

The quantity S_1' is calculated simply: $S_1' = 2r^2 k$, where k is the number of "visible" discs in the visual range $\frac{1}{2r}$. For $r = \frac{1}{13}$ $S_1' = \frac{2}{169}\cdot 88 \approx 1.0435$, i.e.,

$$1 = \frac{1}{2\pi S}\int_{\partial} t\cos\phi\, d\phi\, d\rho = \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S'\cos\phi\, d\phi + \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S'\cos\phi\, d\phi . \qquad (28)$$

The second integral in (27) equals

$$\frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S_2'\cos\phi\, d\phi = 1 - \frac{176}{169\cdot\pi} \approx 0,66497. \qquad (29)$$

Formula (29) allows one to estimate S_2 in (17) from above

$$\frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S_2\cos\phi\, d\phi = \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} \sum_{1>\ell_i>1/2r} r_i \ln\ell_i \cdot \cos\phi\, d\phi \le$$

$$\le \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} \cos\phi\Big(\sum_{1>\ell_i>1/2r} r_i\big(\ell_i - \frac{1}{2r} + \ln\frac{1}{2r}\big)\Big)\, d\phi = \qquad (30)$$

$$= \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S_2'\cos\phi\, d\phi - \frac{1}{2\pi S}\Big(\frac{1}{2r} - \ln\frac{1}{2r}\Big)\int_{-\pi/2}^{\pi/2} \cos\phi\Big(\sum_{i:\ell_i>1/2r} r_i\Big)d\phi .$$

The first integral in (30) is given by formula (29), and

$$\sum_{i:\ell_i>1/2r} r_i = 2\pi r - \sum_{i:\ell_i<1/2r} r_i$$

can be calculated (see above), and the sum is independent of ϕ, then

$$\frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S_2\cos\phi\, d\phi \le \frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} \cos\phi\, S_2'\, d\phi - \frac{1}{\pi S}\Big(\frac{1}{2r} - \ln\frac{1}{2r}\Big)\sum_{i:\ell_i>1/2r} r_i \cdot \qquad (31)$$

For $r = \frac{1}{13}$ we have

$$\frac{1}{2\pi S}\int_{-\pi/2}^{\pi/2} S_2\cos\phi\, d\phi \le 0,66497. \qquad (32)$$

Using formula (13) and collecting (18), (24), (24a), (28) and (32), we get at $r = \frac{1}{13}$

$$0.924 < h < 1.441. \qquad (33)$$

The obtained estimates are good for $r \sim \frac{1}{10}$, though at very small r they become very rough. One can improve estimates from above for h at small r by using formula (27) in a different way.

The internal integral can be represented, as proviously, by the sum

$$\int_{\partial_0} t\, d\rho = \sum_i \ell_i r_i$$

and the sum can be divided into the parts corresponding to the rings

$$k \le \sqrt{x^2+y^2} \le k+1;\quad \sum_{k=0}^{\infty} k\cdot r_k \le \int_{\partial_0} t\, d\rho \le \sum_{k=0}^{\infty} r_k(k+1),$$

where

$$r_k = \sum_{i:k\le\ell_i<k+1} r_i .$$

As has been mentioned before, $r_k \le 8r^2$. From the probability reasons it follows that

$$\sum_{k=0}^{\infty} r_k \ln k \le \frac{\sum_{k=0}^{p} 8r^2\ln k}{\sum_{k=0}^{p} 8r^2 k}\sum_{k=0}^{\infty} r_k\cdot k,$$

where $p = \big[\frac{2\pi r}{8r^2}\big] = \big[\frac{\pi}{4r}\big]$. This

$$\frac{1}{2\pi S}\int_{\partial}\cos\phi\ \ln t\, d\phi\, d\rho \le \frac{2\ln[\pi/4r]!}{[\pi/4r]^2}\cdot\frac{1}{2\pi S}\int_{\partial}\cos\phi\cdot t\cdot d\phi\cdot d\rho \le \qquad (34)$$

$$\le \frac{16r^2}{\pi^2}\ln\pi^2 + \frac{8r}{\pi}\ln\frac{\pi}{4r}.$$

From these estimates there also follow the asymptotic estimates for the entropy as $r \to 0$:

$$C_1 < \frac{h}{r \ln \frac{1}{r}} < C_2 ,$$

where $C_1 = 4$, $C_2 = 2 + \frac{8}{\pi} \approx 4{,}547$. Note, that using the theory of numbers one gets

$$p \approx \left[\frac{2\pi r}{\frac{24}{\pi} r^2}\right] = \left[\frac{\pi^2}{12 r}\right] \qquad \text{and} \qquad C_2 = 2 + \frac{24}{\pi} \approx 4.52.$$

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Чернов Н.И., Федянин В.К., Шведовский В.А. E17-83-236

Вычисление h-энтропии бильярда в замкнутой плоской области с рассеивающей границей

Развит метод вычисления h-энтропии областью, которая во всех точках либо главная, либо плоская, либо выпуклая внутрь.

Асимптотическая точка для энтропии при $r \to 0 - 4 < \frac{h}{r \ln \frac{1}{r}} < 4{,}547$,

где r - радиус дисков, покрывающих область.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Calculation of the Billiards h-Entropy in the Closed Plane Region with the Scattering Boundary

A method is developed for calculating the h-enrtopy of a region which at all points is either smooth or flat or convex inside. The asymptotic estimation of the h-entropy as $r \to 0$ is $4 < \frac{h}{r \ln \frac{1}{r}} < 4.547$, with r the radius of disks covering the region.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983